Review for Elementary Algebra Final Exam

The quadratic formula will be provided on the final exam.

Students are expected to know from memory all other relevant formulas, including:
- Sum of the angles in a triangle
- Pythagorean Theorem
- Area and perimeter of triangles, rectangles, and squares
- All formulas for linear equations (slope formula, slope-intercept and point-slope forms)

In order to be prepared for the final exam, students should be able to do all of the following problems and related problems as well.

Practice problems

1. Translate to an algebraic expression: seven less than twice a number
2. Translate to an algebraic expression: three-fifths of a number
3. Use the commutative law of multiplication to write an expression equivalent to: \(wd + s\)
4. Use the associative law of addition to write an expression equivalent to: \((wd + s) + 3\)
5. Use the distributive law to write an expression equivalent to: \(3(wd + s)\)
6. Simplify:
   a. \(12 - 3(10 - 4)\)
   b. \(|-3 \cdot 4 - 12 \cdot 2| - 8(-7)\)
   c. \((12 - 8)^2\)
   d. \(\frac{4(18 - 8) + 7 \cdot 21}{8^2 - 9^2}\)
   e. \(2 - (x - 1) + 3x\)
   f. \(6[11m - 3(4m - 1)] - (7 - 6m)\)
7. Evaluate: \(\frac{2xy}{x - y^2}\) for \(x = 4\) and \(y = -3\)
8. Add: \((11a^3b + 4a^2b - 3ab - 5) + (2a^3b - 4a^2b - 6)\)
9. Subtract: \((y - 2y^3 + 7y^2) - (4y^2 - 3y + 2y^3)\)
10. Simplify and write the final answers with positive exponents only:

   a. \(5x^0\)
   b. \((5x)^0\)
   c. \((-2)^{-4}\)
   d. \(\frac{z^{-7}}{z^{-4}}\)
   e. \((6a^3b^2c)(-a^2c^5)\)
   f. \((-2n^2v^5)^3\)
   g. \((4xy^4)^{-2}\)
   h. \((3x^{-3}y)^{-2}\)
   i. \((5v^{-5}w^{-3})(-2v^2w^{-5})^4\)
   j. \(\left(\frac{2ab^2}{-5b^3c^4}\right)^3\)
   k. \(\left(\frac{3x^2y^{-3}}{4y^{-4}z}\right)^{-2}\)

11. Multiply:

   a. \(-3x^3(5x^3 - 2x^2 + x - 7)\)
   b. \((8y + 1)(2y - 7)\)
   c. \((6x - 7y)^2\)
   d. \((4a + 5)(a^2 - 5a + 1)\)

12. Divide: \(\left(12xy^6 + 24x^5y^9 - 4xy^3\right) \div (4xy^3)\)

13. Convert to decimal notation: \(7 \cdot 6 \times 10^3\)

14. Convert to scientific notation: \(0.0596\)

15. Multiply and write your answer in scientific notation: \(2 \cdot 75 \times 10^3 \left(4 \cdot 20 \times 10^{-5}\right)\)

16. Divide and write your answer in scientific notation: \(\frac{1\cdot 6 \times 10^{-2}}{6 \cdot 4 \times 10^{-7}}\)
17. Factor completely:
   a. $3x^2 + 24x + 48$
   b. $x^2 - 2xy + x - 2y$
   c. $2x^3 + x^2 - 8x - 4$
   d. $3x^3 - 12x$
   e. $4x^3 + 23x^2 - 6x$
   f. $9x^2 + 16$
   g. $2x^3 - 12x^2 + 16x$
   h. $3x^2 - 11x - 20$

18. Solve for the indicated variable:
   a. $A = \frac{1}{2}bh$, for $b$
   b. $rx - y = t$, for $x$
   c. $Q = \frac{p-q}{2}$, for $q$

