

Review for Intermediate Algebra (MATD 0390) Final Exam

Students are expected to know all relevant formulas, including:

- All special factoring formulas
- All formulas for linear equations (slope formula, slope-intercept and point-slope forms)
- Quadratic formula
- Distance between two points
- Midpoint between two points
- Equation of a circle
- Both forms of equations of parabolas
- All formulas needed to solve applications in this review, such as distance, and direct and inverse variation

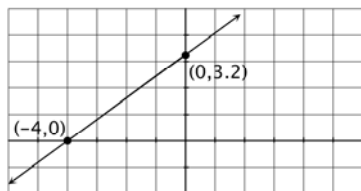
In order to be prepared for the final exam, students should be able to do all of the following problems and related problems as well.

Practice Problems

Simplify. All exponents should be positive integers.

1. $(7x^3y^{-1})(5x^{-1}y^{-4})$ 2. $(4x^2y^{-1})^2$ 3. $\frac{18a^{-7}b^4c^2}{24a^{-1}b^4c^{-3}}$

4. Find the slope, x-intercept, and y-intercept. Then graph the line with equation $-5x + 3y = 15$
5. Find the slope and y-intercept, and use them to graph the line with equation $-2x - 3y = 3$
6. Graph and give the slope for each line:
a. $y = -2$ b. $2x - 8 = 0$
7. Write the slope-intercept equation for the line containing the points $(4, -6)$ and $(-8, 3)$.
8. Identify the slope and y-intercept for this line. Then give the equation in slope-intercept form.



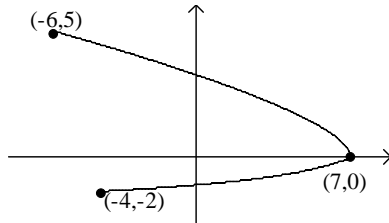
9. Write the equation (in slope-intercept form) for the line through the point $(-3, 5)$ perpendicular to $2y + 3x = 4$.
10. Write the equation (in slope-intercept form) for the line through the point $(2, -5)$ parallel to the line through the points $(1, 2)$ and $(-3, -1)$.
11. Determine without graphing whether the pair of lines is parallel or perpendicular, or neither: $5y + 7 = 8x$ and $-8x + 5y = -15$

12. Determine if the following relations represent functions or not. Give the domain and range of each.

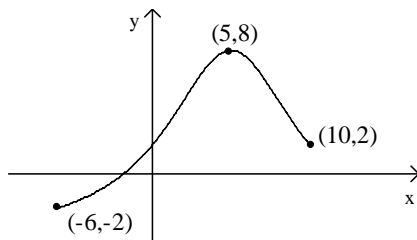
a. $\{(4, 1), (5, 2), (6, 3)\}$

b. $\{(3, 5), (4, 6), (3, 7), (2, 0)\}$

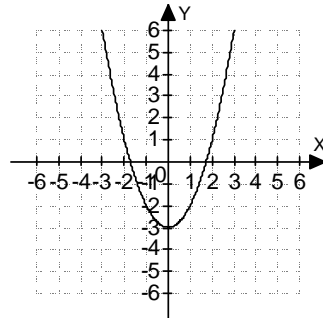
c.



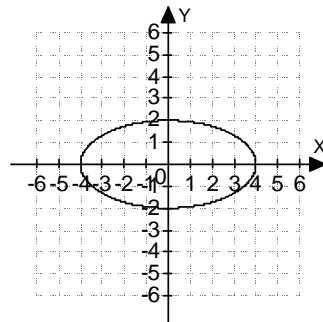
d.



e.



f.



