Review for Intermediate Algebra (MATD 0390) Final Exam
November 2005

Students are expected to know all relevant formulas, including:

- All special factoring formulas
- All formulas for linear equations (slope formula, slope-intercept and point-slope forms)
- Quadratic formula
- Distance between two points
- Midpoint between two points
- Equation of a circle
- Both forms of equations of parabolas
- All formulas needed to solve applications in this review, such as distance, and direct and inverse variation

In order to be prepared for the final exam, students should be able to do all of the following problems and related problems as well.

Practice Problems

For Problems 1 - 3, simplify and express the answer using positive exponents only:

1. \((7x^3y^{-1})(5x^{-1}y^{-1})\)
2. \((4x^2y^{-1})^2\)
3. \(\frac{18a^{-7}b^4c^3}{24a^{-1}b^4c^3}\)

4. Find the slope, x-intercept, and y-intercept. Then graph the line with equation \(-5x + 3y = 15\)

5. Find the slope and y-intercept, and use them to graph the line with equation \(-2x - 3y = 3\)

6. Graph and give the slope for each line:
   a. \(y = -2\)
   b. \(2x - 8 = 0\)

7. Write the equation (in slope-intercept form) for the line containing the points \((4, -6)\) and \((-8, 3)\).

8. Identify the slope and y-intercept for this line. Then give the equation in slope-intercept form.

9. Write the equation (in slope-intercept form) for the line through the point \((-3, 5)\) perpendicular to \(2y + 3x = 4\).

10. Write the equation (in slope-intercept form) for the line through the point \((2, -5)\) parallel to the line through the points \((1, 2)\) and \((-3, -1)\).

11. Determine without graphing whether the pair of lines is parallel or perpendicular, or neither: \(5y + 7 = 8x\) and \(-8x + 5y = -15\)
12. Determine if the following relations represent functions or not. Give the domain and range of each.
   a. \( \{(4, 1), (5, 2), (6, 3)\} \)
   b. \( \{(3, 5), (4, 6), (3, 7), (2, 0)\} \)
   c. \( (-6, 5) \)
   d. \( \begin{array}{c}
   \text{X} \\
   \text{Y} \\
   \text{(-6, -2)} \\
   \text{(7, 0)} \\
   \text{(10, 2)} \\
   \end{array} \)
   e. \( \begin{array}{c}
   \text{X} \\
   \text{Y} \\
   \text{(-6, -2)} \\
   \text{(-4, -5)} \\
   \text{(5, 8)} \\
   \text{(10, 2)} \\
   \end{array} \)
   f. \( \begin{array}{c}
   \text{X} \\
   \text{Y} \\
   \text{(-6, -2)} \\
   \text{(-4, -5)} \\
   \text{(5, 8)} \\
   \text{(10, 2)} \\
   \end{array} \)

13. For \( f(x) = 2x^2 - 3 \) and \( g(x) = 7x - x^2 \), find the following.
   a. \( f(-2) \)
   b. \( (f \cdot g)(a) \)
   c. \( (f + g)(3) \)
   d. \( \left( \frac{f}{g} \right)(x) \)
   e. The domain of \( \frac{f}{g} \)

14. For the function \( f \) shown at right, find the following.
   a. \( f(1) \)
   b. \( f(-2) \)
   c. \( f(2) \)
   d. Which value is larger: \( f(3) \) or \( f(-3) \)?

15. Give the slope and y-intercept, and graph the linear function: \( f(x) = \frac{4}{5}x - 2 \).
16. Suppose 8.2 million pounds of coffee are sold when the price is $6 per pound and 10.6 million pounds are sold at $5 per pound.
   a. Find a linear equation that expresses the amount of coffee sold \( y \) as a function of the price per pound \( x \).
   b. Use the equation from part (a) to predict the consumer demand if the price is $9 per pound.

17. At a particular college, 33 student loans were distributed in 1980 and 63 loans were given during 1990. During this ten-year period, the number of loans disbursed each year followed a linear pattern.
   a. Write a linear equation that expresses the annual number of loans given \( y \) as a function of the year \( x \) since 1980. (That is, \( x = 0 \) corresponds to 1980, \( x = 1 \) corresponds to 1981, etc.)
   b. Use the equation from part (a) to find the number of loans given in 1987.

18. Maria's wages are $2400 plus 3.5% commission on monthly sales.
   a. Write a function expressing the relationship between Maria's wages, \( w \), and her monthly sales, \( x \).
   b. Find her sales for the month if her wages for the month are $3380.

