

Summary of Trigonometric Facts

Formulas Involving Radian Angular Measure

$$1 \text{ deg} = \frac{1}{180} \text{ rad} \quad 1 \text{ rad} = \frac{180}{\pi} \text{ deg} \quad \theta = \frac{s}{r} \quad \omega = \frac{v}{r} \quad A = \frac{1}{2} \theta r^2$$

Trigonometric Function Definitions

$$R = \sqrt{a^2 + b^2}$$

$$\text{sine } x = \sin x = \frac{b}{R} = \frac{\text{opp}}{\text{hyp}}$$

$$\text{cosine } x = \cos x = \frac{a}{R} = \frac{\text{adj}}{\text{hyp}}$$

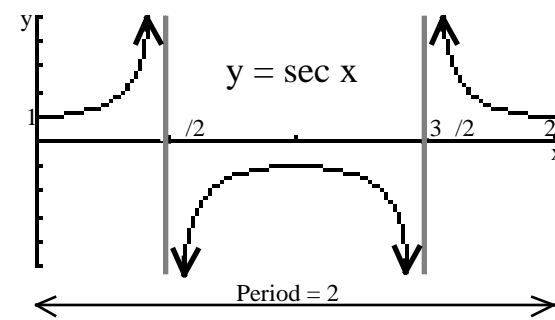
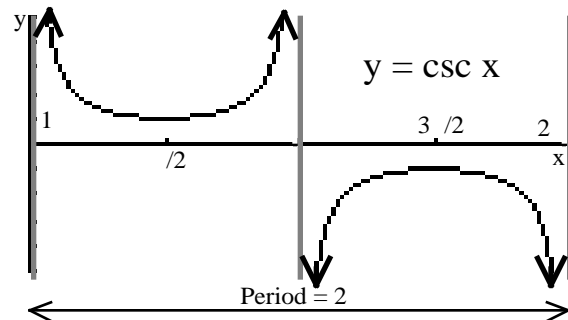
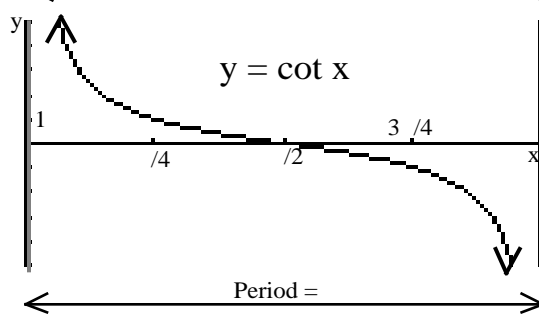
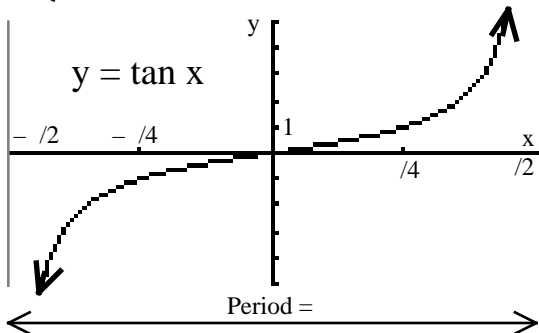
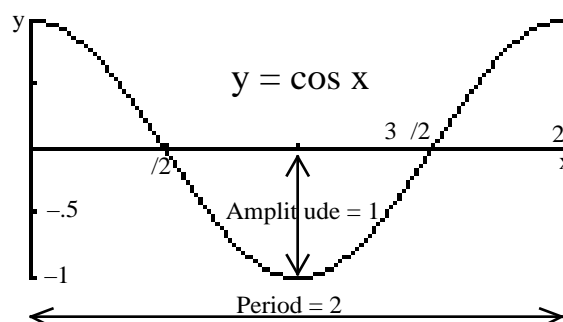
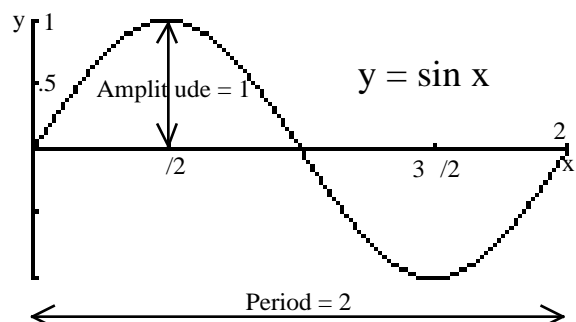
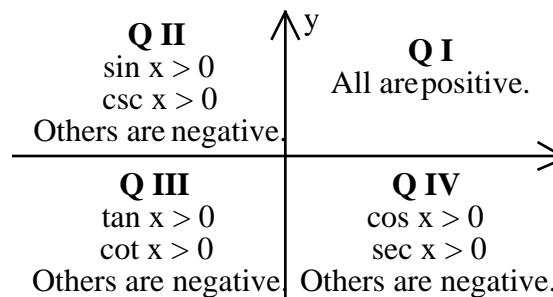
$$\text{tangent } x = \tan x = \frac{b}{a} = \frac{\text{opp}}{\text{adj}}$$