13. For $f(x) = 2x^2 - 3$ and $g(x) = 7x - x^2$, find the following.

a. $f(-2)$

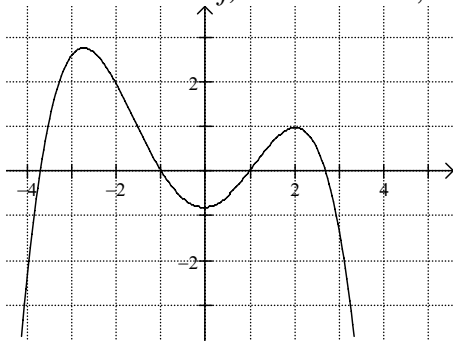
b. $(f \cdot g)(a)$

c. $(f + g)(3)$

d. $\left(\frac{f}{g}\right)(x)$

e. The domain of $\frac{f}{g}$

14. For the function f , shown below, find the following.



a. $f(1)$

b. $f(-2)$

c. For what value(s) of x is $f(x) = 1$

d. Which value is larger: $f(3)$ or $f(-3)$?

15. Give the slope and y intercept, and graph the linear function: $f(x) = -\frac{4}{5}x + 2$.

16. Suppose 8.2 million pounds of coffee are sold when the price is \$6 per pound and 10.6 million pounds are sold at \$5 per pound.
- Find a linear function $D(x)$ that expresses the consumer demand (amount of coffee sold) as a function of the price per pound x .
 - Use this function to predict the consumer demand if the price is \$9 per pound.
 - At what price can you expect to sell no coffee at all? Round your answer to the nearest cent.
17. At a particular college, 33 student loans were distributed in 1980 and 63 loans were given during 1990. During this ten-year period, the number of loans disbursed each year followed a linear pattern.
- Write a linear equation that expresses the annual number of loans given (y) as a function of the number of years (x) since 1980. (That is, $x = 0$ corresponds to 1980, $x = 1$ corresponds to 1981, etc.)
 - Use the equation from part (a) to find the number of loans given in 1987.
18. Maria's wages are \$2400 plus 3.5% commission on monthly sales.
- Write a function expressing the relationship between Maria's wages, w , and her monthly sales, x .
 - Find her sales for the month if her wages for the month are \$3380.
19. Solve the following systems of linear equations in two variables:
- $$\begin{cases} x - 2y = 7 \\ 2x + 5y = 5 \end{cases}$$
 - $$\begin{cases} 4x + 3y = -1 \\ 5x - 2y = 16 \end{cases}$$
 - $$\begin{cases} 2x + 7y = 65 \\ 21y + 6x = 19 \end{cases}$$
20. The sum of two numbers is 95. One of the numbers is 17 more than the other. What are the numbers?
21. A canoe and a motor boat travel toward each other from a distance of 70 miles. The motor boat moves 12 miles per hour faster than the canoe. The two boats pass after 2.5 hours. How fast does each boat move?
22. A cross-country flight traveling with the wind covers 3000 miles in 5 hours. The same plane then changes direction and travels to Texas, against the wind, covering 1500 miles in 3 hours. What are the wind speed, and the speed of the plane in still air?
23. A lab contains 24 ounces of iodine solution of strength 15%. How much pure iodine would you mix with it to get an iodine solution of strength 25%?
24. An investment portfolio of stocks and bonds, with an initial investment of \$120,000, gained \$30,800 over its life. If the stocks gained 32% and the bonds gained 16%, how much was invested in each?

Solve the linear inequalities. Graph the solution and give the answer in interval notation.

25. $3(x - 5) + 2x \geq 2x + 6$ 27. $3(x - 8) < 21$ and $5x + 1 > -14$
26. $-3 \leq -2(x + 0.5) < 4$ 28. $2x + 3 > 19$ or $-3x + 9 \geq 30$

Divide the polynomials:

29.
$$\frac{x^2 - 7x - 6}{x - 4}$$
30.
$$\frac{8x^3 + 8x + 11}{2x + 1}$$

31. Solve the equation for x and simplify, if possible: $|3x - 1| = 8$
32. Graph the solution to the inequality on an xy -coordinate plane. $4x + 3y \geq 24$
33. Graph the solution to each system of inequalities on an xy -coordinate plane.
- a.
$$\begin{cases} 2x + y \leq 4 \\ y \geq 0 \end{cases}$$
- b.
$$\begin{cases} x + y > 4 \\ y \leq 2x - 4 \end{cases}$$