19. Solve the following systems of linear equations in two variables:
   a. \[
   \begin{align*}
   x - 2y &= 7 \\
   2x + 5y &= 5
   \end{align*}
   \]
   b. \[
   \begin{align*}
   4x + 3y &= -1 \\
   5x - 2y &= 16
   \end{align*}
   \]
   c. \[
   \begin{align*}
   2x + 7y &= 65 \\
   21y + 6x &= 19
   \end{align*}
   \]

20. The sum of two numbers is 95. One of the numbers is 17 more than the other. What are the numbers?

21. John’s motor boat took 4 hours to make a trip downstream with a 5-mph current. The return trip against the same current took 6 hr. Find the speed of the boat in still water.

22. A store sells plain sweatshirts for $19.95 each, and Coca-Cola sweatshirts sell for $29.50 each. In one day it sold 120 sweatshirts for a total of $2833.30. How many of each kind were sold?

For Problems 23 - 28, solve the linear inequalities. Graph the solution and give the answer in interval notation.

23. \( 3(x - 5) + 2x \geq 2x + 6 \)
24. \( -3 \leq -2(x + 0.5) < 4 \)
25. \( 3(x - 8) < 21 \text{ and } 5x + 1 > -14 \)
26. \( 2x + 3 > 19 \text{ or } -3x + 9 \geq 30 \)
27. \( |x| \geq 8 \)
28. \( |x| < 5 \)
29. Solve the equation for \( x \) and simplify, if possible: \( |3x - 1| = 8 \)
30. Graph the solution to the inequality on an xy-coordinate plane: \( 4x + 3y \geq 24 \)

31. Graph the solution to each system of inequalities on an xy-coordinate plane:

   \[ \begin{align*}
   &a. \ -4 \leq y < 3 \\
   &b. \ \begin{cases} 2x + y \geq 4 \\ x \geq 0 \end{cases} \\
   &c. \ \begin{cases} x + y > 4 \\ y \leq 2x - 4 \end{cases}
   \end{align*} \]

For Problems 32 - 44, factor completely, if possible:

32. \( x^2 - y^2 \)  
33. \( x^2 + y^2 \)  
34. \( x^3 - y^3 \)  
35. \( x^3 + y^3 \)  
36. \( x^2 - 2xy + y^2 \)  
37. \( x^2 + 2xy + y^2 \)  
38. \( 2x^3y - 4x^2y - 6xy \)  
39. \( a^2(x+1) - 4(x+1) \)  
40. \( 81 - y^4 \)  
41. \( x^4 - 13x^2 + 36 \)  
42. \( 2x^2 + x - 6 \)  
43. \( ab^2 + cb^2 - 4a - 4c \)  
44. \( 5r^3 - 40 \)

For Problems 45 - 47, solve the equations for \( x \) and simplify, if possible.

45. \( 6x^2 = 24 \)  
46. \( 10x^2 + 5x = 0 \)  
47. \( (x - 2)(x + 3) = -4 \)

48. Write a quadratic equation having the given numbers as solutions.
   a. 2, -3  
   b. -4, \( \frac{2}{5} \)

49. Find the domain of the following.
   a. \( f(x) = \frac{x - 3}{x^2 - 3x} \)  
   b. \( g(x) = x^2 + x \)

50. Find the three sides of this right triangle.
   Hint: Use the Pythagorean theorem: \( a^2 + b^2 = c^2 \). 

51. The width of a rectangle is 5 cm less than the length. The area is 104 square cm.
   Find the length and the width.

For Problems 52 - 59, perform the indicated operation and simplify (reduce), if possible.

52. \[ \frac{3x^2 + 3xy}{10x - 20} - \frac{5x^2 - 20}{x^2 + 2xy + y^2} \]  
53. \[ \frac{y + 2}{3 - y} + \frac{y^3 + 8}{3y^2 - 27} \]
54. \( \frac{x^2 - 2}{x - 3} + \frac{x + 4}{3 - x} \) 55. \( \frac{4x + 3}{x^2 + 6x + 8} + \frac{3x}{x + 2} \)

56. \( \frac{5y - 4}{4y - 3} - \frac{2y}{4y + 3} \) 57. \( 1 + \frac{1}{x} \)

58. \( \frac{y - x}{x - 1} \) 59. \( \frac{4x}{3} - \frac{2}{5 + \frac{y}{x}} \)

For Problems 60 - 63, solve the equations for \( x \) and simplify, if possible.

60. \( \frac{6}{2x + 5} = \frac{4}{x - 9} \) 61. \( \frac{x + 2}{x^2 - 5x - 24} + \frac{4}{x - 8} = \frac{2}{x + 3} \)

62. \( \frac{1}{x - 5} + 5 = \frac{x - 4}{x - 5} \) 63. \( \frac{5}{x} - \frac{23}{x} \)

64. Carl, an experienced shipping clerk, can fill a certain order in 6 hours. Tim, a new clerk, needs 8 hours to complete the same job. How long would it take for both working together to fill the order?

