

Resistive Forces

Introduction

In this lab we will investigate the effect of resistive forces on the motion of a falling object. We will investigate two different models for the force due to air resistance referred to as the Linear model and the Quadratic model. The purpose of this lab is to determine if one of these two models can be used to describe the dynamics of a falling object.

Equipment Needed

Computer with Logger Pro SW	Clamp, 3-prong extension
Lab Pro Interface	Clamp, swivel 90°
Motion Detector	Scale, Digital
Coffee Filters	
Ring stand, 120cm	

Theory

Consider the object shown in Figure 1 below. In the absence of air resistance, the object falls with the familiar acceleration due to gravity of 9.8 m/s^2 . In this case we expect the distance to grow quadratically in time and the velocity to grow linearly in time. The situation changes however in the presence of resistive forces. If we analyze the forces shown on the object in Figure 1, Newton's Second Law yields

$$\sum F = ma .$$
$$mg - F(v) = m \frac{dv}{dt} \quad (\text{Equation 1}).$$

We have taken down as the positive direction here for convenience. Note that we have written Newton's second law in terms of the derivative of the velocity. This is useful because the resistive force is



a function of the velocity. In this case, Newton's Second Law yields a differential equation for the velocity. We will now analyze the behavior of the equation qualitatively.

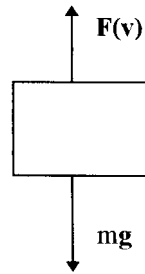


Figure 1 Forces on a falling object to air resistance

If we drop the object from rest, it initially has a velocity of zero, and consequently the air resistance, that is a function of the velocity, is zero. We expect that the object will initially fall freely with an acceleration of g and the distance will grow quadratically in time. As the object speeds up, the resistive force will grow, and we expect the acceleration to decrease. Thus the object's distance will grow less rapidly than that of an object undergoing free fall. Eventually, the object's speed will grow to a point where the resistive force is equal to the weight of the object. At this point, the object will be in equilibrium and will continue to fall at a constant velocity, known as the terminal velocity. We will analyze the terminal velocity for two different models of the resistive force.

The simplest model for a velocity dependent resistive force is that the force is proportional to the velocity (Linear model).

$$F(v) = bv.$$

The constant b depends on the properties of the medium through which the object drops as well as the shape and geometry of the object. If we insert this model into Equation 1, we can solve for the terminal velocity by using the fact that the terminal velocity is an equilibrium solution of the Equation 1. Setting the acceleration equal to 0, we obtain:

$$mg - bv = 0 \quad (\text{Equation 2}).$$

Solving Equation 2 for the terminal velocity yields:

$$v = \frac{mg}{b} \quad (\text{Equation 3}).$$

This result has some interesting consequences. Different objects with the same shape will have different terminal velocities. The more massive an object is, the greater its terminal velocity. Thus for example, when we allow for air resistance a solid ball will fall faster than a hollow ball of the same size and material.

A second and more realistic model, commonly used to model air resistance is a model where the air resistance is proportional to the square of the velocity (Quadratic model). Qualitatively we expect the behavior of the falling object to be similar to that for the case where the air resistance was linear in the velocity. We typically write the resistive force for the case of the quadratic model as:

$$\underline{F(v) = \frac{1}{2}C\rho Av^2}$$

where ρ is the density of the medium through which the object falls, A is the cross sectional area of the object, and C is a constant known as the drag coefficient and is related to the shape and texture of an object. We can again find the terminal velocity by looking for equilibrium solutions of Equation 1. Inserting the quadratic model for the air resistance and setting the acceleration equal to 0 yields:

$$\underline{mg - (1/2)C\rho Av^2 = 0} \quad \text{(Equation 4).}$$

Solving Equation 4 for the terminal velocity yields:

$$v = \left(\frac{2mg}{C\rho A} \right)^{1/2}.$$

Note that again the terminal velocity depends on the mass, only this time it depends on the square root of the mass. The terminal velocity grows with mass more slowly for this model compared to the Linear model. For the Quadratic model to exhibit only a mass dependence it is necessary to ensure that C and A remain constant. Since a coffee filter is very flimsy the shape can change during the run and from run to run, thus causing a change in the values of C and A .

