MATD 0390 INTERMEDIATE ALGEBRA Review for Test 4

Test 4 covers all cumulative material, including new sections 6.3 - 6.8, 7.1, 7.2, 7.4, and 7.5.
Bring a non-graphing calculator and something to write with and erase with.
Be prepared to show your work in order to receive full or partial credit on all test problems.

1. Simplify. Assume all variables represent positive numbers.

   a. $\sqrt{81a^6}$

   b. $\dfrac{\sqrt{500wx^9y^{10}}}{\sqrt{10wx^3y}}$

   c. $\sqrt[3]{-24a^7b^2c^{18}}$

   d. $\sqrt[3]{\dfrac{27x^{10}z^5}{8y^6}}$

   e. $\sqrt[3]{\dfrac{48x^{-5}}{y^2}}$

2. Multiply and simplify. Assume all variables represent positive numbers.

   a. $\sqrt{-10}\sqrt{-30}$

   b. $\sqrt[3]{5x^2}\ \sqrt[3]{50x^4}$

   c. $\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right)$

   d. $\left(\sqrt{7}-\sqrt{2x}\right)^2$

3. Simplify. $14\sqrt{5}-3\sqrt{2}-7\sqrt{45}$

4. Rationalize the denominator Assume all variables represent positive numbers.

   a. $\dfrac{\sqrt{3}}{\sqrt{5}}$

   b. $\dfrac{7}{\sqrt{n}}$

   c. $\sqrt{\dfrac{3z}{x}}$

   d. $\dfrac{2}{7-\sqrt{5}}$

   e. $\dfrac{\sqrt{5}+3}{\sqrt{5}-3}$

   f. $\dfrac{\sqrt{2x}}{\sqrt{x}+\sqrt{y}}$

5. Simplify. Express your answer in the form $a+bi$.

   a. $\left(5-3i\right)-\left(-2+5i\right)$

   b. $\left(4+\sqrt{-9}\right)\left(2-\sqrt{-81}\right)$

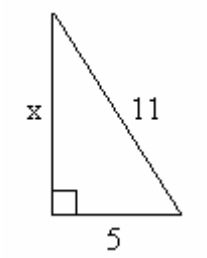
   c. $\left(5+i\right)\left(5-i\right)$

   d. $\dfrac{3-5i}{2i}$

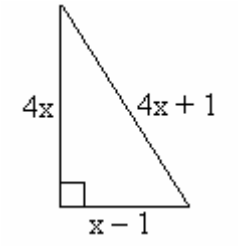
   e. $\dfrac{10-15i}{1+2i}$

6. Find x and determine the length of the missing side(s). Give exact value in simplified form and decimal approximation to three decimal places, if appropriate.

a.



b.



7. Simplify.

a. $\sqrt{-49}$ b. $-\sqrt{-20}$ c. i^{67} d. i^{80}

8. For $\dfrac{2 \pm 3\sqrt{15}}{5}$, find decimal approximations to three decimal places.

9. Solve by **completing the square**. Give an exact value and a decimal approximation to three places, if appropriate. ***You must show your work using this method.***

a. $x^2 + 2x = 3$ b. $x^2 - 6x = 3$

10. Find the x-intercept(s), if any, and y-intercept of the function.

a. $f(x) = 2x^2 - 7x + 5$ b. $g(x) = x^2 - 5x$ c. $h(x) = 4x^2 - 25$

11. Solve by any appropriate method. Give exact values in simplified form only.

a. $2x^2 + 5 = 6x$

b. $5x(x-2) = 1$

c. $(x-5)^2 = -16$

d. $2(x+1)(x-1) = -3x$

e. $(x+7)^2 = 12$

f. $\sqrt{4x+33} + 2 = 7$

g. $\sqrt{x-1} = x - 7$

h. $\sqrt[3]{x+8} = -5$

12. Find the discriminant for the quadratic equation. Then, without solving the equation, determine whether the equation has two unequal rational solutions, two unequal irrational solutions, one repeated real solution, or two complex solutions that are not real.

a. $3x^2 + 6x + 5 = 0$ c. $25x^2 - 10x + 1 = 0$

b. $3x^2 + 6x - 5 = 0$ d. $3x^2 - 13x - 10 = 0$

13. Use transformations and the graph of $y = x^2$ to graph each function.

a. $f(x) = x^2 + 3$

b. $f(x) = (x-2)^2$

c. $f(x) = -2x^2$

d. $f(x) = (x+2)^2 - 3$

14. For the function $f(x) = 2(x+1)^2 - 3$

a. Use transformations and the graph of $y = x^2$ to graph the function.

b. From your graph, determine the vertex, axis of symmetry, and maximum or minimum of the function.

