

# Quadratic Functions

A function that can be put in the form  $f(x) = ax^2 + bx + c$ , where  $a \neq 0$ , is called a **quadratic function**. This arrangement of a quadratic function is called the **general form**. The number  $a$  is called the **leading coefficient**.

Examples:  $f(x) = 6x^2 - 13x + 6$ ,  $f(x) = -3x^2 + 48x$ ,  $f(x) = 2x^2 - 18$

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The graph of the quadratic function  $f(x) = ax^2 + bx + c$ , where  $a \neq 0$  is a U-shaped graph called a **parabola**. The parabola **opens up**  $\cup$  if  $a > 0$  and **opens down**  $\cap$  if  $a < 0$ . The tip of the parabola is called its **vertex**. If the parabola opens up, the vertex is the lowest point on the parabola and the y-coordinate of the vertex is the **minimum value** of the function. If the parabola opens down, the vertex is the highest point on the parabola and the y-coordinate of the vertex is the **maximum value** of the function. The **x-coordinate of the vertex is given by**  $x = -\frac{b}{2a}$ . The **y-coordinate of the vertex is**

**given by**  $f\left(-\frac{b}{2a}\right)$ . The graph of a quadratic function is symmetric about the vertical line that passes through its vertex. (This means that if the graph is folded on the vertical line through the vertex, the two halves of the parabola will lie exactly on top of each other.) This vertical line is called the **axis of symmetry** and its equation is  $x = -\frac{b}{2a}$ , the same as the x-coordinate of the vertex.

Example: Graph  $f(x) = -x^2 + 2x + 8$ . Specify the vertex and axis of symmetry. Which does the function have--a minimum value or a maximum value? What is that "extreme" value?

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The equation for a quadratic function can also be put in the form  $f(x) = a(x - h)^2 + k$ , which is called the **vertex (or standard) form**. As in the general form, the graph is a parabola that **opens up if**  $a > 0$  and **opens down if**  $a < 0$ . The **vertex of the parabola is**  $(h, k)$  and the **axis of symmetry is the vertical line**  $x = h$ .

Example: Graph  $f(x) = 2(x - 4)^2 - 18$ . Specify the vertex and axis of symmetry. Which does the function have--a minimum value or a maximum value? What is that "extreme" value?

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**To convert a quadratic function from vertex form to general form**, perform the indicated square and combine like terms.

Example: Convert  $f(x) = -(x - 5)^2 + 25$  to standard form.

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**To convert a quadratic function from general form to vertex form**, use completing the square. The notation looks simpler if we first replace  $f(x)$  with  $y$ . Then subtract the constant term, if any, from both sides. Next, if the  $x^2$  coefficient is not already one, divide both sides by it. This is needed because completing the square only "works" when the squared term's coefficient is 1. Now complete the square on the right and remember that any operation done to one side of the equation must also be done to the other side. Next, factor the perfect square trinomial that you've created on the right side into the square of a binomial. Then solve the equation for  $y$  and finally replace  $y$  with  $f(x)$ .

Example: Convert  $f(x) = 3x^2 - 18x + 7$  to vertex form.