19. Simplify:
   a. $-\sqrt{121}$
   b. $\sqrt{0}$
   c. $\sqrt{289}$
   d. $\sqrt{300}$

20. Solve the equations for $x$ and simplify your answer if possible:
   a. $0.7(2 - x) = 1.25 - 0.5x$
   b. $\frac{1}{6}\left(\frac{3}{4}x - 2\right) = -\frac{1}{5}$
   c. $7 - 4(x - 1) = 9 - 5x$
   d. $-2(x - 4) = 6(x + 2) + 3x$
   e. $x^2 = 25$
   f. $\frac{3}{4x} + \frac{6}{x} = 1$
   g. $2x^2 + 1 = 19$
   h. $5x^2 + 12x - 20 = -3x$
   i. $x(x + 3) = 10$
   j. $x^2 = 7x - 6$
   k. $x^2 - 7x - 2 = 0$
   l. $2x(x + 2) = x + 3$
   m. $3x^2 - 4x - 2 = 0$

21. Find an equation of the vertical line containing the point $(-4, 5)$ and state its slope.

22. Find an equation of the horizontal line containing the point $(-4, 5)$ and state its slope.

23. For the linear equation $y = 3$, state the slope and graph the line.

24. For the linear equation $x = -2$, state the slope and graph the line.
25. For the linear equation $5x + 3y = -6$:
   a. Find the x and y intercepts
   b. Find the slope
   c. Graph the line

26. Find the slope of the linear equation $y = 3x - 6$

27. Find an equation of the line containing the point $(-1, -3)$ with slope $m = -4$

28. Find the slope of the line from the graph:
   a. [Graph of a line]
   b. [Graph of a line]

29. a. Find the slope of the line passing through the points $4, 2$ and $-2, -7$
   b. Find an equation of the line described in part (a).

30. Solve the system of equations by graphing: \[
\begin{align*}
  y &= -2x + 5 \\
  x - 3y &= 6
\end{align*}
\]

31. Solve the systems of linear equations by any method:
   a. \[
   \begin{align*}
   x &= 3y + 5 \\
   4x - 12y &= 20
   \end{align*}
   \]
   b. \[
   \begin{align*}
   4x - y &= 27 \\
   7x + 2y &= 21
   \end{align*}
   \]
   c. \[
   \begin{align*}
   2x - 10y &= 1 \\
   3x - 15y &= 4
   \end{align*}
   \]

32. In a class of 40 students, 35% work full time. How many students in that class work full time?

33. On a test of 64 items, a student got 56 correct. What percent were correct?
34. A computer is on sale for a 15% reduction. If the sale price is $1572.50, what was the original price?

35. Rent in Austin has increased by 55% since 1990. If monthly rent for a 2 bedroom apartment is now $850, what would rent have been for the same apartment in 1990? Round your answer to the nearest cent.

36. The sum of three consecutive odd integers is – 87. What are the integers?

37. The second angle of a triangle is 4 times as large as the first. The third angle is 45 degrees less than the sum of the other two angles. Find the measure of each angle.

38. The length of a rectangle is 3 feet less than 4 times its width. If the perimeter of the rectangle is 64 feet, what are the dimensions of the rectangle?

39. Two angles are complementary if the sum of their angles is 90 degrees. One of two complementary angles is 12 degrees more than twice the other. Find the measure of each angle.

40. There were 200 tickets sold for a volleyball game. Tickets for students were $2 each and for adults were $3 each. The total amount collected was $530. How many of each type of ticket were sold?

41. Clear Shine window cleaner is 12% alcohol and Sunstream window cleaner is 30% alcohol. How much of each should be used to make 90 ounces of a cleaner that is 20% alcohol?

42. Emilio wishes to mix peanuts worth $2.52 per pound with almonds worth $3.80 per pound to make 480 pounds of a mixture worth $3.44 per pound. How much of each should he use?

43. The length of a rectangle is one foot less than three times the width. Find the length and width if the area is 30 square feet.