We will now conduct an investigation into the free fall behavior of similarly shaped objects with different masses and try to determine which of the models we have posited furnish a better description of the motion

Equipment Procedure

Program File: Under the **File** menu select the **Open** item. The **Experiments** folder will appear, select the **Probes and Sensors** folder and then the **Motion Detector** folder and finally select the **Motion Detector** file. This file will display three time graphs: Position, Velocity and Acceleration. The Position data comes directly from the Motion Detector data while the latter two graphs are derived numerically from the Position data.

Motion Detector: We will employ the Motion Detector to obtain position data for this lab. Insert the motion detector cable into Dig Port 2 the Lab Pro box. The Sensitivity Switch on the motion detector should be in the NORMAL position.

Equipment Set Up: The experimental set up is shown in Figure 2. A meter stick is attached to the Motion Detector using the black clamp mounted on the detector as shown below. Clamp the ultrasonic motion detector so that it faces down, is level, and is positioned as high above the floor as possible. The meterstick, with the motion detector attached, is suspended from the ceiling using paper clips.

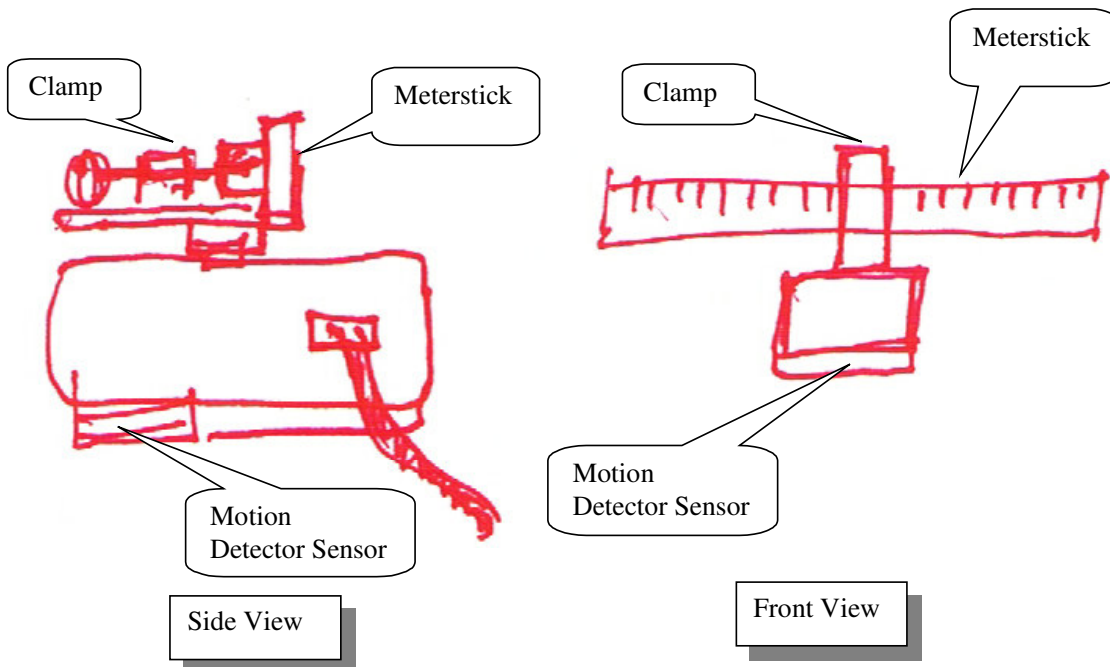


Figure 2 Schematic of experimental set up

The Coffee Filter: We will use a single coffee filter as the object to be dropped in this experiment. These make an ideal object for this experiment since the mass is small and the cross sectional area is large thus enhancing the affect of air resistance. The mass can be easily adjusted by simply placing a few 1.0 gram weights inside the single filter. This allows the mass to be adjusted while maintaining an effectively constant cross sectional area.

Filter Mass: Use the electronic scale to determine the mass of the stack of coffee filters. Scale should be capable of a one-hundredth of a gram precision for measuring a single filter. If the scale only goes to a tenth of a gram then weigh 10 filter and average.

