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Derivation of a Simple Keynesian Model

Y=total production of goods and services; C=consumption spending;

I_g=gross investment spending; G=government spending; X=national exports;
W=national imports; T=taxes collected by government

1   $Y = C + I_g + G + X - W$ (This is true if the economy is in macroeconomic equilibrium.)

2   $W = wY$ ...imports are related to output, Y.

$$0 \prec w \leq 1$$

3   $T = T_p + tY$ ...total tax receipts are related to output, Y, but there are some taxes, T_p, which must be paid even if income is zero, property taxes for example.

$$0 \prec t \leq 1$$

4   $C = a + c(Y - T)$ ...consumption spending is related to disposable income, Y-T, and $0 \prec c \leq 1$, but even if disposable income were zero, still there would be some spending, a, by households.

Plug 2, 3 and 4 into 1 and solve for Y:

$$Y = a + c(Y - T) + I_g + G + X - wY$$

Use 3:

$$Y = a + c(Y - tY - T_p) + I_g + G + X - wY$$

Factor Y:

$$Y = a + c(1 - t)Y - cT_p + I_g + G + X - wY$$

Move all terms containing Y to the left side:

$$Y - c(1 - t)Y + wY = a + I_g + G - cT_p + X$$

Factor Y:

$$Y(1 - c(1 - t) + w) = a + I_g + G - cT_p + X$$

Divide both sides by (1-c(1-t)+w):

$$Y = \left(\frac{1}{1-c(1-t)+w}\right)(a + I_g + G - cT_p + X)$$

This equation summarizes the basic, simple Keynesian Model.

Comments: (All the following terms are important and must be remembered!)

c is called the **"marginal propensity to consume"**.

w is called the **"marginal propensity to import"**.

t is called the **"marginal tax rate"**.

T_p is called **"taxes independent of income"**.

The entire fraction, $\left(\frac{1}{1-c(1-t)+w}\right)$, is called **"the multiplier"**. (Note that the multiplier given in the text is a much simpler (but less realistic) formula which assumes no imports (**w** = 0) and a zero marginal tax rate, **t** = 0. Hence T_p is the only type of taxes--the kind which do not vary with income, like property taxes.

The terms to the right of the fraction, namely $(a + I_g + G - cT_p + X)$, are called **"autonomous spending",** because they are assumed to be constants which are determined outside the model. (They can change of course, it's just that the model itself does not change them.)

Problems Using Keynesian Model Equation

Due:

As assigned in class. This assignment is worth 25 homework penalty points unless otherwise announced.

Problems:

1) Estimate the correct sizes of all the parameters in the model Keynesian model as derived above. In essence you will be making wild guesses. To calculate, you will need the spreadsheet called A KEYNESIAN MODEL. The spreadsheet is available on my web-site under "Courses", and you will need it to answer this question. Use the Web to collect your "data", probably starting either at the St. Louis Fed data web site: http://research.stlouisfed.org/fred2/ , or at the Bureau of Economic Analysis: http://www.bea.gov/ . Look at the available data and make guesses of the values of the various parameters in our version of the Keynesian model, I mean, guess the values of c, t, w, X, a, I_g and G. Then plug your guesses into the spreadsheet and see what values of Y, W (imports) and T (total tax revenue) are calculated by the model. Also collect actual values for Y, T and W. (The model does not need these three, they are calculated by the model.) If your guesses are good, the model will calculate values for Y, T and W which are close to the actual values. (Remember that T is total Federal + State + Local tax revenue.) Probably there will be discrepancies, so start adjusting your guesses until calculated Y, T and W are close to the actual values from the web., trying to reduce or eliminate the inconsistencies--make small changes and see whether things improve. (Usually, each change will make some things better, but other things worse!) When you are finished, your collected actual values of Y, W and T should be reasonably similar to the values for Y,W and T calculated by the model.

    Part of your assignment is to summarize your exact sources of all your data and also summarize what adjustments you needed to make. This must be a maximum of 2 single spaced pages and probably should be less than one page.   There is no once correct answer for this part 1) of the exercise. Enjoy the fun of trying to make the model work and stop when you have learned enough.

2) Using your current best guesses for parameter values as determined in problem 1 above, report the value of your "multiplier". (This is easy. Read it out of the spreadsheet!)

3) Assuming your Keynesian model parameter values are valid, compute the GDP impact of a $200 billion increase in Government spending. (Increase G by $200 billion and see how GDP changes.)

4) Assuming the assumptions of your Keynesian model are valid, what will be the impact on GDP of a Bush tax cut which reduces tax revenue by $200 billion per year? (In making this calculation, look at your model's calculated value of $T=T_p + tY$, then temporarily pretend Y is not changing and reduce little t by just the amount which would reduce T by $200 billion if Y didn't change. (Actually, since Y might change (the Keynesians think it will) tax revenues might decline less or more than this. For example, if the tax cut stimulates the economy, Y will increase and T will therefore end up declining less than the original size of the tax cut.)

5) Now combine a tax **increase** of $200 billion (the exact opposite of problem 4) with a $200 billion increase in G (same as given in problem 3). This is a so-called "balanced budget" stimulus package. Then answer these questions:

    A. Under your Keynesian model will the government deficit be the same, or will it improve, and by how much?

    B. Under your Keynesian model, how much will the economy be stimulated by this fiscal policy?