MATH 2413 - Calculus I
Maximum/Minimum Word Problems

1. Point A is 10 miles due north of Point B on a straight shoreline. Point C is on the edge of an island and is 8 miles due east of Point B. A communications cable is to be run from Point A to Point C. The cable will go under ground along the shoreline from Point A to a Point P between Points A and B. The cable will then go under water from Point P to Point C. The cost to run the cable under ground is $200 per mile while the cost to run the cable under water is $300 per mile. (a) How far from Point A should Point P be located in order to minimize the total cost of laying the cable? (b) What is the minimum total cost?

2. A page is to contain 30 square inches of print. The margins at the top and the bottom of the page are each 2 inches wide. The margins on each side are only 1 inch wide. Find the dimensions of the page so that the least paper is used.

3. A wire 12 inches long is cut into two pieces. One piece is bent to form an equilateral triangle. The other piece is bent to form a circle. Find the length of wire to be used in each shape in order to make the sum of the enclosed areas minimum.

4. James Bond is on an island 6 miles from the nearest point A on a beach with a straight shoreline. He desperately needs to get to the point B which is 10 miles up the beach from A. If he can swim 6 miles an hour and run 12 miles an hour, where on the beach should he come ashore in order to reach B in the least possible time?

5. A cylindrical tank, open at the top, is to be built to hold 1500 $\pi$ cubic feet. The cost of the material for the bottom is $3 per square foot while the cost of the material for the lateral surface is $2 per square foot. Find the radius and height that the tank should have in order to minimize the cost of the tank. What is the minimum cost?

6. Atlas Aquariums, Inc. is planning to produce an 8 cubic foot aquarium with a rectangular base that is twice as long as it is wide. The material for the base costs $.75 per square foot and the glass for the sides costs $1 per square foot. (a) Find the dimensions of the aquarium that will minimize the cost of the material used in its construction. (b) What is the minimum cost?

7. At noon Ship A, steaming east at the rate of 20 miles per hour, is directly south of Ship B steaming south at 16 miles per hour, the distance between them being 82 miles. (a) When are the ships nearest each other and (b) what is the minimum distance?

8. A Norman window is a rectangle surmounted by a semicircle. The perimeter of such a window is to be 20 feet. If the semicircular part of the window is tinted so as to admit only half as much light as the rectangular part, find the dimensions of the window that will admit the most light.

9. An open box is to be made from a rectangular piece of material by cutting equal squares from each corner and turning up the sides. Find the dimensions of the box of maximum volume if the material has dimensions 6 in. by 6 in.

10. The American Hangnail Association (A.H.A.) moves from city to city to raise money for research regarding hangnails. They have found that if they stay in a city for $x$ days, the percentage of the population that will make a donation is given by $P = 1 - e^{-0.01x}$. They have also found that the average donation received is $2 and that their daily costs for carrying on their fund-raising activities is $1,000. (a) How long should the A.H.A. stay in Podunk, Texas, a city of 100,000 people, in order to maximize its net proceeds? (b) What is their maximum possible net in Podunk?
Answers

1. (a) $10 - \frac{16}{\sqrt{5}}$, (b) $3788.85$

2. $2 + \sqrt{15}$ inches, $4 + 2\sqrt{15}$ inches

3. $\frac{108}{\pi \sqrt{3} + 9}$ inches for the triangle, $\frac{12\pi \sqrt{3}}{\pi \sqrt{3} + 9}$ inches for the circle

4. $2\sqrt{3}$ miles

5. radius is 10 ft, height is 15 ft, minimum cost is $900\pi \approx 2827.43$

6. (a) 2 ft by 4 ft by 1 ft, (b) $18$

7. (a) 2 hr, (b) $10\sqrt{41}$ mi

8. $\frac{40}{8 + 3\pi}$ ft is radius of semicircle, $\frac{80}{8 + 3\pi}$ ft is width of rectangle, $\frac{40 + 10\pi}{8 + 3\pi}$ ft is height of rectangle

9. 4 in. by 4 in. by 1 in.

10. (a) 69.315 days, (b) $30,685.28$