Experimental Procedure

Collecting the Data: Begin by holding the filter as high as possible directly under the motion detector. Click on the **Collect** button. When the data collection starts the motion detector will begin emitting a clicking sound and you should then release the filter. The program will automatically collect position data for approximately 10 seconds and then the **Stop** button will change back to the **Collect** button and the data collection will end.

Dropping the Filter: It is important for the filter to drop without rocking back and forth. Support the filter from the flat bottom of the filter on your finger tips. To launch the filter quickly move your hand down and then pull your hand back and out of the cone of the emitted ultrasonic sound pulses.

Data Graphs: You should now have a Distance versus Time graph similar to Figure 3. We now want to extract relevant data from our graphs. Figure 3 shows qualitatively what a Distance graph might look like. Initially the distance grows quadratically in time, but as the terminal velocity is approached, the distance tends to grow linearly in time. The slope of the linear portion of the distance versus time graph is the terminal velocity. One method would be to extract the value of this slope and thus obtain an experimental value of the terminal velocity.

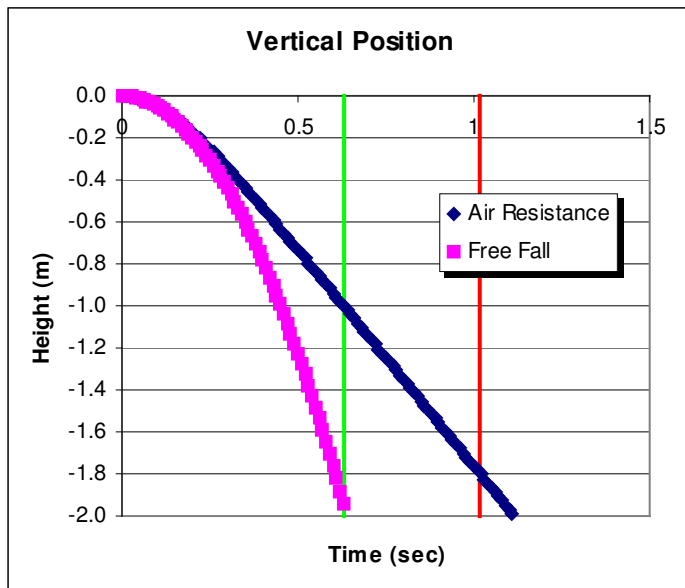


Figure 3.
Schematic of “Typical” Data

The time constant for this freefall problem is called τ (“tau” - Greek letter) and is represented by the following expression.

$$\tau = \left(\frac{2m}{gC\rho A} \right)^{1/2}$$

The time constant represents the characteristic time interval in the problem. This is the size of the time interval over which important event in the problem take place. It takes a time interval about three (3) time constants in length for the velocity of the filter to reach its terminal velocity. This time is indicated by the first vertical line in the Distance and Velocity graphs. The second vertical line represents the total fall time of the filter. The interval between these two lines is the time period available for making measurements of the terminal velocity.

The problem with using the Distance graph is the difficulty in distinguishing the quadratic portion of the curve from the linear portion of the curve. NOTE: The use of the terms quadratic and linear here pertain to the time dependence of the position of the filter and do not relate to the Air Resistance models that you are testing.

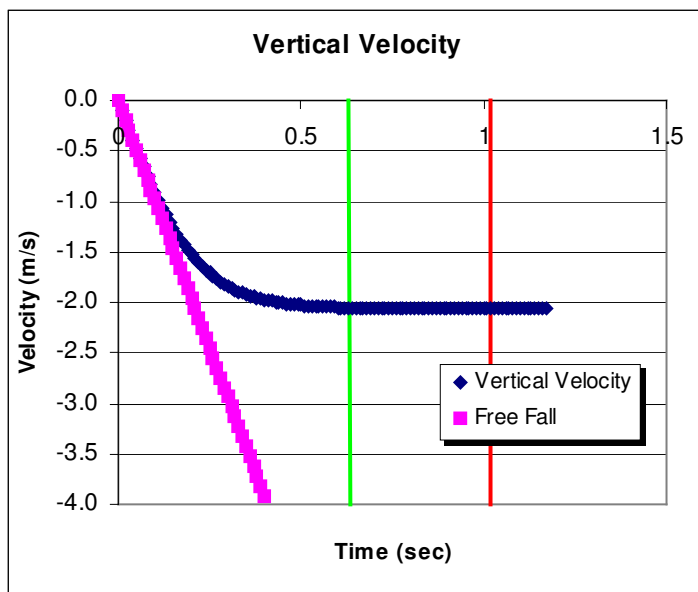


Figure 4. Typical Velocity versus Time Graph Window

On the Velocity versus Time graph the velocity starts to grow linearly ($v = gt$) and then flattens out as it approaches the terminal velocity. This graph is easier to deal with in that you can more readily distinguish between the linear portion and the flat portion of the graph.

