MATD 0390 INTERMEDIATE ALGEBRA

Test 3 covers all cumulative material, including new sections 4.4-4.8, 5.1-5.4, 5.6-5.7, and 6.1-6.2. Bring a non-graphing calculator and something to write with and erase with. Be prepared to show your work on all test problems in order to receive full or partial credit.

1. For the line given by the equation \( x - 3y = 6 \)
   a. Find the x- and y-intercepts.
   b. Find the slope of the line.

2. Find a linear function with slope \(-\frac{3}{4}\), passing through the point \((-4, -2)\).

3. \( y \) varies directly as the square of \( x \). When \( x = 4 \), then \( y \) is 48. Find \( y \) when \( x = 16 \).

4. \( y \) varies inversely with \( x \). When \( x = 4 \), then \( y \) is 5. Find the variation constant, \( k \).

5. \( y \) varies directly with \( x \). When \( x = 16 \), then \( y = 20 \). Find \( y \) when \( x = 36 \).

6. Hooke’s law states that the distance \( D \) a spring stretches varies directly with the weight \( W \) attached to the spring. When a weight of 40 pounds is attached, the spring stretches 5 inches.
   a. Express the distance that a spring stretches as a function of the weight attached.
   b. Find the distance that the spring stretches from a 65 pound weight attached.

7. Ohm’s law states that the amount of current that flows through a wire varies inversely as the resistance. If a current of 9 amps flows through a wire with a resistance of 2 ohms, then what current flows through a wire with a resistance of 3 ohms?

8. Factor completely. If the polynomial cannot be factored, state that it is prime.
   a. \(-6a^3b + 3a^3b^2\)
   b. \(2x^3 + x^2 - 18x - 9\)
   c. \(9x^2 - 25y^2\)
   d. \(3x^3 - 12x\)
   e. \(9x^2 + 16\)
   f. \(27s^3 + 64t^3\)
   g. \(8 - 125y^3\)
   h. \(2x^3 - 12x^2 + 16x\)
   i. \(6x^2 + 43xy - 40y^2\)
   j. \(8x^2 + 20x - 12\)
   k. \(15x^2 + 8x - 12\)
   l. \(49x^2 - 84x + 36\)
   a. \( x^2 - 3x - 28 = 0 \)
   b. \( 2x^2 = 6x \)
   c. \( 3x(x + 2) = 20 - x \)
   d. \( x(2x + 5) = 3 \)
   e. \( 9x^2 = 16 \)
   f. \( x(x - 3) = 4x^2 - 30x + 60 \)

10. For the function \( P(x) = 10x^2 - 17x \), solve \( P(x) = -3 \).

11. A rectangular garden has an area of 65 square feet. Its length is 3 feet less than twice its width. What are the dimensions of the garden?

12. The base of a triangular sail is seven meters less than the height. If the area is 30 square meters, what are the base and height?

13. The outside border of a picture frame has a length of 24 inches and a height of 18 inches. Inside the frame, an area of 216 square inches of picture is visible. The frame is the same width all the way around the picture. Find the width of the frame.

14. The height \( h(t) \), in feet, of a penny \( t \) seconds after it is dropped from the top of a building is given by \( h(t) = 100 - 16t^2 \). How much time will pass before it hits the ground? (Hint: The height at the ground is zero.)

15. Simplify.
   a. \( \frac{24x^2y^9}{36x^5y} \)
   b. \( \frac{y - 5}{10 - 2y} \)
   c. \( \frac{2a^2 - 5a - 3}{2a^2 - 12a + 18} \)
   d. \( \frac{8x - 8}{6x^2 - 6} \)

