MATH 2413, CALCULUS I
Review for test 2, Spring 2007

Dates of test	**Wednesday, March 7 in class**
Material covered	Up to 3.2
Allowable materials	Calculator (no TI-89, 92 or equivalent)

Sample problems

1. Find slope-intercept equation of the tangent line to the curve $f(x) = x^2 - x$ at $a = -1$

2. Find the slope-intercept equation of the normal line to the curve $p(t) = e^t$ at $t = -1$. Do not use decimal approximations.

3. Sketch the graph of a function that satisfies the conditions: $\lim_{x \to -2^+} f(x) = \infty$, $f(0) = 3$, $f'(0) = 0$, $f'(1) = -1$, $\lim_{x \to \infty} f(x) = 1$

4. Evaluate the limits if they exist. If they are infinite, indicate whether the limit is $+\infty$ or $-\infty$

   a. $\lim_{x \to \infty} \frac{2x^2 - x}{3x^2 + 1}$

   b. $\lim_{x \to -\infty} \frac{2x - 1}{x^2 + 3}$

   c. $\lim_{x \to -\infty} \frac{x(x-1)}{x+3}$

   d. $\lim_{x \to 1} \frac{x - 1}{|x - 1|}$

   e. $\lim_{x \to \infty} \frac{4x^3 - x}{3 - 2x^3}$

   f. $\lim_{x \to 4^+} \frac{x}{4 - x}$

   g. $\lim_{x \to 4^-} \frac{x}{4 - x}$

   h. $\lim_{x \to \infty} e^{-x^2}$

   i. $\lim_{x \to \infty} \sin x$

5. Evaluate the limit and prove your result using the squeeze theorem. $\lim_{x \to \infty} \frac{x}{3x + \sin x}$

6. A model rocket is fired straight up from the ground, and its height in meters t seconds after it is fired is described by the curve $h(t) = -4.9t^2 + 90t$.

   a. What is the average velocity from the start until it reaches its highest point?

   b. What is the instantaneous velocity 3 seconds after it is fired?

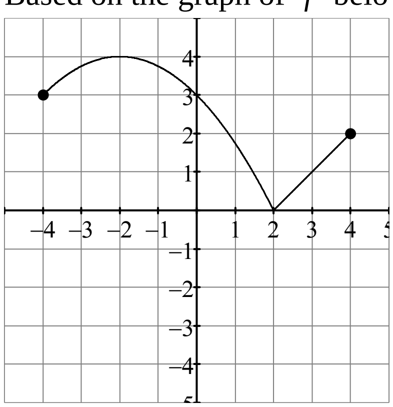
7. The table below depicts the minimum wage each year that an increase took effect since 1950.

Year	Minimum Wage Adjusted for Inflation
1950	\$4.96
1956	\$5.88
1963	\$6.54
1968	\$7.36
1974	\$6.49
1979	\$6.39
1981	\$5.90
1997	\$5.15

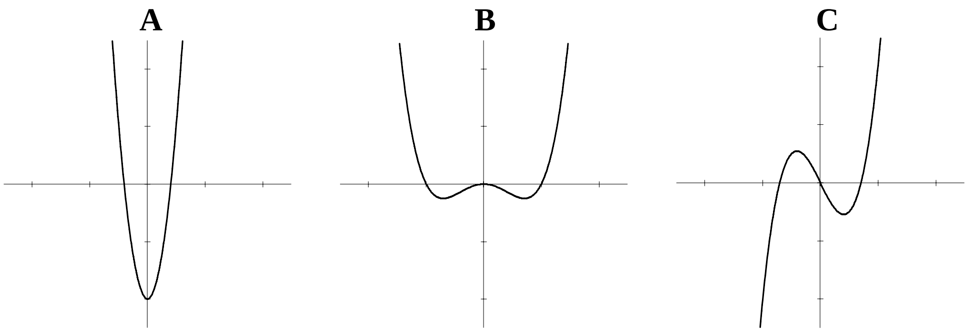
   a. What was the average rate of decline of the Minimum Wage Adjusted for Inflation from 1968 to 1981?

   b. After 1968 the Minimum Wage Adjusted for Inflation started to decline. Between what years was the average rate of decline the greatest? What was the average rate of decline between those years?

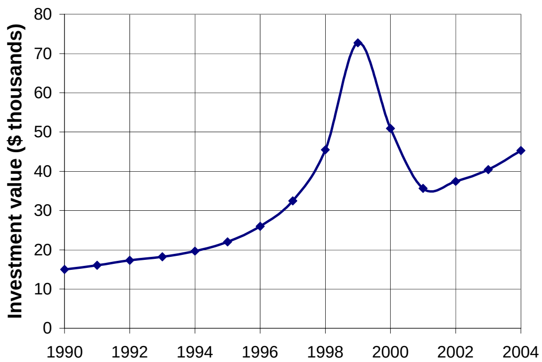
8. Based on the graph of f below, sketch a graph of f'.