Selection of the Terminal Velocity Data: You need to select the data points on the Velocity versus Time graph that you will analyze - you need to select the “flat” section. There will be some noise in this data that will appear as vertical excursions about a horizontal line. At the beginning of this selected region you should be to the right of the linear region. At the end of this selected region you should be to the left of the time when the filter hit the ground (i.e. distance graph went flat and the velocity dropped).

Measuring the Terminal Velocity: Click on the **Stats** button and the program will average the data points in that portion of the Velocity versus Time graph that you selected. A box will appear with the results of the statistical analysis of the selected data and you will be interested in the Mean value. The Mean will be the terminal velocity for this run.

Be sure to record the terminal velocity and the mass of the filter plus weights for each drop in your Data Table. With this same mass, repeat the drop two (2) more times for a total of three terminal velocity measurements for each mass.

Repeat your series of three (3) velocity measurements **four more times**, each time increasing the mass of the filter by placing an additional 1.0 gram weight inside the filter.

Calculate the average terminal velocity for each of the masses along with the standard deviation and record their values in the Data Table. When you are done the Data Table should contain five masses with five corresponding average terminal velocities.

Data Table

Mass (kg)	V ₁ (m/s)	V ₂ (m/s)	V ₃ (m/s)	V _{Average} (m/s)	Std Dev

Data Analysis

The information you wish to extract from your data is a determination of model for air resistance better describes the data. The Linear model for air resistance gives a result where the terminal velocity is a linear function of the mass, and the quadratic model yields a terminal velocity that is proportional to the square root of the mass. In either case, we expect a result of the form

$$v = km^n \quad \text{(Equation 4)}$$

where k is some constant and n is some power. For the Linear model $n=1$; while for the Quadratic model $n= \frac{1}{2}$. We wish to determine the value of this exponent.

A common data analysis tool used to extract exponent information is the log-log plot. On a log-log plot instead of plotting y vs. x we plot $\log(y)$ vs. $\log(x)$. The log can be to any base, but typically base 10 is chosen. If we consider $v = km^n$, let us examine what happens when we take the log of both sides.

Remember the important properties of logarithms:

$$\text{i) } \log(AB) = \log(A) + \log(B) \quad \text{and} \quad \text{ii) } \log(A^n) = n \log(A).$$

Taking the log of both sides of Equation 4 and using the properties of the logarithms yields

$$\log(v) = n \log(m) + \log(k).$$

Thus if we plot $\log(v)$ vs. $\log(m)$, we obtain a line with slope equal to the exponent that is the power to which the mass is raised in Equation 4. From this exponent we can determine which of our models, if either, better describes our data. A slope close to $\frac{1}{2}$ will indicate that the Quadratic model is a better fit to the data, whereas a value closer to 1.0 will favor the Linear model.

- Graph $\log(V_T)$ vs. $\log(m)$ and perform a linear regression analysis to determine the slope. In Excel use the X-Y Scatter chart and NOT the Line chart.
- Determine the uncertainty in your slope using the worksheet function LINEST.
- Is your slope with uncertainty consistent with either of the models discussed in this lab? Make a reasonable attempt to account for any discrepancies.

Report Format

Follow the “Format for Formal Lab Reports” document

Problems

Spikes in the Distance versus Time graphs: The spike, if interpreted literally, means that the object whose motion is being measured suddenly increased in velocity and then quickly decelerated and actually rose instead of falling. The probable source of these spikes is that the motion detector is picking up signals from the bottom and the sides of the coffee filter. You can minimize this effect by dropping the filter so that it doesn't rock back and forth, as it falls to the floor.

Drift: Air currents in the room, due to the movements of individuals or due to the air conditioning system, can cause the falling filter to drift sideways. This can affect the distance measurement.