Multiplying Fractions
a visual approach

Remember that one way to understand a fraction is to interpret the bottom as the number of equal-sized pieces in one cake, pizza, gallon, inch, hour, or whatever. Then, the top represents the number of pieces you’re talking about. For example, \( \frac{3}{4} \) would mean 3 slices out of a pizza cut into 4 equal-sized pieces.

How can we understand what it means to multiply two fractions?

\[
\frac{2}{3} \times \frac{4}{5} = ?
\]

We can think of this as saying “two-thirds of four-fifths”.

\[
\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}
\]

What about the other way around?

\[
\frac{4}{5} \times \frac{2}{3} = ?
\]

We can think of this as saying “four-fifths of two-thirds”.

\[
\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}
\]

Notice that in either case, we multiply numerators to get the numerator and multiply denominators to get the denominator.

So, we get the same answer either way:

Multiplication of fractions is commutative.
Multiplying Fractions
a more careful explanation

We can understand multiplication with fractions by understanding three ideas:

(a) When you multiply a unit fraction by a whole number, the numerator of the product is the whole number.

\[ \frac{2}{5} \times \frac{1}{5} = \frac{2}{5} \]

“Two one-fifths is two-fifths.”

\[ \frac{1}{5} \times 2 = \frac{2}{5} \]

“One-fifth of two is two-fifths.”

(b) To multiply two unit fractions, just multiply the denominators.

\[ \frac{1}{3} \times \frac{1}{5} = \frac{1}{15} \]

“One-third of one-fifth is one-fifteenth.”

(c) Every fraction is a whole number times a unit fraction.

\[ \frac{2}{3} \times \frac{1}{5} = \left( 2 \times \frac{1}{3} \right) \times \frac{1}{5} \]

\[ = 2 \times \left( \frac{1}{3} \times \frac{1}{5} \right) \]

\[ = 2 \times \frac{1}{15} \]

\[ = \frac{2}{15} \]