One of the basic ideas in any geometry is the concept of distance. We measure distance between two points along the shortest path between those points. In Euclidean geometry, for example, we measure distance along straight lines. In taxicab geometry, we measure distance along “zig-zag” lines. For spherical geometry, it is harder to define distance because it is not immediately clear what the shortest paths look like. Since we are restricted to paths on the sphere, there are no straight paths. Remember, sphere means just the surface of a ball, not the ball itself.

Since paths on a sphere are not straight, we will try to find the most direct routes possible, ones that are most nearly straight. A “straightest” curve is called a geodesic. In some geometries, like Euclidean geometry, there is exactly one geodesic through any two different points. On the other hand, in taxicab geometry, there are usually many different geodesics between two given points.

The most direct route between two points is the one with the least curvature. In order to compare paths, we would like a way to measure curvature. The curvature of a path can be different at different points along the path. In general, then, a geodesic is a curve with minimum curvature at every point.

At any point of interest, consider the circle that best matches the path there. This "hugging" or "kissing" circle is called the osculating circle at that point. The radius of the osculating circle is called the turning radius. At points where the turning radius is smaller, we say that the path has higher curvature. Where the turning radius is larger, we say that the path has lower curvature. From these observations, we can compare curvature at different points.

On a sphere, the minimum curvature is achieved by having the maximum turning radius.

On a sphere, a circle having the largest possible radius is called a great circle. How big can it be? It cannot be wider than the entire sphere. Here are some other ways to describe a great circle: a circle on a sphere with the same radius as the sphere, a circle on a sphere with the same diameter as the sphere, a circle on a sphere with the same center as the sphere, or a circle that divides the sphere into two equal halves. Can you think of other ways to describe a great circle?

In spherical geometry, then, geodesics are on the sphere’s great circles.
THE EARTH AND MAPS

The earth is almost spherical, with diameter approximately 8000 miles.

Geographical coordinates are traditionally given in terms of latitude and longitude. Latitude is measured in degrees North or South of the equator. Longitude is measured in degrees West or East of a semicircle running through the North Pole and Greenwich, England.

Curves of constant latitude are called parallels and are circles of different sizes, all of which have their centers along the North-South axis. The equator is the parallel with the largest radius. The Arctic Circle is another parallel. Is it a great circle? Can you name any other parallels? Which of the parallels is a great circle?

Curves of constant longitude are called meridians and are semicircles, all of which have the same radius as the earth. The meridian through Greenwich is called the Prime Meridian.

It is impossible to make a flat map which preserves distances, directions, angles, and areas of regions all at the same time. This is because of intrinsic differences in the geometry of a sphere and the geometry of a plane. Still, it is convenient to work store, transport, and use flat maps, and so map-makers have attempted to faithfully represent some aspects of the earth while distorting others. Which aspects you should represent accurately depends on your application.