15. For the function $f(x) = -\frac{1}{2}x^2 - 1$

a. Determine whether the parabola opens up or down.

b. Find the vertex and the axis of symmetry.

c. Find the y-intercept.

d. Find the x-intercept(s), if any.

e. Find the maximum or minimum value. State whether it is a maximum or minimum.

f. Graph the function.

16. For the function $f(x) = -x^2 + 6x - 4$

a. Determine whether the parabola opens up or down.

b. Find the vertex and the axis of symmetry.

c. Find the y-intercept.

d. Find the x-intercept(s), if any.

e. Find the maximum or minimum value. State whether it is a maximum or minimum.

f. Graph the function.

17. Multiply and simplify. Express your solution in **radical notation**.

$\sqrt{3} \bullet \sqrt[3]{5}$

18. For the line described by the equation $5x - y = 10$

    a. What is the x-intercept?

    b. What is the y-intercept?

    c. What is the slope?

    d. Graph the line.

19. Solve. Express your solution in **interval notation**. $-1 < 3 - 2x \le 9$

20. For the function $g(x) = \sqrt[5]{7x+10}$

    a. Evaluate $g(-6)$

    b. What point is on the graph of g?

    c. Find the domain

21. Find $f(3)$ and find the domain.

    a. $f(x) = \sqrt{4x-8}$

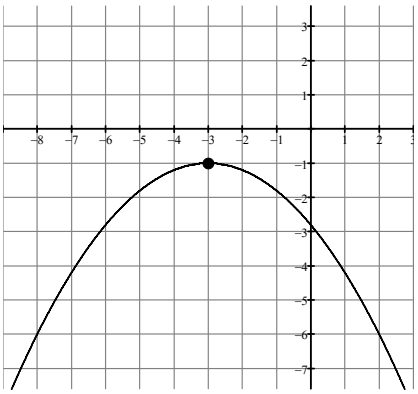
    b. $f(x) = \dfrac{x+1}{x-5}$

22. The profit, in dollars, from sales of grand pianos for a manufacturer is given by the function $P(x) = -1.5x^2 + 5100x - 60000$, where x is the number of pianos produced and sold in a given month.

    a. How many pianos should the manufacturer produce to maximize profit?

    b. What is the maximum profit attainable?

23. Choose the appropriate formula for the quadratic function whose graph is drawn below.



    a. $f(x) = -5(x-3)^2 - 1$

    b. $f(x) = \dfrac{1}{5}(x+3)^2 - 1$

    c. $f(x) = -\dfrac{1}{5}(x-3)^2 - 1$

    d. $f(x) = -\dfrac{1}{5}(x+3)^2 - 1$

24. Harvey wants to create 20 oz of iodine solution of strength 15%. He has on hand some iodine solution of strengths 18% and 10%. How much of each should she use?

25. Keshia wants an extra $1938 per year in income. She has $24,000 to invest and wants to split the investment between an A-rated bond that pays 6.5% per year and a B-rated bond that pays 10% per year. How much should she invest in each type of bond?

26. Solve for the indicated variable.

a. r: $V = \frac{1}{3}\pi r^2 h$ (volume of a cone)

b. t: $d = \frac{at^2}{2} + s$ (distance traveled in a straight line)

c. g: $T = 2\pi\sqrt{\frac{\ell}{g}}$ (a pendulum formula)

For problems 27 – 33, you must define variables, and set up and solve equations. Show your work.

