

# Simple Harmonic Motion

## Introduction

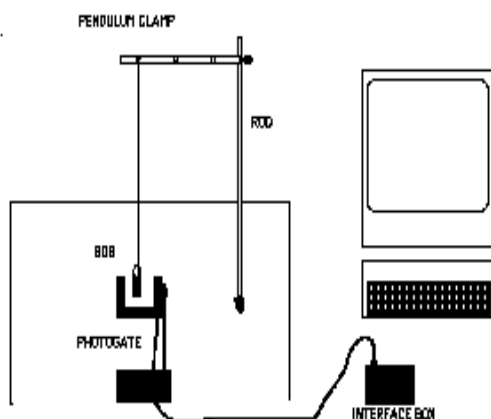
An object that experiences a linear restoring force about its equilibrium position exhibits a periodic motion called Simple Harmonic Motion (SHM). This behavior is observed in both the mass-spring system and the rubber band that obey Hooke's Law.

In this lab we will study two systems that exhibit SHM, the simple pendulum and the mass-spring system. From the equation of motion of a simple harmonic oscillator the angular frequency,  $\omega$ , of the motion can be determined. Since  $\omega = 2\pi/T$  this same equation of motion gives a relationship for the period of the motion.

The purpose of this lab is to test the dependency of the period relationships on the system parameters for the pendulum and the mass-spring systems.

## Equipment - Pendulum

Computer with Logger Pro S/W	Pendulum Bob	Protractor
Interface with Cables	Pendulum Clamp	Meter Stick
Photogate with support hardware	Support Rod	String



*Figure 1*

## Theory - Pendulum

A motion that repeats itself is called periodic. The period is the time it takes for an object exhibiting periodic motion to return to its starting point. If the period of that motion is exactly the same for each recurrence of the motion then that object has the basis of a time keeping device. The period of a pendulum was used for centuries as an accurate timekeeper.

Simple pendulum motion can be demonstrated by a point-like mass (called a bob) on the end of a string. The length is measured from the pivot point to the center of mass of the bob. For small displacement angles, i.e. less than  $15^\circ$ , the period of a pendulum depends only on the length of the pendulum and on no other physical characteristic of the pendulum.

The period of oscillation for a pendulum, this is given by

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{or} \quad T^2 = 4\pi^2\frac{L}{g}$$

where “L” is the length of the pendulum and “g” is the acceleration due to gravity.

### **Equipment Procedure - Pendulum** (Refer to Figure 1)

1. Assemble the pendulum by attaching the top end of the string to the pendulum clamp and attaching the pendulum bob to the lower end.
2. Orient the photogate so that the center of mass of the pendulum bob passes through the photogate beam, at the lowest point of its swing.
3. Open the **Pendulum** file. (Click on the **File** menu and then on the **Open** menu item. Under the **Probes and Sensors** folder, double click on the **Photogate** folder. In the **Photogate** folder, double click on the **Pendulum Timer** file.)
4. The nature of the Pendulum Program: The Pendulum file is a program that uses the photogate to implement an event timer specific to pendulum motion. The specific nature of pendulum motion, and simple harmonic motion in general, is that in the course of moving through one cycle of its motion it will pass through the equilibrium point of its motion two times. The first pass initiates the event timer, the second pass is ignored, and the third pass will terminate the timer when the photogate beam is broken the third time. The time recorded is the time interval from the 1<sup>st</sup> breaking of the beam to the third breaking of the beam (the beginning of the next cycle). This will produce an experimental estimate for the period of the simple pendulum.

