## Atwood's Machine

## Introduction

In this lab we will investigate the application of Newton's Second Law to a simple physical system known as Atwood's Machine. Under the right circumstances it is possible to use this machine to measure accelerations very accurately. The purpose of this lab is to use Atwood's Machine for making a determination of "g", the acceleration due to gravity.

## Equipment Needed

Clamp, swivel $90^{\circ}$
Weight Hanger, 50g(2)
Mass Set, Gram
Ringstand, 120 cm
String (1.5m)

Computer with Data Pro Software
Photogate
Smart Pulley
LabPro Interface


## Theory

Our version of Atwood's Machine consists of two masses suspended over a pulley as shown in the figures above and below. If the two masses are equal in value, then the system will be in equilibrium and no motion will take place. If the two masses are close to each other in value, then the acceleration of the masses will be small, and can be accurately measured. This makes

Atwood's Machine a technologically useful device for the determination of the acceleration due to gravity. The acceleration of the two masses is given by

$$
a=\frac{m_{1}-m_{2}}{m_{1}+m_{2}} g . \text { (Equation 1) }
$$

You will derive this formula from the application of Newton's $2^{\text {nd }}$ Law to the Atwood Machine and include your derivation as part of your lab report. Masses can be measured very accurately, and if we can measure the acceleration accurately as well, then Atwood's Machine can be used to give an accurate value for $g$. Our measured values of these accelerations will be used to calculate an estimate of " g ".

## Figure 1 Atwood's Machine



## Procedure

Motivated by the discussion above, we need to measure the acceleration of the masses. We will utilize a device known as a smart pulley to measure these accelerations. For now, we can think of the smart pulley as a device that measures the speed of the string as it rides on the rotating pulley. Later in the semester, we will investigate its operation in more detail.

1. Set up

Feed a 1.5 m length of string through the smart pulley. There should be small loops tied in both ends of the string. Hang a weight hanger on a loop at each end of the string. Clamp the smart pulley so that the masses hang over the end of the table and can move freely without hitting table.
2. Preliminary Observations (Predict the results of the following experiment.)

| Exp Procedure | Prediction \& Why | Experimental Result | Prediction Correct? |
| :--- | :--- | :--- | :--- |
| Place equal 50g <br> masses on both <br> weight hangers. <br> Position the masses at <br> equal heights and let <br> them go. |  |  |  |
| Place equal 50g <br> masses on both <br> weight hangers. <br> Position one mass <br> above the other and <br> then let go. |  |  |  |
| Place a 50g mass on <br> one weight hanger <br> and a 52g mass on the <br> other. Position the <br> 50 g mass as a height <br> greater than the 52g <br> mass and then let go. |  |  |  |
| Place a 50g mass on <br> one weight hanger <br> and a 52g mass on the <br> other. Position the <br> $50 g$ mass at a height <br> lower than the 52g <br> mass and then let go. |  |  |  |

Write a general statement about what determines the motion of the two masses based on your observations.

## Data Acquisition

Note: Don't forget to include the mass of the weight hanger.
Connect the smart pulley to channel DG1 of the ULI and make sure that the ULI is turned on and connected to the computer via the USB cable. Open the LoggerPro program and then open the file called Pulley. (File menu item $=>$ Open file $=>$ Probes \& Sensors folder $=>$ Photogate folder $=>$ Pulley). The experimental set-up is shown in Figure 2.

## Experimental Procedure - Constant System Mass

Place 50 g on one of the weight hangers and 52 g on the other. The 50 g side should contain at least four (4) 1 g masses. Click on the Collect button to start the data collection and then release the masses. Make sure that the masses do not swing during the experiment.

Click on the Stop button to end the data collection. To optimize the graphs for easier viewing click on the "A" (Auto-Size) button at the top of the screen.
Although Distance, Velocity and Acceleration graphs are displayed we will be using the velocity graph to determine the acceleration of the masses.

- Click on the velocity graph and drag a box around the relevant velocity data. This will be the straightest part of the positively sloping graph. Avoid the first few points and stop selecting points before the graph starts to turn down.
- Choose the Linear Fit button along the top of the screen. This will calculate the slope of the linear fit line to the velocity data. This slope is the experimental value of the acceleration of the masses.
- Record the value of the slope in the data table along with the value both masses.


## Include Slope Uncertainty Data:

With the cursor over the Linear Fit data box
Apple: Hold down the Control key and press enter or click on the data box with the mouse.

