Example 1:
The large square is 3 times as wide and 3 times as tall as the small square.
The large square has 9 times the area of the small square.

Example 2:
The large triangle is 3 times as wide and 3 times as tall as the small triangle.
The large triangle has 9 times the area of the small triangle.

Example 3:
The large circle is 3 times as wide and 3 times as tall as the small circle.
The large circle has 9 times the area of the small circle.
In each of these examples, the shape was maintained without distortion by multiplying every length by a scaling factor (SF) of 3, but the area ends up multiplied by 9, not by 3.

Here's why:

Area is a 2-dimensional measure, meaning that calculating area involves multiplying two lengths.

For triangles, \( \text{area} = \frac{1}{2} (\text{base} \times \text{height}) \) . For circles, \( \text{area} = \pi (\text{radius} \times \text{radius}) \).

The area of any bounded region must be some part of the area inside the enclosing rectangle, so every area formula looks like \( \text{area} = (p)(\text{height})(\text{width}) \), so this is true for all shapes, not just squares, triangles, and circles.

The large blob has 9 times the area of the small blob.

When we magnify or shrink a shape without distortion, we multiply both the height and the width of the rectangle by the scaling factor, so

\[
\text{NewArea} = \text{OldArea} \times (SF)^2
\]

The same type of reasoning shows that when scaling a 3-dimensional object,

\[
\text{NewVolume} = \text{OldVolume} \times (SF)^3
\]

Example 4:
Gena is 5 feet tall. Xena has exactly the same build as Gena, but Xena is 6 feet tall. Assume that weight grows like volume. (How good an assumption is that?)

The scaling factor from Gena to Xena is 1.2
If Gena's waist measures 25 inches, then Xena's waist measures 30 inches.
If Gena requires 6.25 square feet of armor, then Xena requires 9 square feet of armor.
If Gena's lung capacity is 2 liters, then Xena's lung capacity is 3.456 liters.
If Gena weighs 100 pounds, then Xena weighs 172.8 pounds.

Example 5:
You are designing a 4-foot-tall solid chocolate swan centerpiece. It took 2 cups of melted chocolate to make a 6-inch-tall scale model of the swan.

The scaling factor from the model to the centerpiece is 8. The centerpiece will contain \(8^3\) times as much chocolate as the model. The full-scale swan contains 1024 cups, or 64 gallons, of chocolate.