

# Methods of Solving Quadratic Equations

An equation that can be put in the form  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , is called a **quadratic equation**.

Examples:  $6x^2 = 13x - 6$ ,  $3x^2 = 48x$ ,  $2x^2 - 18 = 0$ ,  $x^2 = 7$ ,  $3(x - 5)^2 = 2$

## Solving by Factoring

Arrange the equation so that it is in the form  $ax^2 + bx + c = 0$ . Factor, if possible, the left-hand side. Then set each factor equal to zero and solve the resulting equations.

Examples: Solve (a)  $3x^2 = 48x$ , (b)  $x^2 = 2x + 8$ , (c)  $6x^2 = 13x - 6$

## Solving by the Principle of Square Roots

This method only works if the quadratic equation consists only of a squared expression and a constant term and is based on the fact that if  $(\text{expression})^2 = k$ , where  $k$  is a constant, then  $\text{expression} = \pm \sqrt{k}$ .

Examples: Solve (a)  $9x^2 = 25$ , (b)  $x^2 - 7 = 0$ , (c)  $3(x - 5)^2 = 2$

## Solving by Completing the Square

Arrange the quadratic equation so that it is in the form  $ax^2 + bx = c$ ; in other words, with the constant term by itself on one side.

If  $a$  is not one, divide both sides by  $a$  to make the  $x^2$  coefficient equal to 1. You should now have an equation of the form  $x^2 + mx = p$ .

Complete the square on left side and be sure to add the same constant to the right side also.

On the left side factor the trinomial into the square of a binomial and on the right side combine the constants. You should now have an equation of the form  $(x + n)^2 = k$ .

Now you can use the Principle of Square Roots to complete the solution.

Examples: Solve (a)  $2x^2 + 6 = 10x$ ,  $3x^2 = 5x + 4$

## Solving by Using the Quadratic Formula

Arrange the equation so that it is in the form  $ax^2 + bx + c = 0$  and determine the values of  $a$ ,  $b$ , and  $c$ .

Then substitute those values into the Quadratic Formula  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  and simplify (remove any perfect squares from under the radical and reduce the fraction, if possible).

Examples: Solve:  $16x^2 = 16x - 1$ ,  $2x^2 - 18 = 0$ ,  $5x^2 = 7x$