Windows PC: Right Click on the data box
This will bring up a Pop-Up box - select the "Linear Fit Options" choice. Another Pop-Up box will appear - put a check in the box next to "Show Uncertainties."

Click OK. The original Linear Fit data box will now display the Slope and the YIntercepts and their respective uncertainties. Record the slope uncertainty in your data table.

Repeat the experiment and analysis four more times, each time moving an additional 1 g mass from the lighter side to the heavier side. There should be a total of 5 data sets when you are done.

Figure 2 Experimental Set-up


## Data Analysis

Use MS Excel to perform the following analysis.

1. Construct a graph of $a$ (Vertical axis)vs. $\frac{\left(m_{1}-m_{2}\right)}{\left(m_{1}+m_{2}\right)}$. (Horizontal axis). The measured acceleration " $a$ " should be plotted on the $y$-axis and the mass difference ratio should be plotted on the x -axis

Based on your theory given in equation (1), what would you expect the slope and $y$ intercept of this graph to be? Include the best-fit line on your graph. Print a copy of your graph.
2. Find the standard error estimate for the slope of the best-fit line of $a$ vs. $\frac{\left(m_{1}-m_{2}\right)}{\left(m_{1}+m_{2}\right)}$.
3. Record your best-fit slope and $y$ intercept along with the standard error estimates in the data table.

If you are using Excel then you will need to use the LINEST Worksheet function to calculate the uncertainties in the slope. Start by selecting a 2 -column by 2 -row area of cells on the spreadsheet. Then, under the Inset menu item select the Function option - or - click on the " $\mathbf{f}_{\mathbf{x}}$ "
button on one of the tool bars. The Paste Function dialogue box will appear. In the left window select the Statistical Function category. In the right window scroll down to find the LINEST function and select it. Then click OK. The LINEST dialogue box will appear with four input items: "Known y's", "Known x's", "Const", "Stats". Place the y and x cell ranges in the first two input boxes. If the "Const" item is TRUE or omitted the y-intercept will be calculated. It will be set to zero if "Const" is set to FALSE. "Stats" is set to TRUE to get the array of statistics to be displayed in the $2 \times 2$ array. Now this is an ARRAY Function and the "Ctrl" and "Shift" keys need to be held down when pressing the "Enter" key or clicking the OK button.

| slope | y-intercept |
| :---: | :---: |
| std error in slope | std error in y-intercept |


| 9.8 | -0.08318 |
| ---: | ---: |
| 0.196611 | 0.009233 |

## Error Analysis

Determine the 1 standard error interval about the best-fit slope value of your acceleration graph. Does the accepted value of " $g$ ", the acceleration due to gravity, lie within this interval. If it does, then you can say that your experimental slope value agrees with the accepted value of "g," within your experimental error. If not, then there is a systematic error. If there is a systematic error, then try to identify the source. Examine the assumptions that went into your theory. Did they seem to hold experimentally? Did you observe any irregularities with the operation of the equipment? Describe on these in your lab report. Did you make any errors in operating the equipment? Discuss the impact of these. Do your best. Accounting for systematic error can be very difficult, but it is an important skill that all scientists and engineers must develop to do their jobs. It is unacceptable to just say that the experiment had human and experimental errors.

## Report Format

This lab will require a full write-up from each student individually. You should follow the format specified in the handout Format for Formal Lab Reports. You should include the derivation of the acceleration for the Atwood's Machine, including relevant sketches and free body diagrams in the Introduction section of the report. Your report should include your graph and cleaned up data tables. Your preliminary observations should be attached as an appendix. In other words, staple your lab hand out to the back of your write up.

## Data Table

| $\mathbf{m}_{1}(\mathbf{k g})$ | $\mathbf{m}_{\mathbf{2}}(\mathbf{k g})$ | ABS( $\left.\mathbf{m}_{1}-\mathbf{m}_{2}\right) /$ <br> $\left(\mathbf{m}_{1}+\mathbf{m}_{2}\right)$ | $\mathbf{a}_{\text {meas }}$ <br> $\left(\mathbf{m} / \mathbf{s}^{2}\right)$ | $\mathbf{a}_{\text {meas }}$ <br> Uncertainty <br> $\left(\mathbf{m} / \mathbf{s}^{2}\right)$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Slope $\qquad$ $\pm$ Uncertainy $\qquad$
\% Uncertainty $=$ Uncertainty *100/Slope $\qquad$

