

College Mathematics

loans

The amount of a loan is the *single amount* that would have the same future value as the entire series of payments. This is called the *present value* of the annuity.

Example 1 (solving for the payment):

Suppose you have been approved for a \$9000 car loan with an interest rate of 7.5%, compounded monthly. If you want to make equal payments at the *end* of every month to pay off the loan in 4 years, what must be the monthly payment?

Solution:

The future value of the entire series of payments is
$$= \text{PMT} [(1.00625^{48} - 1) / (1.00625 - 1)]$$

The future value of the loan amount is $\$9000(1.00625)^{48}$.

This must be equal to the future value of all the payments:

$$\$9000(1.00625)^{48} = \text{PMT} [(1.00625^{48} - 1) / (1.00625 - 1)]$$

$$\begin{aligned} \text{PMT} &= \$9000(1.00625)^{48} / [(1.00625^{48} - 1) / 0.00625] \\ &= \$217.61 \end{aligned}$$

Your monthly payment will be \$217.61

Example 2 (solving for the loan amount):

Suppose you can afford to make \$500 payments at the *end* of every month toward a house loan. If the annual interest rate is 6%, compounded monthly, and you plan on taking 30 years to pay off the loan, how big a loan can you afford?

Solution:

The future value of the entire series of payments is
$$\begin{aligned} &\$500 [(1.005)^{359} + \dots + (1.005)^2 + (1.005)^1 + 1] \\ &= \$500 [((1.005)^{360} - 1) / (1.005 - 1)] \\ &= \$502,257.52 \end{aligned}$$

The future value of the loan amount is $PV(1.005)^{360}$.

This must be equal to the future value of all the payments:

$$PV(1.005)^{360} = \$502,257.52$$

$$\begin{aligned} PV &= \$502,257.52 / (1.005)^{360} \\ &= \$83,395.81 \end{aligned}$$

You can afford a mortgage loan of \$83,395.81