Date of test  
Wednesday, April 25 to Tuesday, May 1

Material covered  
Up to 4.6

Allowable materials  
Calculator (no TI-89, 92 or equivalent)

Sample problems

1. The graph below represents $f'$ over the interval $[0, 5]$.

   ![Graph of f']

   a. At what $x$-value does $f$ have a local maximum, if any?
   b. At what $x$-value does $f$ have a local minimum, if any?
   c. Over what interval(s) is $f'$ concave down?

2. The graph of $f''$ is drawn below.

   ![Graph of f'']

   a. At what $x$-values does $f$ have inflection points, if any?
   b. If $f'$ has the following critical values, determine whether $f$ has a local minimum or maximum at each.
      i. $x = -2$
      ii. $x = 4$
3. The graph of $f'$ is drawn below. Assume the domain of $f'$ is $(-\infty, \infty)$ and that the graph of $f''$ does not cross the $x$-axis outside of this window. Over what intervals is $f^\prime$:

- a. increasing?
- b. decreasing?
- c. concave up?
- d. concave down?

4. Find the point on the parabola $2x^2 - y = 3$ that is closest to the origin.
   (hint: minimize $d^2$ from the distance formula)

5. Identify the exact intervals (not decimal approximations) over which the function $G(x) = x^3 - 6x$ is
   - a. Increasing
   - b. Decreasing
   - c. Concave up
   - d. Concave down

6. For the function $f(x) = x \sin x$, $0 \leq x \leq 2\pi$, over what interval is $f$ concave down? Use a graphing calculator to solve the equation you obtain using calculus. Express the interval using decimal approximations to four places.

7. A 13 foot ladder leaning against a wall is sliding away from the wall at a rate of 0.6 feet per second. At what rate is the top of the ladder sliding down along the wall when the base is 5 feet away from the wall?
8. A cone-shaped coffee filter releases coffee at a constant rate of 500 cubic centimeters per minute. The radius and the height of the cone are equal. At what rate is the coffee level inside the cone changing when the level is 5 centimeters? Give an exact value.

9. A cylindric barrel is to be constructed to hold 6 cubic feet of ale. The material for the sides costs $2.00 per square foot, and the material for the ends costs $3.00 per square foot. Find the radius and the total cost of the cheapest such barrel.

10. Find the absolute maximum and minimum of the function \( f(x) = x^3 - 9x + 1 \) over the interval \([0, 2] \)

11. For the function \( f(x) = \sin x \cos x \), over the interval \([0, \pi] \): (give exact values)

   a. At what x-values does \( f \) have a local maximum?

   b. At what x-values does \( f \) have a local minimum?

   c. What are the inflection points of \( f \) ?

12. Evaluate the following limits symbolically, using L’Hospital’s Rule if it applies. Show your steps.

   a. \( \lim_{x \to \infty} \frac{e^{2x}}{x} \)

   b. \( \lim_{x \to -\infty} \frac{e^{2x}}{x} \)

   c. \( \lim_{x \to \infty} \left(x^{1/x}\right) \)

   d. \( \lim_{x \to 0^+} x^2 \ln x \)

   e. \( \lim_{x \to 0^+} \left(\frac{1}{x} + \ln x\right) \)

   f. \( \lim_{x \to 0} (\csc x - \cot x) \)

   g. \( \lim_{x \to 0} (1 + 2x)^{1/x} \)

   h. \( \lim_{x \to 0^+} \frac{\ln x}{x} \)

13. Find the relative extrema, if any, of the function \( f(x) = x^2 + \frac{16}{x} \)

14. Find the area of the largest possible rectangle that has its base on the \( x \)-axis, and its other two vertices lying on the curve \( y = e^{-x^2} \).
15. For the function whose graph is drawn below, identify the x-values in the open interval $(-4, 7)$ where the function:

- a. has a critical value
- b. has a relative minimum
- c. has a relative maximum
- d. attains its absolute maximum, if any
- e. attains its absolute minimum, if any

16. Find all of the following for the function $f(x) = \frac{3x^2 + 2x + 1}{x^2}$

- a. Local extrema, if any
- b. Horizontal asymptote, if any
- c. Vertical asymptotes, if any
- d. Inflection points, if any
- e. Draw a graph of $f$ that captures all of the above aspects of the function. (You may use a graphing calculator).
ANSWERS:

1.  
   a.  \( x = 4 \)  
   b.  none  
   c.  \((1,5)\)

2.  
   a.  \( x = -3 \) and \( x = 3 \)  
   b.  
      i.  max at \( x = -2 \)  
      ii. min at \( x = 4 \)

3.  
   a.  \((-\infty,-2)\) and \((2,\infty)\)  
   b.  \((-2,2)\)  
   c.  \((-\infty,-3)\) and \((0,3)\)  
   d.  \((-3,0)\) and \((3,\infty)\)

4.  
   \(\left(-\frac{\sqrt{11}}{8} - \frac{1}{4}\right)\) and \(\left(\frac{\sqrt{11}}{8} - \frac{1}{4}\right)\)

5.  
   a.  \((-\infty,-\sqrt{2}),(\sqrt{2},\infty)\)  
   b.  \((-\sqrt{2},\sqrt{2})\)  
   c.  \((0,\infty)\)  
   d.  \((-\infty,0)\)

6.  \((1.0769, 3.6436)\)

7.  \(0.25 \text{ feet per second}\)

8.  \(\frac{20}{\pi} \text{ cm/min}\)

9.  \(r = \frac{3^2}{\pi}\) feet  
    Minimum cost = \$41.85

10. absolute max = 1 at \( x = 0 \)  
    absolute min = -9.392 at \( x = \sqrt{3} \)

11.  
   a.  \(\frac{\pi}{4}\)  
   b.  \(\frac{3\pi}{4}\)  
   c.  \((\frac{\pi}{2},0)\)

12.  
   a.  \(+\infty\)  
   b.  0 (L’Hospital does not apply)  
   c.  1  
   d.  0  
   e.  \(+\infty\)  
   f.  0
g. $e^2$

h. $-\infty$ (L’Hospital does not apply)

13. relative min of 12 at $x=2$

14. $\sqrt{\frac{2}{e}}$

15.
   a. critical values at $x = -2, 0, 0.8, 2.2, 5$
   b. relative min at $x = 0, 2.2$
   c. relative max at $x = -2, 0.8$
   d. no absolute max
   e. absolute min at $x = 2.2$

16.
   a. local min: $(-1, 2)$
   b. horizontal asymptote: $y = 3$
   c. vertical asymptote at: $x = 0$
   d. inflection point: $\left(-\frac{3}{2}, \frac{19}{9}\right)$

E. viewing window: $[-6, 6] \times [0, 6]$