65. Caleb's average driving speed is 12 kilometers per hour slower than Ling's. In the same length of time it takes Caleb to drive 231 km, Ling drives 297 km. What is Caleb's average speed?

66. a. Solve for \( R \): \( I = \frac{nE}{R + nr} \) b. Solve for \( r \): \( rL = H(r + k) \)

For Problems 67 - 70, simplify. Assume all variables represent nonnegative real numbers.

67. \( (x^6)^{1/3} \) 68. \( \sqrt{75} \) 69. \( \sqrt{36b^6c^9} \) 70. \( 2\sqrt{27} - 5\sqrt{300} \)

For Problems 71 - 73, multiply and simplify. Assume all variables represent nonnegative real numbers.

71. \( (6 - \sqrt{3})(2 + \sqrt{3}) \) 72. \( \sqrt{6x} \sqrt{3xy} \) 73. \( \sqrt[3]{54x^5y^2} \sqrt[3]{27x^{15}y^{10}} \)

For Problems 74 - 76, evaluate:

74. \( 8^{2/3} \) 75. \( 16^{-1/4} \) 76. \( (-64)^{-1/3} \)

77. Solve the equation for \( x \) and simplify, if possible: \( x - 1 = \sqrt{3x + 7} \)

For Problems 78 - 80, perform the indicated operation and express the answer in \( a + bi \) form:

78. \( \sqrt{-25} (3 - 2i) \) 79a. \( (2 - 3i)(-4 - 5i) \) 79b. \( \frac{2 + 3i}{1 - 5i} \)

80. \( (3 - 6i) + (2 + 3i) - i^3 \)
81. Solve each equation for \( x \) and simplify, if possible.

\begin{align*}
\text{a.} \quad x^2 + 2x &= 4 \\
\text{b.} \quad 2x^2 + 16 &= 0 \\
\text{c.} \quad 1 - \frac{3}{x} - \frac{7}{x^2} &= 0 \\
\text{d.} \quad 3x^2 - 6x + 5 &= 0
\end{align*}

82. For each equation, use the discriminant to determine the type of solutions and how many solutions exist:

\begin{align*}
\text{a.} \quad x^2 + 4x + 6 &= 0 \\
\text{b.} \quad x^2 - 7x + 5 &= 0 \\
\text{c.} \quad 6x^2 + 5x - 4 &= 0
\end{align*}

83. Graph \( f(x) = -2(x+1)^2 + 4 \) and find the following:

\begin{align*}
\text{a.} \quad \text{the vertex} & \quad \text{b.} \quad \text{the axis of symmetry} \\
\text{c.} \quad \text{the maximum or the minimum of the function}
\end{align*}

84. For the function \( f(x) = x^2 - 6x + 4 \), find the following:

\begin{align*}
\text{a.} \quad \text{the axis of symmetry} & \quad \text{b.} \quad \text{the vertex} \\
\text{c.} \quad \text{the} \ y\text{-intercept} & \quad \text{d.} \quad \text{the} \ x\text{-intercept(s), if any} \\
\text{e.} \quad \text{the graph}
\end{align*}

85. In business, the total profit \( P \) is the difference between the revenue \( R \) and the cost \( C \). For \( x \) units produced, \( R(x) = 1000x - x^2 \) and \( C(x) = 3000 + 2x \), where \( R(x) \) and \( C(x) \) are dollar amounts. Find the following:

\begin{align*}
\text{a.} \quad \text{the total profit function} \ P(x) \\
\text{b.} \quad \text{the value of} \ x \ (\text{number of units}) \ \text{at which the maximum total profit occurs} \\
\text{c.} \quad \text{the maximum total profit}
\end{align*}

86. The stopping distance \( d \) of a car after the brakes have been applied varies directly as the square of the speed \( r \). If a car traveling 60 mph can stop in 200 ft, what stopping distance corresponds to a speed of 36 mph?

87. The current \( I \) in an electrical conductor varies inversely as the resistance \( R \) of the conductor. If the current is 0.5 amperes when the resistance is 240 ohms, what is the current when the resistance is 960 ohms?

88. Graph the following functions: \ a. \quad y = 4^x \quad \text{b.} \quad y = \left(\frac{1}{4}\right)^x

89. Given the points \( A(2, -3) \) and \( B(4, -2) \), find the distance from \( A \) to \( B \) and find the midpoint of the line segment joining \( A \) and \( B \).

90. Find the center and radius of each circle:

\begin{align*}
\text{a.} \quad (x + 5)^2 + (y - 3)^2 &= 64 \\
\text{b.} \quad x^2 + y^2 - 16x + 4y + 5 &= 0
\end{align*}