16. Multiply or divide, as indicated, and simplify your answer.
   a. \( \frac{x^2}{9x^2 - 1} \cdot \frac{9x^2 + 6x + 1}{3x^2 - 5x} \)
   b. \( \frac{3a^2c}{a + 1} \cdot \frac{a^2 + a}{27c^5} \)
   c. \( \frac{9p^2 - 16q^2}{9p + 6q} \div \frac{6p + 8q}{6p + 3q} \)
17. Simplify.
   \[
   \frac{x^2}{x+1} \quad \frac{x^2 - 2x}{x^2 + x} \\
   a. \quad \frac{x}{2} \\
   b. \quad \frac{x}{x} \\
   \]

   \[
   \frac{x + 4}{x - 2} - \frac{x}{x + 2} \\
   1 - \frac{x - 4}{x + 2} \\
   a. \quad \frac{1}{x} - \frac{3}{x} \\
   b. \quad \frac{1}{y} + \frac{2}{x} \\
   \]

19. Add or subtract, as indicated, and simplify your answer.
   a. \[
   \frac{x}{x^2 - 1} + \frac{1}{x^2 - 1} \\
   b. \quad \frac{4}{x - 2} + \frac{3x - 2}{2 - x} \\
   c. \quad \frac{5}{x - 5} - \frac{3}{x} \\
   d. \quad \frac{4}{x^2 - 1} - \frac{3}{x^2 + x} \\
   \]

20. For the function \( f(x) = \frac{2}{x - 1} \)
   a. What is the domain?
   b. Find all values \( a \) such that \( f(a) = -3 \)

21. Solve.
   a. \[
   \frac{3}{x} = \frac{5}{x - 2} \\
   \]
   b. \[
   \frac{x}{x - 1} = \frac{5}{x + 1} + 1 \\
   \]

22. Solve for the indicated variable.
   a. \( z: \quad 2x - 3z = y \)
   b. \( y: \quad x = \frac{3}{z - 2y} \)
   c. \( R: \quad \frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} \)
   d. \( s_1: \quad v = \frac{s_2 - s_1}{t} \)
23. Simplify. Use absolute value notation where appropriate.
   a. \(-\sqrt{\frac{81}{25}}\)  
   b. \(\sqrt{(x - 3)^2}\)  
   c. \(\frac{3\sqrt[3]{-8y^3}}{}\)  
   d. \(\sqrt[5]{-32}\)

24. Evaluate.
   a. \(\left(\frac{1}{8}\right)^{-2/3}\)  
   b. \(16^{3/4}\)

25. Simplify, and express your answer using positive rational exponents, if appropriate.
   a. \(\frac{x^{1/8}}{x^{-3/8}}\)  
   b. \(\left(h^{2/5} \cdot h^{-1/2}\right)^3\)  
   c. \((81c)^{3/4}\)  
   d. \((-125a^6)^{1/3}\)

For problems 26 – 31, you must define variables, and set up and solve equations. Show your work.

26. Robert can paint a room in 12 hours. Julio can paint the same room in 8 hours. How long will it take them to paint the room together?

27. An airplane travels at a rate of 300 miles per hour in still air. One day it travels 1200 miles against the wind in the same amount of time it takes to travel 1800 miles with the wind. What is the wind speed?

28. The current of a river flows at a rate of 8 miles per hour. A rowboat travels 16 miles downstream in the same amount of time it takes to travel 6 miles upstream. What is the still water speed of the rowboat?

29. John rides his bicycle at a speed 4 miles per hour slower than his friend Alice. If Alice rides 40 miles in the same time that John rides 24 miles, how fast does each of them ride?

30. A man who is 10 feet tall casts a shadow that is 4 feet long. At the same time and place, a sign casts a shadow that is 14 feet long. How tall is the sign?

31. A pool can be emptied in 6 hours with both a strong pump and a weak pump working together. The weak pump alone requires 5 hours more than the strong pump alone to empty the pool. How long does it take each pump alone to empty the pool?