27. The still water speed of a motorboat is 15 miles per hour. The motorboat travels upstream, to a point 8 miles away, and then back to its starting point. The round trip takes 1.2 hours. What is the speed of the current?

28. The current of a river flows at a rate of 5 miles per hour. A rowboat travels 16 miles downstream in the same amount of time it takes to travel 6 miles upstream, what is the still water speed of the rowboat?

29. Joe rides his bicycle 40 miles to a vista point at the top of a hill. His average biking speed up the hill is 12 miles per hour slower than his average biking speed down the hill. The total time for his round trip is 7 hours. Find Joe's average speed in each direction.

30. Mia beats the morning rush hour on her way to work and drives back home from work during the evening rush hour. In the same time that she travels the 20 miles to work, she can travel only 16 miles of the trip back home. Her average speed coming home is 7 miles per hour slower than her average speed going to work. What is Mia's average speed each way?

31. A rectangular garden has **area** 90 square feet. Its length is 3 feet less than twice its width. What are the dimensions (length and width)?

32. The height *s*, in feet, of a toy rocket after *t* seconds when fired straight up with an initial speed of 160 feet per second from an initial height of 56 feet can be modeled by the function $s(t) = -16t^2 + 160t + 56$. When will the height of the rocket be 200 feet?

33. Lucas needs to wash the windows on his house. He has a 15-foot ladder and places the base of the ladder 9 feet from the wall of the house. How far up the wall will the ladder reach?

ANSWERS:

1.
   a. $9a^3$
   b. $5x^3y^4\sqrt{2y}$
   c. $-2a^2c^6\sqrt[3]{3ab^2}$
   d. $\dfrac{3x^3z\sqrt[3]{xz^2}}{2y^2}$
   e. $\dfrac{2}{x}\sqrt[3]{\dfrac{6}{x^2y^2}}$

2.
   a. $i\sqrt{10}\bullet i\sqrt{30}=i^2\sqrt{300}=-10\sqrt{3}$
   b. $5x^2\sqrt[3]{2}$
   c. 4
   d. $7-2\sqrt{14x}+2x$

3. $-7\sqrt{5}-3\sqrt{2}$

4.
   a. $\dfrac{\sqrt{15}}{5}$
   b. $\dfrac{7\sqrt{n}}{n}$
   c. $\dfrac{\sqrt{3xz}}{x}$
   d. $\dfrac{7+\sqrt{5}}{22}$
   e. $\dfrac{7+3\sqrt{5}}{2}$
   f. $\dfrac{x\sqrt{2}-\sqrt{2xy}}{x-y}$

5.
   a. $7-8i$
   b. $35-30i$
   c. 26
   d. $-\dfrac{5}{2}-\dfrac{3}{2}i$
   e. $-4-7i$

6.
   a. x = $4\sqrt{6}\approx 9.798$ units
   b. x = 10; Sides are 9, 40, & 41 units.

7.
   a. $7i$
   b. $-2i\sqrt{5}$ or $-2\sqrt{5}\,i$
   c. $-i$
   d. 1

8. $\dfrac{2+3\sqrt{15}}{5}\approx 2.724$

   $\dfrac{2-3\sqrt{15}}{5}\approx -1.924$

9.
   a.
$$\begin{cases} x^2+2x+\left(\frac{2}{2}\right)^2=3+\left(\frac{2}{2}\right)^2 \\ (x+1)^2=4 \\ \sqrt{(x+1)^2}=\pm\sqrt{4} \\ x+1=\pm 2 \\ x=-1\pm 2 \\ x=1,\ \ x=-3 \end{cases}$$
   b.
$$\begin{cases} x^2-6x+\left(\frac{-6}{2}\right)^2=3+\left(\frac{-6}{2}\right)^2 \\ \text{(See 9}a\text{ above for steps.)} \\ x=3+2\sqrt{3}\approx 6.464 \\ x=3-2\sqrt{3}\approx -0.464 \end{cases}$$

10. a. x-int: $(1,0),\left(\dfrac{5}{2},0\right)$ y-int: $(0,5)$
    b. x-int: $(0,0),(5,0)$ y-int: $(0,0)$
    c. x: $\left(-\dfrac{5}{2},0\right),\left(\dfrac{5}{2},0\right)$ y: $(0,-25)$