9. Three graphs are drawn below. Determine which is f, f', and f''.



10. The value of an investment is described by the graph below:



a. In what year was the growth rate the highest? In what year was the rate of decline the highest?

b. Which is greater, the average growth from 1996 to 1998, or the instantaneous growth rate in 1998?

c. Draw a graph that represents the growth rate as a function of the year (consider a decline to be negative growth).

11. Give an example of a function that

a. is continuous but not differentiable at $x = 0$

b. has a jump discontinuity at $x = 1$

c. has an infinite discontinuity at $x = -2$

d. has a hole at $x = 2$

12. Use the intermediate value theorem to prove that there is a solution to the equation $\cos x = 2x$ between $x = 0$ and $x = \frac{\pi}{2}$.

13. Use the definition of the derivative to evaluate the following (you must use this method and show all of your work.)

    a. $\frac{d}{dx}\left[x^2+5\right]$

    b. $\frac{dy}{dx}$ if $y=\frac{1}{x-1}$

    c. $f'(x)$ if $f(x)=\sqrt{2x}$

14. Find the following derivatives:

    a. $\frac{d}{dx}\left[x^2-\frac{2}{x}\right]$

    b. $f'(x)$ if $f(x)=\frac{e^x+1}{\sqrt{x}}$

    c. $\frac{d}{dx}\left[x^2e^x\right]$

    d. $\left.\frac{df}{dx}\right]_{x=2}$ if $f(x)=\frac{x+1}{x-1}$

    e. $s'(t)$ and $s''(t)$ if $s(t)=\frac{a}{2}t^2+v_0t+s_0$, where a, v_0, and s_0 are constants

15. Motion of a particle is described by the function $s=f(t)=t^3-15t^2+72t$, $0\le t\le 10$, where *s* is the position in meters and *t* is time in seconds.
    a. Find an expression for the velocity after *t* seconds
    b. What is the velocity after 5 seconds?
    c. At what times is the particle
        i. At rest?
        ii. Moving forwards?
        iii. Moving backwards?
        iv. Accelerating?
        v. Decelerating?

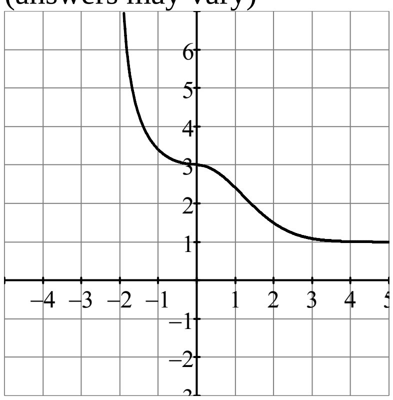
16. For the function $f(x)=x^3-4x^2+4x-7$, at what values of x does the tangent line have slope -1? Show your work solving symbolically.

ANSWERS:

1. $y = -3x - 1$

2. $y = -et - e + \frac{1}{e}$

3. (answers may vary)



4.
   a. $\frac{2}{3}$
   b. 0
   c. $-\infty$
   d. does not exist (left lim $\neq$ right lim)
   e. -2
   f. $-\infty$
   g. $+\infty$
   h. 0
   i. does not exist (oscillation)

5. $-1 \le \sin x \le 1$. Therefore:

$$\frac{x}{3x+1} \le \frac{x}{3x+\sin x} \le \frac{x}{3x-1}.$$

By the squeeze theorem:

$$\lim_{x\to\infty} \frac{x}{3x+1} \le \lim_{x\to\infty} \frac{x}{3x+\sin x} \le \lim_{x\to\infty} \frac{x}{3x-1}$$

The outer limits:

$$\lim_{x\to\infty} \frac{x}{3x+1} = \lim_{x\to\infty} \frac{x}{3x+1} \frac{\div x}{\div x} = \lim_{x\to\infty} \frac{1}{3+1/x} = \frac{1}{3}$$

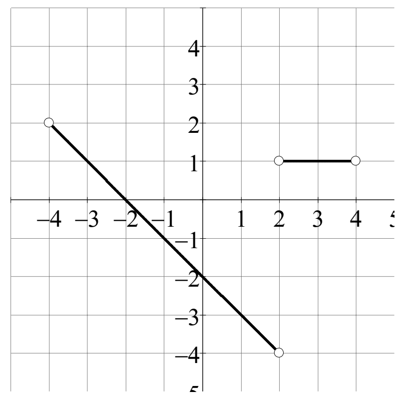
$$\lim_{x\to\infty} \frac{x}{3x-1} = \lim_{x\to\infty} \frac{x}{3x-1} \frac{\div x}{\div x} = \lim_{x\to\infty} \frac{1}{3-1/x} = \frac{1}{3}$$