Factor completely, if possible:

34. $16x^2 - 49y^2$
35. $x^2 + y^2$
36. $x^3 + y^3$
37. $x^2 - 2xy + y^2$
38. $49x^2 + 112x + 64$
39. $2x^3y - 4x^2y - 6xy$
40. $81 - y^4$
41. $2x^2 + xy - 6y^2$
42. $12x^2 + 52x - 9$
43. $ab^2 + cb^2 - 4a - 4c$
44. $5r^3 - 40$

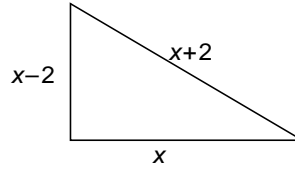
Solve the equations for x and simplify, if possible.

45. $6x^2 = 24$
46. $10x^2 + 5x = 0$
47. $(x - 2)(x + 3) = -4$
48. Find the domain of the following.
- a. $f(x) = \frac{x - 3}{x^2 - 3x}$
- b. $g(x) = x^2 + x$

Perform the indicated operation and simplify (reduce), if possible.

49. $\frac{3x^2 + 3xy}{10x - 20} \cdot \frac{5x^2 - 20}{x^2 + 2xy + y^2}$
50. $\frac{y + 2}{3 - y} \div \frac{y^3 + 8}{3y^2 - 27}$
51. $\frac{x^2 - 2}{x - 3} + \frac{x + 4}{3 - x}$
52. $\frac{4x + 3}{x^2 + 6x + 8} + \frac{3x}{x + 2}$
53. $\frac{5y - 4}{4y - 3} - \frac{2y}{4y + 3}$
54. $1 + \frac{1}{x}$
55. $\frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{x} - \frac{1}{y}}$
56. $\frac{\frac{4x}{3} - 2}{5 + \frac{y}{x}}$

57. Find the three sides of this right triangle.
Hint: Use the Pythagorean theorem:
 $a^2 + b^2 = c^2$.



58. The length of a rectangle is 5 cm less than twice the width. The area is 75 square cm. Find the length and the width.

Solve the equations for x and simplify, if possible.

59. $\frac{6}{2x+5} = \frac{4}{x-9}$

61. $\frac{1}{x-5} - \frac{2}{x} = \frac{5}{x^2-5x}$

60. $\frac{x+2}{x^2-5x-24} + \frac{4}{x-8} = \frac{2}{x+3}$

62. $x - \frac{5}{x} = \frac{23}{x}$

63. Carl, an experienced shipping clerk, can fill a certain order in 6 hours. Tim, a new clerk, needs 8 hours to complete the same job. How long would it take for both working together to fill the order?

64. Caleb's average driving speed is 12 kilometers per hour slower than Ling's. In the same length of time it takes Caleb to drive 231 km, Ling drives 297 km. What is Caleb's average speed?

65. Solve for R : $I = \frac{nE}{R+nr}$

66. Solve for r : $rL = H(r+k)$

Simplify. Assume all variables are greater than or equal to zero.

67. $(x^6)^{1/3}$

69. $\sqrt{36b^6c^9}$

68. $\sqrt[3]{54x^8y^2z^9}$

70. $2\sqrt{27} - 5\sqrt{300}$

Multiply and simplify. Assume all variables are greater than or equal to zero

71. $(6-\sqrt{3})(2+\sqrt{3})$

72. $\sqrt{6x}(\sqrt{3x}-\sqrt{2})$

Evaluate:

73. $8^{2/3}$

74. $16^{-1/4}$

75. $(-64)^{-1/3}$

76. Solve the equation for x and simplify, if possible: $x-1 = \sqrt{3x+7}$

Perform the indicated operation and express the answer in standard form, $a+bi$:

77. $\sqrt{-25}(3-2i)$

80. $(3-6i) + (2+3i) - i^3$

78. $(2-3i)(-4-5i)$

79. $\frac{2+3i}{1-5i}$

Solve each equation for x and simplify, if possible.