11.

a. $\left\{\frac{3}{2}+\frac{1}{2}i,\ \frac{3}{2}-\frac{1}{2}i\right\}$

b. $\left\{\frac{5+\sqrt{30}}{5}, \frac{5-\sqrt{30}}{5}\right\}$

c. $\{5+4i,\ 5-4i\}$

d. $\left\{-2, \frac{1}{2}\right\}$

e. $\{-7+2\sqrt{3},\ -7-2\sqrt{3}\}$

f. $\{-2\}$

g. $\{10\}$ [Note: 5 is extraneous.]

h. $\{-133\}$

12. [Refer to Example 5 in 7.2 on page 566.]

a. Discriminant = -24 (negative)
Two complex solutions that are not real

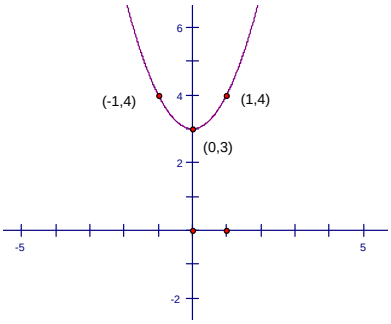
b. Discriminant = 96 (not a perfect square)
Two unequal irrational solutions

c. Discriminant = 0 (zero)
One repeated real solution

d. Discriminant = $289 = 17^2$ (perfect square)
Two unequal rational solutions

13.

a.



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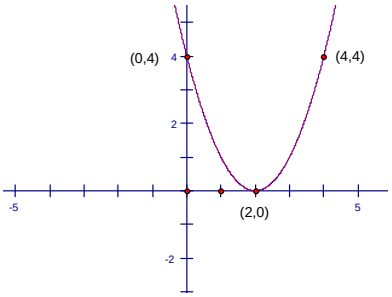
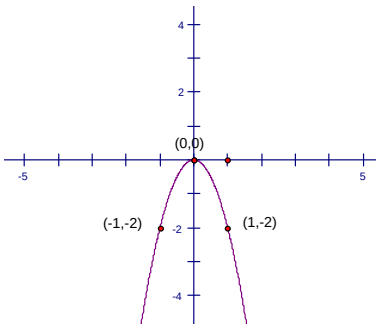
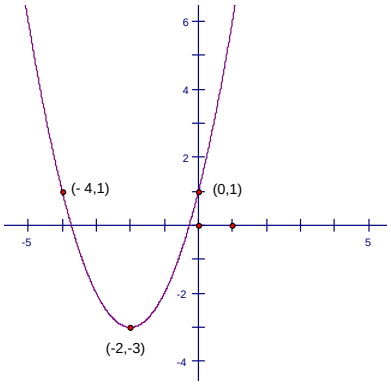


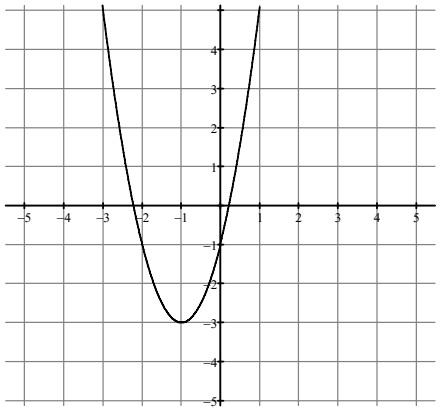
c.



d.



14. a.



b. vertex: $(-1, -3)$
axis of symmetry: $x = -1$
minimum $= -3$

15.

a. down

b. vertex: $(0,-1)$

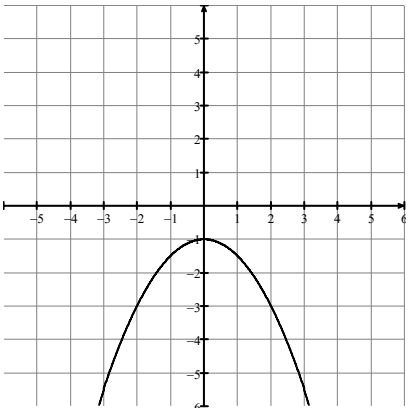
axis of symmetry: $x=0$ (y-axis)

c. y-intercept: $(0,-1)$

d. x-intercept(s): none

e. maximum $=-1$

f.