44. A water pipe runs diagonally under a rectangular garden that is 7 feet longer than it is wide. If the pipe is 13 feet long, what are the dimensions of the garden?

45. The length of one leg of a right triangle is 12 meters. The length of the hypotenuse is 8 meters longer than the other leg. Find the lengths of the hypotenuse and the other leg of the triangle.

46. Two angles are supplementary if the sum of their angles is 180 degrees. One of two supplementary angles is 8 degrees less than three times the other. Find the measure of each angle.

47. A sample of 125 light bulbs contained 3 defective bulbs. How many defective bulbs would you expect in a batch of 3000 light bulbs?

48. Solve and graph the solution on a number line:
   a. $x - 3 > 1$
   b. $2 - 4w \geq 7$
   c. $9 + 2y \leq 4y + 5$
49. List all value(s) of x for which each rational expression is undefined:
   a. \[ \frac{12}{-3x} \]  
   b. \[ \frac{x + 2}{x - 5} \]  
   c. \[ \frac{x^2 - 9}{x^2 - 3x - 10} \]

50. Simplify (reduce), if possible:
   a. \[ \frac{42x^4}{35x^6} \] 
   b. \[ \frac{y - 5}{5 - y} \] 
   c. \[ \frac{a^2 + a - 20}{2a^2 + 4a - 30} \]

51. Perform the indicated operation. If possible, simplify (reduce) your answer:
   a. \[ \frac{7a}{4} + \frac{a - 3}{4} \] 
   b. \[ \frac{4}{y + 5} + \frac{1}{y + 5} \] 
   c. \[ \frac{5 + 3x}{4x} - \frac{2x + 1}{4x} \] 
   d. \[ \frac{a^2}{a + 3} - \frac{2a + 15}{a + 3} \] 
   e. \[ \frac{2}{3x} + \frac{5}{x + 1} \] 
   f. \[ \frac{10x}{9x^4y^7} \cdot \frac{3y^2}{25y} \] 
   g. \[ \frac{5a + 5}{4a - 8} \cdot \frac{2a^2 - 8a + 8}{a^2 - 1} \] 
   h. \[ \frac{x^2 + 5x + 4}{x^2 - 4} \div \frac{x^2 + 8x + 7}{x^2 + 5x - 14} \] 
   i. \[ (2y - 1) \div \frac{2y^2 - 11y + 5}{4y^2 - 1} \]
Answers

1. \(2n - 7\)

2. \(\frac{3}{5}n\)

3. \(dw + s\)

4. \(wd + (s + 3)\)

5. \(3wd + 3s\)

6.
   a. \(-6\)
   b. 92
   c. 16
   d. \(-11\)
   e. \(2x + 3\)
   f. 11

7. \(\frac{24}{5}\)

8. \(13a^3b - 3ab - 11\)

9. \(-4y^3 + 3y^2 + 4y\)

10.
   a. 5
   b. 1
   c. \(\frac{1}{16}\)
   d. \(\frac{1}{z^3}\)
   e. \(-6a^5b^2c^6\)

f. \(-8n^6v^{15}\)

g. \(\frac{1}{16x^2y^8}\)

h. \(\frac{x^6}{9y^2}\)

i. \(\frac{80v^3}{w^{23}}\)

j. \(-\frac{8a^3}{125b^3c^{12}}\)

k. \(\frac{16z^2}{9x^4y^2}\)

11.
   a. \(-15x^6 + 6x^5 - 3x^4 + 21x^3\)
   b. \(16y^2 - 54y - 7\)
   c. \(36x^2 - 84xy + 49y^2\)
   d. \(4a^3 - 15a^2 - 21a + 5\)

12. \(3y^3 + 6x^4y^6 - 1\)

13. 7600

14. \(5.96 \times 10^{-2}\)

15. \(1.16 \times 10^{-1}\)