81. $x^2 + 2x = 4$

82. $2x^2 + 16 = 0$

83. $1 - \frac{3}{x} - \frac{7}{x^2} = 0$

84. $3x^2 - 6x + 5 = 0$

85. For each equation, use the discriminant to determine whether the quadratic equation has two unequal rational solutions, two unequal irrational solutions, one repeated real solution, or two complex solutions that are not real.

a. $x^2 + 4x + 6 = 0$

b. $x^2 - 7x + 5 = 0$

c. $6x^2 + 5x - 4 = 0$

86. Find the x -intercepts, if any, and y -intercept of the function.

a. $f(x) = x^2 - 6x + 4$

c. $h(x) = x^2 - 6x + 10$

b. $g(x) = 2x^2 - 5x + 3$

87. Graph the function $f(x) = -(x+1)^2 + 4$. From the graph, give the axis of symmetry and the vertex.

88. Graph the function $f(x) = x^2 - 6x + 4$ and determine whether it has a minimum or maximum value. Then find the minimum or maximum value.

89. In business, the total profit P is the difference between the revenue R and the cost C . For x units produced, $R(x) = 1000x - x^2$ and $C(x) = 3000 + 2x$, where $R(x)$ and $C(x)$ are dollar amounts. Find the following:

a. the total profit function $P(x)$

b. the value of x (number of units) at which the maximum total profit occurs

c. the maximum total profit.

90. The stopping distance d of a car after the brakes have been applied varies directly as the square of the speed r . A car traveling 60 mph can stop in 200 ft.

a. Express stopping distance as a function of speed.

b. What stopping distance corresponds to a speed of 36 mph?

91. The current I in an electrical conductor varies inversely as the resistance R of the conductor. If the current is 0.5 amperes when the resistance is 240 ohms, what is the current when the resistance is 960 ohms?

92. Given the points $A:(2, -3)$ and $B:(4, -2)$, find the distance from A to B and find the midpoint of the line segment joining A and B .

93. Find an equation for the circle with center $(-5, 1)$ and radius 7.

94. Find the center and radius of each circle:

a. $(x+5)^2 + (y-3)^2 = 64$

b. $x^2 + y^2 - 16x + 4y + 5 = 0$

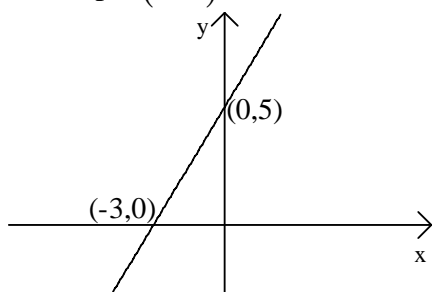
ANSWERS

1. $\frac{35x^2}{y^5}$

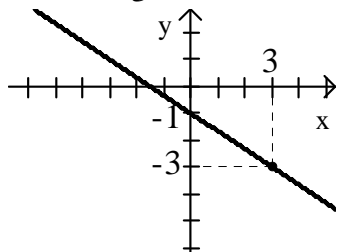
2. $\frac{16x^4}{y^2}$

3. $\frac{3c^5}{4a^6}$

4. Slope: $\frac{5}{3}$; x -intercept: $(-3,0)$; y -intercept: $(0,5)$

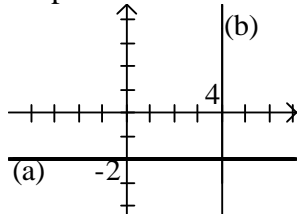


5. Slope: $-\frac{2}{3}$; y -intercept: $(0,-1)$



6. a. Slope: 0

b. Slope: undefined



7. $y = -\frac{3}{4}x - 3$

8. Slope: 0.8; y -intercept: $(0,3.2)$
 $y = 0.8x + 3.2$

9. $y = \frac{2}{3}x + 7$

10. $y = \frac{3}{4}x - \frac{13}{2}$

11. Parallel

12.