### **Experimental Procedure - Pendulum**

5. Measure the length of the pendulum string and record it in the Pendulum Data Table. The length of the pendulum is measured from the top end of the string where it emerges from the bottom of the pendulum clamp to the center of mass of the pendulum bob.
6. To begin an experimental run, move the pendulum bob about 15 degrees from the vertical and release it. The pendulum bob should travel in a plane and not trace out an elliptical path. If the bob moves in an elliptical path, start it again.
7. Click on the **Collect** button to start the measurement process.
8. Measure at least 20 complete cycles and then click the **Stop** button.
9. Measure the Average Period using the graph on the computer screen and the Logger Pro **Statistics** functions. Select a region of the graph that encompasses at least 20 period measurements. Click on the **Statistics** button and pick the **Mean** or **Average** option. Record this Average Period in the Pendulum Data Table.

10. Repeat the experimental procedure for 7 different string lengths ranging from 10 to 70 cm, releasing the pendulum bob each time from an angle of 15 degrees.
11. Plot a graph of  $T^2$  vs.  $L$ .
12. Perform a best fit to the data with a Linear Trendline. (Don't force the trendline through the origin). Display the equation of the trendline and the correlation coefficient.
13. Using the Excel worksheet function LINEST calculate the uncertainty (Std Error) in the value of the slope of the trendline.
14. Calculate the value of "g" from the slope. The slope is equal to  $4\pi^2/g$  and therefore  $g = 4\pi^2/\text{slope}$ .
15. The uncertainty in  $g = (\text{Slope Uncertainty}/\text{Slope}) * g$  (explained below "Uncertainty Analysis")
16. Compare to the accepted  $g = 9.81 \text{ m/s}^2$  and calculate the % error.

**Pendulum Data Table**

Length L (m)	Average Period T (s)	$T^2(\text{s}^2)$

Slope = \_\_\_\_\_ +/- Uncertainty \_\_\_\_\_  $\text{s}^2/\text{m}$

"g" from slope;  $g = 4\pi^2/\text{slope}$  \_\_\_\_\_ +/- Uncertainty \_\_\_\_\_  $\text{m/s}^2$

% error: \_\_\_\_\_

**Uncertainty Analysis:**

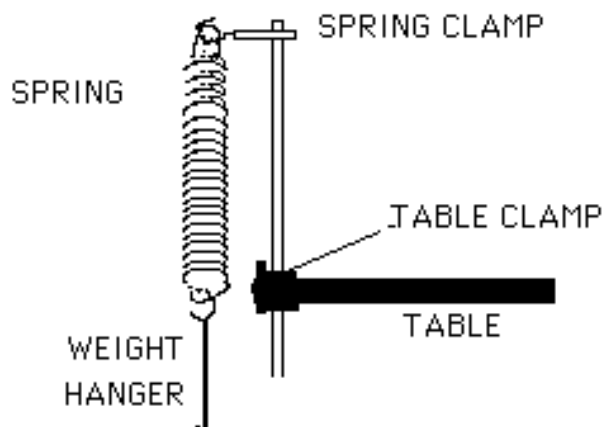
The uncertainties (std errors) computed by the Excel worksheet function LINEST are the uncertainties in the slope. In this lab we are interested in quantities calculated from these slope values. The questions arises: How do these slope uncertainties propagate into the final calculated quantities? The quick answer is to express the uncertainties as a fraction of their respective slopes and then apply these fractions to the respective calculated quantities to determine their uncertainties. With uncertainties in the low single digit percentage range the errors in this approximation are negligible.

### Theory - Spring

An example of simple harmonic motion also includes the oscillations of a mass attached to the end of a spring. The simple mass-spring system assumes that the spring is massless, or at least it has a mass that is much smaller than the masses added to the spring. The period of the oscillation depends on the parameters of the system, namely the spring constant, **k**, and the added mass, **m**. For the simple mass-spring system, this period is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where **m** is the suspended mass, and **k** is the spring constant.



