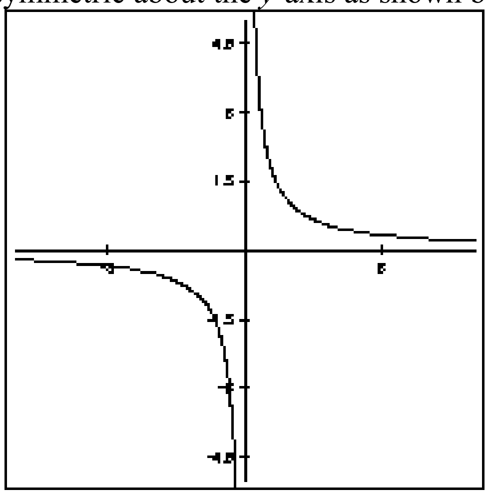
MATH 1314 College Algebra - Notes for Section 4.6 – Rational Functions

The formula for a **rational function** is a fraction in which the numerator and denominator are polynomials.

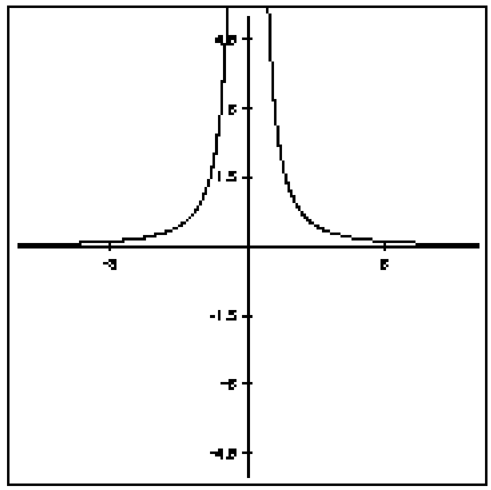
Examples: $f(x)=\dfrac{3}{x}$, $g(x)=\dfrac{4x-3}{7x+9}$, $h(x)=\dfrac{3x-4}{x^2-9}$, $k(x)=\dfrac{2x^2-4}{3x^2+4}$

The **domain** of a rational function is the set of all real numbers except those that would make the denominator equal to zero. For the examples above, the domain of f is $\{x \mid x \neq 0\}$, the domain of g is $\left\{x \middle| x \neq -\dfrac{9}{7}\right\}$, the domain of h is $\{x|x \neq \pm 3\}$, and the domain of k is $(-\infty,\infty)$.

Two of the simplest rational functions are $y=1/x$ and $y=1/x^2$. For $y=1/x$, changing the sign on x changes the sign on y so it is an odd function and it's graph is symmetric about the origin as shown below. For $y=1/x^2$, changing the sign on x doesn't change the value of y so it is an even function and it's graph is symmetric about the y-axis as shown below.



$y=1/x$



$y=1/x^2$

Notice that in both cases, as x gets closer and closer to being zero, the graph gets closer and closer to the y-axis, but never touches it. Also, in both cases, as $x \to -\infty$ or $x \to \infty$, the graph gets closer and closer to the x-axis, but never touches it. Such barriers to a graph are called **asymptotes**. In both of these examples, the x-axis (whose equation is $y=0$) is a **horizontal asymptote** and the y-axis (whose equation is $x=0$) is a **vertical asymptote.**

As with other functions, the graph of a rational function can be transformed by reflecting it, shifting it, or scaling it. For example, compared to the graph of $y=\dfrac{1}{x}$, the graph of $y=\dfrac{1}{x-2}$ is shifted 2 units to the right. Compared to the graph of $y=\dfrac{1}{x^2}$, the graph of $y=\dfrac{1}{x^2}-2$ is shifted down 2 units.

In this course, a rational function will have a **vertical asymptote** at any value of x not in the domain. For example, $h(x)=\dfrac{3x-4}{x^2-9}$ has the vertical lines $x=-3$ and $x=3$ as vertical asymptotes.

The **horizontal asymptote** of the graph of a rational function depends on the degrees of the numerator and denominator of its formula. *If the degree of the numerator is less than the degree* of the denominator, then $y=0$ is the horizontal asymptote. *If the degree of the numerator is equal to the degree of the denominator*, then $y=\dfrac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$ is the horizontal asymptote. *If the degree of the numerator is greater than the degree of the denominator*, then there is no horizontal asymptote.