You should be able to do all of the following things:

Given a description of a graph in words, symbols, or as a table, make a drawing of the graph.
Given a situation or problem where a graph is useful, make a drawing of the graph.

Given a drawing of a graph, give a description of the graph in words, symbols, or as a table.
Given two drawings of graphs, discuss (using words, drawings, and or tables) how to tell whether the two graphs are equivalent or not.

Given a drawing of a graph,
identify each vertex (plural: vertices) and edge,
tell the degree of each vertex,
tell whether the graph is connected or not,
tell whether the graph is a tree or not,
tell whether the graph is complete or not, and
describe any path or circuit in the graph.

Explain how a tree is minimally connected, while a complete graph is maximally connected.
Given the number of vertices in a tree, calculate the number of edges it has.
Given the number of vertices in a complete graph, calculate the number of edges it has.

Explain in words and/or by examples what is meant by each of the following terms: path, circuit, Euler path, Euler circuit, Hamilton circuit. Given a graph and a description of a circuit, tell whether that circuit is an Euler circuit. Given a graph and a description of a circuit, tell whether that circuit is a Hamilton circuit.

Given any description of a connected graph (words, symbols, or table), determine whether the graph has an Euler circuit, an Euler path, or neither. If a graph has an Euler circuit or an Euler path, find one and describe it by numbering the edges.

Given a graph that has Hamilton circuits, find one or more Hamilton circuits and describe each by listing the vertices in the order traveled (like, ABEDCA). Name categories of graphs that have Hamilton circuits and categories of graphs that do not have Hamilton circuits.

Know what is meant by a Traveling Salesman Problem (TSP). Know what it means to solve one.
Given a complete graph with costs on the edges, solve the TSP (by “brute force”).
Given the number of vertices in a complete graph, calculate the number of different TSP “cases” that must be considered. Knowing the time it takes to check each TSP case, calculate the length of time needed to check all cases.

Given a complete graph with costs on the edges, apply the nearest-neighbor algorithm to find an approximate TSP solution.

Given a connected graph with costs on the edges, find a minimum spanning tree (MST).

Give clear, correct, complete answers to all of the questions on the next page.
1. Can two graphs have the same number of vertices but not be equivalent?
   If so, give an example.
2. Can two graphs have the same number of vertices and the same number of edges but not be equivalent?
   If so, give an example.
3. Can two graphs have the same number of vertices, the same number of edges, and the same degree sequence, but not be equivalent? If so, give an example.
4. Explain how having a vertex with an odd degree makes it impossible for a graph to have an Euler circuit.
5. Why must an Euler path start at one odd-degree vertex and end at the other one?
6. Give an example of a graph that has an Euler circuit but does not have a Hamilton circuit.
7. Give an example of a graph that has a Hamilton circuit but does not have an Euler circuit.
8. What is the minimum number of edges needed to build a connected graph with 100 vertices?
9. Give three different definitions of tree.
10. Does every connected graph have to be complete?
11. Does every complete graph have to be connected?
12. How many edges are needed to build a complete graph with 100 vertices?
13. Explain why the sum of degrees in a graph equals two times the number of edges.
14. Can the sum of degrees in a graph ever be an odd number? Why or why not?
15. Which graphs can be drawn without any edges crossing? Which graphs cannot?
16. In all of Topoland, there are only 8 towns.
    One of these towns, Noparity, has exactly 3 roads connected to it.
    None of the towns is isolated. You should not assume anything else about Topoland.
    Explain, using everyday words, what would happen, step by step, if you tried to make a grand tour of Topoland that used every road exactly once, without backtracking, and finished where it started.
    First, discuss the case of starting at Noparity.
    Then discuss the case of starting at a town other than Noparity.
17. Is there a general theory that, given any connected graph, can tell whether or not that graph has an Euler circuit?
    If so, give a detailed description of how to use the theory, being careful to explain when you have an Euler circuit and when you do not.
    If not, name three types of graphs, and tell which have Euler circuits and which do not.
18. Is there a general theory that, given any graph, can tell whether or not that graph has a Hamilton circuit?
    If so, give a detailed description of how to use the theory, being careful to explain when you have a Hamilton circuit and when you do not.
    If not, name three types of graphs, and tell which have Hamilton circuits and which don't.
19. What is a Traveling Salesman Problem (TSP)?
    Is it possible, in principle, to solve any given TSP? How?
    Does the nearest-neighbor algorithm solve TSPs?
    If so, explain how. If not, explain why not.
20. In computational mathematics, what is meant by “brute force”?
    Explain how to solve a TSP using "brute force".
    Why aren't all TSPs solved this way?
    How would having a faster computer help? How would it not help?
    Why do we use algorithms like nearest neighbor or cheapest link?