*Figure 2*

### Equipment - Spring

Computer with Logger Pro S/W	Coil Spring	Slotted Weights
Interface with Cables	Spring Clamp	Meter Stick
Photogate with support hardware	Support Rod	Digital Scale
Weight Hanger		

### Equipment Procedure - Spring: Refer to Figure 2

1. Measure and record the mass of a spring,  $M_{\text{spring}}$ .
2. Attach the smaller end of the spring to the spring clamp and at the other end of the spring attach a 50-gram weight hanger plus an extra 50 grams..
3. **Photogate Positioning:** The photogate is mounted in a horizontal plane for this portion of the lab. It should be positioned vertically, relative to the mass-spring system, so that the beam line will be centered vertically on the middle of the weights when the system is in its equilibrium position. The photogate beam line needs to be offset so that the vertical pole of the weight hanger does not break the beam. In the course of its periodic motion the weights need to clear the beam by passing alternately above and below the plane of the

beam. This arrangement is necessary so that the pendulum program will be able to measure the period of the mass-spring system.

**Experimental Procedure - Spring:**

4. To begin an experimental run, pull the weight hanger down, until the top of the added weights clears the path of the photogate beam, and then release it. This will start the spring oscillating. Click the **Collect** button to start the measurement process. Let the system oscillate for at least 20 cycles and then click on the **Stop** button to terminate the measurement process.
5. Repeat the procedure for five additional masses (120, 140, 160, 180, 200 grams) by placing an additional 20-grams to the hanger for each trial.
6. Measure the Average Period using the graph on the computer screen and the Logger Pro **Statistics** functions. Select a region of the graph that encompasses at least 20 measurements. Click on the **Statistics** button and pick the **Mean** or **Average** option. Record this Average Period in the Spring Data Table.
7. Calculate the Average Period squared for each mass and record the results in the Pendulum Data Table.
8. Plot **T<sup>2</sup> vs. Mass** (Mass is the hanging mass) including the mass of the weight hanger.
9. Perform a best fit with a Linear Trendline. (Don't force the trendline through the origin). Display the equation of the trendline and the correlation coefficient. Using the Excel worksheet function LINEST calculate the uncertainty (Std Error) in the value of the slope of the trendline.
10. From  $T^2 = (4\pi^2/k)m$ , find **k**.  $k = (4\pi^2/\text{slope})$
11. Carry out the uncertainty analysis as described in the Pendulum section.

## Spring Data Table

Mass of the spring  $M_{\text{spring}}$  : \_\_\_\_\_(kg)

Hanging mass (kg)	Average Period T(s)	$T^2(s^2)$
0.100		
0.120		
0.140		
0.160		
0.180		
0.200		

Slope: \_\_\_\_\_ +/- Uncertainty \_\_\_\_\_ m/N;

$$\% \text{Uncertainty} = \text{Uncertainty} * 100 / \text{Slope} = \underline{\hspace{2cm}}$$

Spring Constant,  $k$ , calculated from the slope.

$$k = (4\pi^2 / \text{slope}) = \underline{\hspace{2cm}} \text{ +/- Uncertainty } \underline{\hspace{2cm}} \text{ N/m } (= k * \% \text{Uncertainty})$$

## Spring Mass Correction

During this lab we have assumed that the spring is massless. In actual fact the spring has a mass of about 160 grams and that is approximately equal to the total amount of mass that was added to the spring. Experimentally this can be seen in the non-zero value of the constant term in the trendline equation. This term should be zero since there is no constant term in the period equation. The trendline equation is telling us that we have under stated the mass of the system. The size of the missing-mass can be estimated by setting  $T^2$  equal to zero, in the trendline equation, and then solve for  $m$ . This will be a negative quantity because this is the mass that is missing. If this amount of mass was added to each of our data points the new trendline would pass through the origin. If this added mass is divided into the mass of the spring,  $M_{\text{spring}}$ , the result should be approximately 3.0. A straight forward theoretical analysis of the massive spring problem states that the correct mass to use is the added mass plus one-third of the spring mass,  $M_{\text{spring}}$ .

## Lab Report

Your report should follow the instructions in the document "Format for Formal Lab Reports."