MATH 1314 – College Algebra
Graphing Rational Functions

To graph a rational function:

1. Determine the domain by setting the denominator equal to 0 and solving for x. The domain is the set of all real numbers *except* those that make the denominator equal to 0.

2. Completely reduce, if possible, the rational function and **use the reduced formula for the remaining steps**.

3. Determine the intercepts.

   (a) To determine any x intercepts substitute 0 for y, and solve for x. (Shortcut: Just set the numerator equal to 0 and solve for x.)

   (b) If 0 is not in the domain of the original (unreduced) formula for the function, there is no y intercept. If 0 is in the domain of the original formula, substitute 0 for x and solve for y.

4. To determine the vertical asymptotes, determine the values of x that make the denominator equal to zero. There are vertical asymptotes at these values of x. Draw them with dashed lines.

5. To determine the asymptote that the tail ends of the graph get close to:

   (a) If the degree of the numerator is less than the degree of the denominator, then $y = 0$ (the x-axis) is the horizontal asymptote.

   (b) If the degree of the numerator is equal to the degree of the denominator, then the horizontal line whose equation is given by

$$y = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$$

   is the horizontal asymptote.

   (c) If the degree of the numerator is more than the degree of the denominator, there is no horizontal asymptote. Instead there is a slant or curved asymptote that the tail ends of the graph approach. To determine its equation, use polynomial long division to divide the numerator by the denominator. Then $y =$ the quotient of this division (ignore the remainder) is the formula for this asymptote.

6. If the graph produced in Steps 1-5 exists at any values of x that were not in the original domain (determined in Step 1), these points must be removed from the graph. Draw an empty circle at each such point to indicate that there is a "hole" in the graph.