College Mathematics
Quiz 21: Euclidean and non-Euclidean geometries; topology of surfaces

1. King Kong has exactly the same shape and materials as an ordinary gorilla, but he is 20 times as tall as an ordinary gorilla.

Fill in each blank with an appropriate number.

(a) King Kong's footprint should cover _____ times the area of an ordinary gorilla's.
(b) King Kong's waist measurement should be _____ times an ordinary gorilla's.
(c) King Kong should be _____ times the weight of an ordinary gorilla.

2. Describe the differences between the properties of a cylinder and a Möbius band.

3. Can a Klein bottle exist in 3-dimensional space? Discuss.

4. What do you get when you cut a Möbius band "into thirds"?

   (A) three cylinders, each one-third as wide as the Möbius band
   (B) three Möbius bands, each one-third as wide as the original
   (C) one Möbius band, one-third as wide and three times as long as the original
   (D) one band, one-third as wide and three times as long, with one-and-a-half (540°) twists
   (E) none of the above

5. (a) Draw one more "O" to make 5 in a row.
     (b) Draw one more "X" to make 5 in a row.
6. Fill in each blank with the most appropriate word or phrase.

(a) In any geometry, a shortest possible path between two points is called a _________________.
(b) The surface of a ball is called a _________________.
(c) Among curved paths, the shortest path is the one with minimum _________________.
(d) The radius of the osculating circle at a point is called the _________________ _________________.
(e) A largest possible circle on a sphere is called a _________________ _________________.

7. Calculate the spherical distance from the North Pole to the South Pole.
Assume the diameter of Earth is 8000 miles. You may approximate pi as 3.14
Round your answer to the nearest hundred miles.

8. Given two points on a sphere, is there always a great circle that goes through both points?
If so, explain how to draw that circle. If not, explain why not.

9. Fill in each blank with the most appropriate word or phrase.

(a) In Euclidean geometry, the shortest path between two points is along a _________________.
(b) In spherical geometry, a shortest path between two points is along a _________________.
(c) The Euclidean distance between two points is usually ______________ than the taxicab distance.
(d) In taxicab geometry, the set of all points that are a fixed distance away from a specific point is called a ________________ _________________.
(e) In 3-dimensional Euclidean geometry, the set of all points that are a fixed distance away from a specific point is called a _________________.

10. Circle the most appropriate word or phrase to make each statement true.

(a) In Euclidean geometry, the sum of angles inside a triangle is (less than / equal to / more than) 180 degrees.
(b) In spherical geometry, the sum of angles inside a triangle is (less than / equal to / more than) 180 degrees.

BONUS:
(a) Describe situations in taxicab geometry in which there is only one geodesic between two points.
(b) Describe situations in spherical geometry in which there is more than one geodesic between two points.