MATH 1314 – College Algebra
Matrix Arithmetic

Two matrices are **equal** if and only if their dimensions are the same and corresponding elements are equal. For example, if we are told that $\begin{bmatrix} 2 & 5 & -3 \\ y & 7 & z \end{bmatrix} = \begin{bmatrix} 2 & x & -3 \\ -1 & 7 & 8 \end{bmatrix}$, then it follows that $x = 5$, $y = -1$, and $z = 8$.

Two matrices with the same dimensions can be **added** (or **subtracted**) by adding (or subtracting) their corresponding elements. For example,

$$\begin{bmatrix} 3 & -1 & 0 \\ -5 & 2 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 6 & 8 \\ 1 & -9 & 0 \end{bmatrix} = \begin{bmatrix} 3+(-7) & -1+6 & 0+8 \\ -5+1 & 2+(-9) & 4+0 \end{bmatrix} = \begin{bmatrix} -4 & 5 & 8 \\ -4 & -7 & 4 \end{bmatrix}$$

and

$$\begin{bmatrix} 3 & -1 & 0 \\ -5 & 2 & 4 \end{bmatrix} - \begin{bmatrix} -7 & 6 & 8 \\ 1 & -9 & 0 \end{bmatrix} = \begin{bmatrix} 3-(-7) & -1-6 & 0-8 \\ -5-1 & 2-(-9) & 4-0 \end{bmatrix} = \begin{bmatrix} 10 & -7 & -8 \\ -6 & 11 & 4 \end{bmatrix}.$$

A single real number is sometimes referred to as a **scalar** to distinguish it from a matrix, which is a rectangular arrangement of numbers. A matrix can be multiplied by a scalar by multiplying every element in the matrix by the scalar. For example, if $A = \begin{bmatrix} -4 & 2 \\ 5 & 0 \\ 3 & -7 \end{bmatrix}$, then

$$6A = \begin{bmatrix} 6(-4) & 6(2) \\ 6(5) & 6(0) \\ 6(3) & 6(-7) \end{bmatrix} = \begin{bmatrix} -24 & 12 \\ 30 & 0 \\ 18 & -42 \end{bmatrix}.$$

An $m \times n$ matrix can be **multiplied** with an $n \times p$ matrix. The result is an $m \times p$ matrix. To determine the *i*-th row, *j*-th column element of the product matrix go across the *i*-th row of the first matrix and down the *j*-th column of the second matrix and multiply corresponding elements. For example,

$$\begin{bmatrix} -1 & 0 & 3 \\ 9 & -2 & 8 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 5 & -4 \\ -7 & -9 \end{bmatrix} = \begin{bmatrix} -1(2)+0(5)+3(-7) & -1(6)+0(-4)+3(-9) \\ 9(2)+(-2)(5)+8(-7) & 9(6)+(-2)(-4)+8(-9) \end{bmatrix} = \begin{bmatrix} -2+0-21 & -6+0-27 \\ 18-10-56 & 54+8-72 \end{bmatrix}$$

$$= \begin{bmatrix} -23 & -33 \\ -48 & -10 \end{bmatrix}.$$

Properties of Matrix Arithmetic

1. Matrix addition is commutative: $A + B = B + A$
2. Matrix multiplication is *not* commutative: $AB \neq BA$
3. Matrix addition is associative: $A + (B + C) = (A + B) + C$
4. Matrix multiplication is associative: $A(BC) = (AB)C$
5. Matrix multiplication distributes over matrix addition or subtraction: $A(B \pm C) = AB \pm AC$