a. Function.
 Domain: $\{4, 5, 6\}$
 Range: $\{1, 2, 3\}$

b. Not function.
 Domain: $\{2, 3, 4\}$
 Range: $\{0, 5, 6, 7\}$

c. Not function.
 D: $\{x | -6 \leq x \leq 7\}$ or $[-6, 7]$
 R: $\{y | -2 \leq y \leq 5\}$ or $[-2, 5]$

d. Function.
 D: $\{x | -6 \leq x \leq 10\}$ or $[-6, 10]$
 R: $\{y | -2 \leq y \leq 8\}$ or $[-2, 8]$

e. Function.
 D: \mathbb{R} or $(-\infty, \infty)$
 R: $\{y | y \geq -3\}$ or $[-3, \infty)$

f. Not function.
 D: $\{x | -4 \leq x \leq 4\}$ or $[-4, 4]$
 R: $\{y | -2 \leq y \leq 2\}$ or $[-2, 2]$

13.

a. 5

b. $-2a^4 + 14a^3 + 3a^2 - 21a$

c. 27

d. $\frac{2x^2 - 3}{7x - x^2}$

e. $\{x | x \neq 0, 7\}$

14.

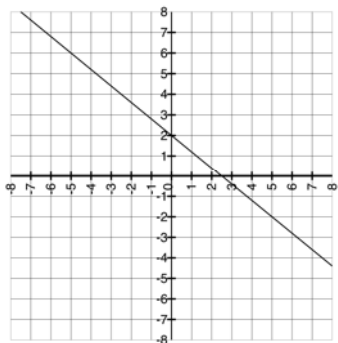
a. 0

b. 2

c. $\{-3.5, -1.5, 2\}$

d. $f(-3)$

15. Slope: $-\frac{4}{5}$; y-intercept: $(0,2)$



16.

a. $D(x) = -2.4x + 22.6$

b. 1 million pounds

c. \$9.42

17.

a. $y = 3x + 33$

b. 54 loans

18.

a. $w = 2400 + 0.035x$

b. \$28,000

19.

a. $(5, -1)$

b. $(2, -3)$

c. No solution

20. 39, 56

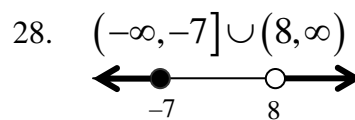
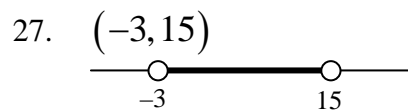
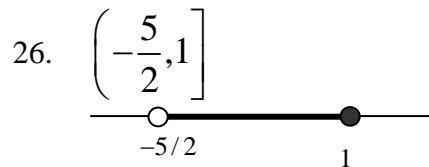
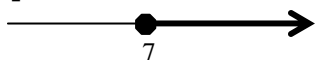
21. The canoe moves 8 mph.
The motor boat moves 20 mph.

22. Still air speed is 550 mph.
Wind speed is 50 mph

23. 3.2 ounces

24. \$72,500 in stocks
\$47,500 in bonds.

25. $[7, \infty)$

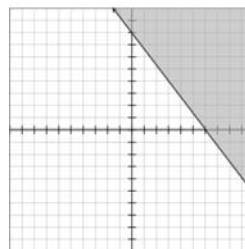


29. $x - 3 - \frac{18}{x - 4}$

30. $4x^2 - 2x + 5 + \frac{6}{2x + 1}$

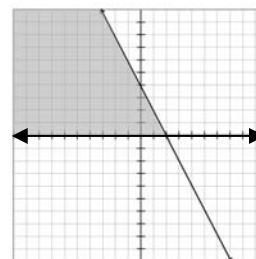
31. $\left\{3, -\frac{7}{3}\right\}$

32.

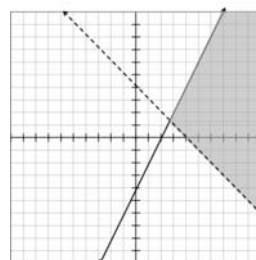


33.

a.



b.