16. \(2.5 \times 10^4\)
17.
a. \(3(x + 4)^2\)
b. \((x - 2y)(x + 1)\)
c. \((x + 2)(x - 2)(2x + 1)\)
d. \(3x(x + 2)(x - 2)\)
e. \(x(4x - 1)(x + 6)\)
f. cannot be factored
g. \(2x(x - 4)(x - 2)\)
h. \((3x + 4)(x - 5)\)

18.
a. \(b = \frac{2A}{h}\)
b. \(x = \frac{y + t}{r}\)
c. \(q = p - 2Q\)

19.
a. \(-11\)
b. \(0\)
c. \(17\)
d. \(10\sqrt{3}\)

20.
a. \(0.75\)
b. \(\frac{16}{15}\) or \(1\frac{1}{15}\)
c. \(-2\)
d. \(-\frac{4}{11}\)
e. \(-5, 5\)
f. \(\frac{27}{4}\) or \(6\frac{3}{4}\)

g. \(-3, 3\)
h. \(-4, 1\)
i. \(-5, 2\)
j. \(6, 1\)
k. \(\frac{7 \pm \sqrt{57}}{2}\)
l. \(\frac{-3 \pm \sqrt{33}}{4}\)
m. \(\frac{2 \pm \sqrt{10}}{3}\)

21. equation: \(x = -4\)
slope: undefined

22. equation: \(y = 5\)
slope: 0

23. slope: 0

24. slope: undefined
25.
   a. x-intercept = \( \left( -\frac{6}{5}, 0 \right) \)
      y-intercept = \( (0, -2) \)
   b. slope = \( -\frac{5}{3} \)
   c. 

26. 3

27. \( y = -4x - 7 \)

28.
   a. \( -\frac{1}{2} \)
   b. undefined

29.
   a. \( m = \frac{3}{2} \)
   b. \( y = \frac{3}{2}x - 4 \)

30.

31.
   a. Dependent (infinitely many solutions)
   b. \( (5, -7) \)
   c. Inconsistent (no solution)

32. 14 students work full time

33. 87.5%

34. $1850

35. $548.39

36. −31, −29, and −27

37. 22.5, 90, and 67.5 degrees

38. width: 7 feet
   length: 25 feet

39. 26 and 64 degrees

40. 70 student tickets and 130 adult tickets
41. 50 oz. of Clear Shine (12%) and 40 oz. of Sunstream (30%)

42. 135 pounds of peanuts and 345 pounds of almonds

43. length: 9 feet
   width: \( \frac{10}{3} \) or \( 3 \frac{1}{3} \) feet

44. width: 5 feet
   length: 12 feet

45. hypotenuse: 13 m
   other leg: 5 m

46. 47 and 133 degrees

47. 72 defective light bulbs

48. a. \( x > 4 \) or \( \{ x \mid x > 4 \} \)

   b. \( w \leq - \frac{5}{4} \) or \( \{ w \mid w \leq - \frac{5}{4} \} \)

   c. \( y \geq 2 \) or \( \{ y \mid y \geq 2 \} \)

49. a. 0
   b. 5
   c. \(-2, 5\)

50. a. \( \frac{6}{5x^2} \)
   b. \(-1\)
   c. \( \frac{a - 4}{2a - 6} \) or \( \frac{a - 4}{2(a - 3)} \)

51. a. \( \frac{8a - 3}{4} \)
   b. \( \frac{5}{y + 5} \)
   c. \( \frac{x + 4}{4x} \)
   d. \( a - 5 \)
   e. \( \frac{17x + 2}{3x(x + 1)} \) or \( \frac{17x + 2}{3x^2 + 3x} \)
   f. \( \frac{2}{15x^3 y^6} \)
   g. \( \frac{5a - 10}{2a - 2} \) or \( \frac{5(a - 2)}{2(a - 1)} \)
   h. \( \frac{x + 4}{x + 2} \)
   i. \( \frac{4y^2 - 1}{y - 5} \) or \( \frac{(2y + 1)(2y - 1)}{y - 5} \)