16.

a. down

b. vertex: $(3,5)$

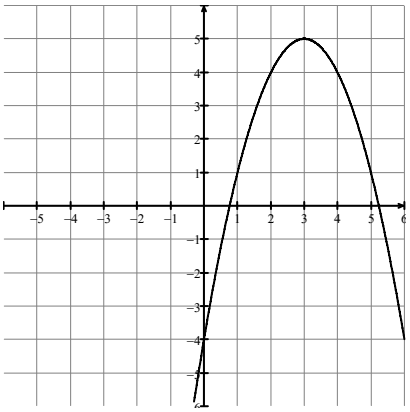
axis of symmetry: $x=3$

c. y-intercept: $(0,-4)$

d. x-intercepts: $\left(3+\sqrt{5}\approx 5.236,\,0\right)$, $\left(3-\sqrt{5}\approx 0.764,\,0\right)$

e. maximum $=5$

f.



17. $3^{1/2}\bullet 5^{1/3}=3^{3/6}\bullet 5^{2/6}=\left(3^3\bullet 5^2\right)^{1/6}$

$=\left(27\bullet 25\right)^{1/6}=\left(675\right)^{1/6}=\sqrt[6]{675}$

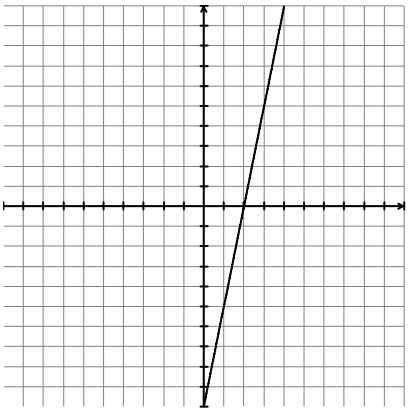
18.

a. $(2,0)$

b. $(0,-10)$

c. 5

d.



19. $[-3,2)$

20.

a. $g(-6)=-2$

b. $(-6,-2)$

c. $\mathbb{R}$ (because index is odd)

21.

a. $f(3)=2;\ [2,+\infty)$

b. $f(3)=-2;\ \{x\,|\,x\neq 5\}$

22. a. 1700 pianos (Use $x=\dfrac{-b}{2a}$)

b. $P(1700)=\$4{,}275{,}000$

23. d

24. Equations: $\begin{cases} x+y=20 \\ .18x+.10y=(20)(.15) \end{cases}$

12.5 oz of strength 18%, and
7.5 oz of strength 10%

25. Equations: $\begin{cases} A+B=24000 \\ 0.065A+0.10B=1938 \end{cases}$

\$13,200 in A-rated bonds at 6.5%, and
\$10,800 in B-rated bonds at 10%

26.

a. $r = \sqrt{\frac{3V}{\pi h}}$

b. $t = \sqrt{\frac{2(d-s)}{a}}$

c. $g = \frac{4\pi^2 \ell}{T^2}$

27. Equation: $\frac{8}{15+c} + \frac{8}{15-c} = 1.2$

The speed of the current is 5 mph.

28. Equation: $\frac{16}{s+5} = \frac{6}{s-5}$

The still water speed is 11 miles per hour.

29. Equation: $\frac{40}{x} + \frac{40}{x-12} = 7$

Joe's average speed is 8 mph uphill and 20 mph downhill.

30. Equation: $\frac{20}{x} = \frac{16}{x-7}$

Mia's average speed is 35 mph going to work and 28 mph going back.

31. $L = 2w - 3$

Equation: $w(2w-3) = 90$

Width = 7.5 feet, and
Length = 12 feet

32. Equation: $200 = -16t^2 + 160t + 56$

The rocket will reach a height of 200 feet after 1 second and again after 9 seconds.

33. Equation: $h^2 + 9^2 = 15^2$

The ladder will reach a height of 12 feet.