Therefore $\lim_{x\to\infty} \frac{x}{3x+\sin x} = \frac{1}{3}$

6.
   a. 45 m/s
   b. 60.6 m/s

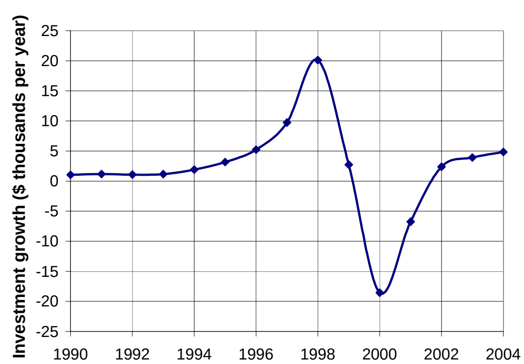
7.
   a. \$0.11 per year
   b. \$0.245 per year from 1979 to 1981

8.



9.
$f \to B$
$f' \to C$
$f'' \to A$

10.
   a. 1998
   b. instantaneous growth rate in 1998
   c.



11. (answers may vary)
   a. $f(x) = |x|$
   b. $f(x) = \frac{|x-1|}{x-1}$
   c. $f(x) = \frac{1}{x+2}$

d. $f(x) = \dfrac{x(x-2)}{x-2}$

12. Let $f(x) = \cos x - 2x$.
f is continuous over its domain.
$f(0) = 1 - 0 = 1$
$f\left(\dfrac{\pi}{2}\right) = 0 - \pi = -\pi$
Since $f\left(\dfrac{\pi}{2}\right) \le 0 \le f(0)$, the intermediate value theorem says there is a value c between $x = 0$ and $x = \dfrac{\pi}{2}$ such that $f(c) = 0$.
$\cos c - 2c = 0 \;\Rightarrow\; \cos c = 2c$, and therefore $x = c$ is a solution to the equation.

13.

a.
$$\lim_{h\to 0} \frac{(x+h)^2 + 5 - (x^2+5)}{h}$$
$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 5 - x^2 - 5}{h}$$
$$= \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0} \frac{h(2x+h)}{h}$$
$$\lim_{h\to 0} (2x+h) = \boxed{2x}$$

b.
$$\lim_{h\to 0} \frac{\dfrac{1}{x+h-1} - \dfrac{1}{x-1}}{h}$$
$$= \lim_{h\to 0} \frac{\dfrac{x-1-(x+h-1)}{(x+h-1)(x-1)}}{h}$$
$$= \lim_{h\to 0} \frac{\dfrac{x-1-x-h+1}{(x+h-1)(x-1)}}{h} \cdot \frac{\dfrac{1}{h}}{\dfrac{1}{h}}$$
$$= \lim_{h\to 0} \frac{-h}{(x+h-1)(x-1)h}$$
$$= \lim_{h\to 0} \frac{-1}{(x+h-1)(x-1)} = \boxed{-\frac{1}{(x-1)^2}}$$

c.
$$\lim_{h\to 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{2x+2h} - \sqrt{2x}}{h} \cdot \frac{\sqrt{2x+2h} + \sqrt{2x}}{\sqrt{2x+2h} + \sqrt{2x}}$$
$$= \lim_{h\to 0} \frac{2x+2h-2x}{h\left(\sqrt{2x+2h} + \sqrt{2x}\right)}$$
$$= \lim_{h\to 0} \frac{2h}{h\left(\sqrt{2x+2h} + \sqrt{2x}\right)}$$
$$= \lim_{h\to 0} \frac{2}{\sqrt{2x+2h} + \sqrt{2x}}$$
$$\frac{2}{\sqrt{2x} + \sqrt{2x}} = \frac{2}{2\sqrt{2x}} = \boxed{\frac{1}{\sqrt{2x}}}$$

14.

a. $2x + \dfrac{2}{x^2}$

b. $\dfrac{e^x}{\sqrt{x}} - \dfrac{e^x + 1}{2x\sqrt{x}}$ or $\dfrac{2xe^x - e^x - 1}{2x\sqrt{x}}$

c. $2xe^x + x^2e^x$

d. $-\dfrac{2}{(x-1)^2}\Bigg]_{x=2} = -2$

e. $s'(t) = at + v_0$
$s''(t) = a$

15.

a. $v(t) = 3t^2 - 30t + 72$

b. –3 m/s

c.
- i. at $t = 4$ and $t = 6$ seconds
- ii. from 0 to 4 seconds, and from 6 to 10 seconds
- iii. from 4 to 6 seconds
- iv. from 5 to 10 seconds
- v. from 0 to 5 seconds

16. $x = \dfrac{5}{3}$, $x = 1$