$$\text{cosecant } x = \csc x = \frac{R}{b} = \frac{\text{hyp}}{\text{opp}}$$

$$\text{secant } x = \sec x = \frac{R}{a} = \frac{\text{hyp}}{\text{adj}}$$

$$\text{cotangent } x = \cot x = \frac{a}{b} = \frac{\text{adj}}{\text{opp}}$$

| A | 0° | 30° | 45° | 60° | 90° |
|-------|----|------|------|------|---------|
| | 0 | 1/6 | 1/4 | 1/3 | 1/2 |
| sin A | 0 | 1/2 | √2/2 | √3/2 | 1 |
| cos A | 1 | √3/2 | √2/2 | 1/2 | 0 |
| tan A | 0 | √3/3 | 1 | √3 | undef'd |



Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent Identities

$$\frac{\sin x}{\cos x} = \tan x \quad \frac{\cos x}{\sin x} = \cot x$$

Pythagorean Identities

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \sec^2 x \quad 1 + \cot^2 x = \csc^2 x \end{aligned}$$

Cofunction Identities

$$\begin{aligned} \sin x &= \cos\left(\frac{\pi}{2} - x\right) & \cos x &= \sin\left(\frac{\pi}{2} - x\right) \\ \tan x &= \cot\left(\frac{\pi}{2} - x\right) & \csc x &= \sec\left(\frac{\pi}{2} - x\right) \\ \sec x &= \csc\left(\frac{\pi}{2} - x\right) & \cot x &= \tan\left(\frac{\pi}{2} - x\right) \end{aligned}$$

Even-Odd Identities

$$\begin{aligned} \sin(-x) &= -\sin x & \csc(-x) &= -\csc x \\ \cos(-x) &= \cos x & \sec(-x) &= \sec x \\ \tan(-x) &= -\tan x & \cot(-x) &= -\cot x \end{aligned}$$

Half Angle Formulas

$$\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}} \quad \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}} \quad \tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

Product to Sum or Difference Formulas

$$\begin{aligned} \sin x \cos y &= \frac{1}{2} [(\sin(x+y)) + \sin(x-y)] & \cos x \sin y &= \frac{1}{2} [\sin(x+y) - \sin(x-y)] \\ \sin x \sin y &= \frac{1}{2} [\cos(x-y) - \cos(x+y)] & \cos x \cos y &= \frac{1}{2} [\cos(x+y) + \cos(x-y)] \end{aligned}$$

Inverse Trigonometric Functions

$$\begin{aligned} \arcsin x \text{ or } \sin^{-1}x & \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] & \operatorname{arccsc} x \text{ or } \csc^{-1}x & \left[-\frac{\pi}{2}, 0 \right) \cup \left(0, \frac{\pi}{2} \right] \\ \arccos x \text{ or } \cos^{-1}x & [0, \pi] & \operatorname{arcsec} x \text{ or } \sec^{-1}x & \left[0, \frac{\pi}{2} \right) \cup \left(\frac{\pi}{2}, \pi \right] \\ \arctan x \text{ or } \tan^{-1}x & \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) & \operatorname{arccot} x \text{ or } \cot^{-1}x & \left(-\frac{\pi}{2}, 0 \right) \cup \left(0, \frac{\pi}{2} \right) \end{aligned}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ b^2 &= a^2 + c^2 - 2ac \cos B \\ c^2 &= a^2 + b^2 - 2ab \cos C \end{aligned}$$

Area of a Triangle

$$\begin{aligned} \text{area} &= \frac{1}{2} bc \sin A \\ \text{area} &= \frac{1}{2} ac \sin B \\ \text{area} &= \frac{1}{2} ab \sin C \end{aligned}$$

Heron's Formula

$$\begin{aligned} \text{area} &= \sqrt{s(s-a)(s-b)(s-c)}, \\ \text{where } s &= \frac{a+b+c}{2} \end{aligned}$$

Sum of Sine and Cosine with Same Period

$$a \sin cx + b \cos cx = A \sin(cx + \phi),$$

$$\text{where } A = \sqrt{a^2 + b^2}, \quad \sin \phi = \frac{b}{\sqrt{a^2 + b^2}}, \quad \text{and } \cos \phi = \frac{a}{\sqrt{a^2 + b^2}}$$

Sum and Difference Formulas

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Double Angle Formulas

$$\begin{aligned} \sin 2x &= 2 \sin x \cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2 \cos^2 x - 1 \\ &= 1 - 2 \sin^2 x \end{aligned}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$