32. The triangles below are similar. Find the length of the sides marked \(x\) and \(y\).

![Diagram of similar triangles with sides labeled 18, 48, x, y, 30, 24]
ANSWERS:

1. x-int = (6, 0)  
y-int = (0, -2)  
b. slope = $\frac{1}{3}$

2. $f(x) = -\frac{3}{4}x - 5$

3. $y = 768$

4. $k = 20$

5. $y = 45$

6. a. $D = \frac{1}{8}W$ or $D = 0.125W$  
b. $8\frac{1}{8}$ or $8.125$ inches

7. 6 amps

8. a. $-3a^3b(2a - b)$  
b. $(x + 3)(x - 3)(2x + 1)$  
c. $(3x + 5y)(3x - 5y)$  
d. $3x(x + 2)(x - 2)$  
e. prime (sum of squares)  
f. $(3s + 4t)(9s^2 - 12st + 16t^2)$  
g. $(2 - 5y)(4 + 10y + 25y^2)$  
h. $2x(x - 4)(x - 2)$  
i. $(6x - 5y)(x + 8y)$  
j. $4(2x - 1)(x + 3)$  
k. $(3x - 2)(5x + 6)$  
l. $(7x - 6)^2$

9. a. $\{-4, 7\}$  
b. $\{0, 3\}$  
c. $\{-4, \frac{5}{3}\}$  
d. $\{\frac{1}{2}, -3\}$  
e. $\{-\frac{4}{3}, \frac{4}{3}\}$  
f. $\{4, 5\}$

10. $\left\{\frac{1}{5}, \frac{3}{2}\right\}$

11. width = 6.5 feet  
    length = 10 feet

12. base = 5 meters  
    height = 12 meters

13. 3 inches

14. 2.5 seconds  
    (At the ground, height is zero. 
    Equation: $0 = 100 - 16t^2$)

15. a. $\frac{2y^8}{3x^3}$  
b. $-\frac{1}{2}$  
c. $\frac{2a + 1}{2(a - 3)}$  
d. $\frac{4}{3(x + 1)}$

16. a. $\frac{x(3x + 1)}{(3x - 1)(3x - 5)}$  
b. $\frac{a^3}{9c^4}$  
c. $\frac{(3p - 4q)(2p + q)}{2(3p + 2q)}$
17. a. \( \frac{2x}{(x+1)^2} \)  
   b. \( \frac{1}{x(x+2)^2} \) 

18. a. \( \frac{4(x+1)}{3(x-2)} \)  
   b. \( \frac{y-3x}{y+2x} \) 

19. a. \( \frac{1}{x-1} \)  
   b. -3  
   c. \( \frac{2x+15}{x(x-5)} \)  
   d. \( \frac{x+3}{x(x+1)(x-1)} \) 

20. a. \( \{x\mid x \neq 1\} \)  
   b. \( \left\{ \frac{1}{3} \right\} \) 

21. a. \( \{-3\} \)  
   b. \( \left\{ \frac{3}{2} \right\} \)  
   c. \( \{-6\} \) 

22. a. \( z = \frac{2x-y}{3} \)  
   b. \( y = \frac{xz-3}{2x} \)  
   c. \( R = \frac{r_1r_2}{r_1 + r_2} \)  
   d. \( s_1 = s_2 - vt \) 

23. a. \( -\frac{9}{5} \)  
   b. \( |x-3| \)  
   c. \( -2y \)  
   d. \(-2\) 

24. a. 4  
   b. 8 

25. a. \( x^{1/2} \)  
   b. \( \frac{1}{h^{3/10}} \)  
   c. \( 27c^{3/4} \)  
   d. \(-5a^2\) 

26. \( 4\frac{4}{5} \) hours 

27. 60 miles per hour 

28. 17.6 miles per hour 

29. John rides 6 miles per hour, and Alice rides 10 miles per hour. 

30. 35 feet tall 

31. The strong pump alone can empty the pool in 10 hours, and the weak pump alone can empty the pool in 15 hours. 

32. \( x = 38.4 \) and \( y = 11.25 \)