34. $(4x + 7y)(4x - 7y)$
35. Prime (Sum of squares)
36. $(x + y)(x^2 - xy + y^2)$
37. $(x - y)^2$
38. $(7x + 8)^2$
39. $2xy(x + 1)(x - 3)$
40. $(3 - y)(3 + y)(9 + y^2)$
41. $(2x - 3y)(x + 2y)$
42. $(6x - 1)(2x + 9)$
43. $(a + c)(b - 2)(b + 2)$
44. $5(r - 2)(r^2 + 2r + 4)$
45. ± 2
46. $\left\{0, -\frac{1}{2}\right\}$
47. $\{-2, 1\}$
- 48.
- a. $\{x | x \neq 0, 3\}$
- b. all real numbers
49. $\frac{3x(x + 2)}{2(x + y)}$
50. $\frac{-3(y + 3)}{y^2 - 2y + 4}$
51. $x + 2$
52. $\frac{3x^2 + 16x + 3}{(x + 2)(x + 4)}$
53. $\frac{12y^2 + 5y - 12}{(4y + 3)(4y - 3)}$
54. $\frac{x + 1}{x}$
55. $y + x$
56. $\frac{2x(2x - 3)}{3(5x + y)}$
57. 6, 8, 10
58. $L = 10$ cm, $W = 7.5$ cm
59. -37
60. -10
61. No Solution (5 is not in domain)
62. $\pm 2\sqrt{7}$
63. $3\frac{3}{7}$ hours
64. 42 km/h
65. $R = \frac{nE - nrI}{I}$
66. $r = \frac{Hk}{L - H}$
67. x^2
68. $3x^2z^3\sqrt[3]{2x^2y^2}$
69. $6b^3c^4\sqrt{c}$
70. $-44\sqrt{3}$
71. $9 + 4\sqrt{3}$
72. $3x\sqrt{2} - 2\sqrt{3x}$
73. 4
74. $\frac{1}{2}$

75. $-\frac{1}{4}$

76. 6 (-1 is extraneous)

77. $10 + 15i$

78. $-23 + 2i$

79. $-\frac{1}{2} + \frac{1}{2}i$

80. $5 - 2i$

81. $-1 \pm \sqrt{5}$

82. $\pm 2i\sqrt{2}$

83. $\frac{3 \pm \sqrt{37}}{2}$

84. $\frac{3 \pm i\sqrt{6}}{3}$

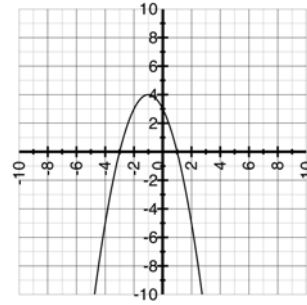
85.

- a. Two imaginary solutions
- b. Two real (irrational) solutions
- c. Two real (rational) solutions

86.

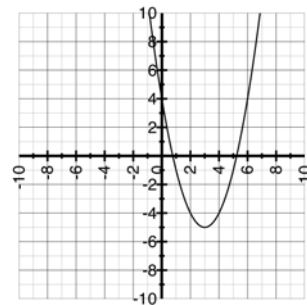
- a. x -intercepts: $(3 + \sqrt{5}, 0)$,
 $(3 - \sqrt{5}, 0)$
 y -intercept: $(0, 4)$
- b. x -intercepts: $(\frac{3}{2}, 0)$, $(1, 0)$
 y -intercept: $(0, 3)$
- c. No x -intercepts;
 y -intercept: $(0, 10)$

87.



Axis of symmetry: $x = -1$
Vertex: $(-1, 4)$

88.



The function f has a minimum value of -5.

89. $P(x) = -x^2 + 998x - 3000$

- a. 499 units
- b. \$246,001

90.

a. $d = \frac{1}{18}r^2$

b. 72 ft

91. 0.125 amperes

92. $d = \sqrt{5}$; midpoint is $(3, -\frac{5}{2})$

93. $(x + 5)^2 + (y - 1)^2 = 49$

94.

a. Center: $(-5, 3)$; Radius: 8

b. Center: $(8, -2)$; Radius: $3\sqrt{7}$