# College Mathematics
## annuities

An annuity is a series of payments at regular intervals.

The future value of an annuity is the total value of all payments at the end of the payment schedule.

Example:

Suppose you deposit \$250 into an account at the *end* of *every* month for 40 years. If the account has an 8.4% annual growth rate, compounded monthly, what is the future value of your annuity at the end of the 40 years?

Solution:

There are 480 payments in all.

Notice that for the first month, nothing is in the account.
Also notice that the last payment goes in at the end of the 40 years and is immediately taken out.
This makes the calculation a bit simpler than if we made payments at the beginning of each month.

Let's analyze each payment *separately*:

Imagine if you made the *first* payment but no other payment.
What would be the future value of the payment?
The payment would be in the account for 479 months, so the future value is $\$250(1.007)^{479}$.

The second payment stays in the account for 478 months, so the FV (future value) is $\$250(1.007)^{478}$.

The third payment stays in the account for 477 months, so the FV (future value) is $\$250(1.007)^{477}$.

and so on…

By the way, what about the *last* payment?
Remember, the last payment goes in at the end of the 480th month and comes out right after that.
It grows for 0 months, so the FV is $\$250(1.007)^{0}$, or simply \$250.

What we want is the *grand total* of all these future values. In other words, we want to calculate

FV = $250(1.007)^{479}+250(1.007)^{478}+250(1.007)^{477}+\ldots+250(1.007)^{1}+250(1.007)^{0}$

FV = $250\left[(1.007)^{479}+(1.007)^{478}+(1.007)^{477}+\ldots+(1.007)^{2}+(1.007)^{1}+1\right]$ *

FV = $250\times\left[\dfrac{(1.007)^{480}-1}{(1.007)-1}\right]$

= about \$980,500.

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*At this point, it's helpful to know a shortcut for adding up successive powers of a number.

$$M^{17}+M^{16}+M^{15}+M^{14}+\ldots+M^{3}+M^{2}+M+1=\frac{M^{18}-1}{M-1}$$