Conservation of Energy Problems

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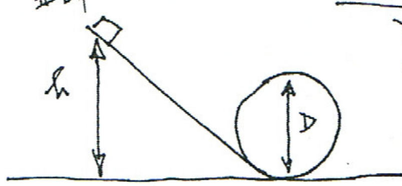
CHAPTER 6

#84



GIVEN:

$D = 20.0\,m$

$m = 988\,kg$

$h = 40.0\,m$

NO FRICTION

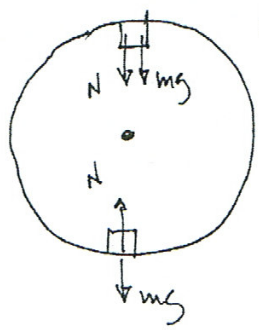
NO AIR RESISTANCE

QUES:

(a) WHAT IS V AT TOP OF LOOP?

(b.) FORCE EXERTED ON CAR BY TRACK AT TOP OF LOOP?

(c.) MIN HEIGHT h TO MAKE IT OVER THE TOP?



(a.) V AT BOTTOM OF INCLINE IS V_f

$$mgh = \tfrac{1}{2}mv_f^2 \longrightarrow v_f^2 = 2gh$$

AT TOP $\quad \tfrac{1}{2}mv_f^2 = mgD + \tfrac{1}{2}mv^2$

MULTIPLY BY $\frac{2}{m}$ $\quad v_f^2 = 2gD + v^2$

$$v^2 = v_f^2 - 2gD = 2gh - 2gD = 2g(h-D)$$

$$v = \sqrt{2(9.8)(40-20)} = 19.8\,m/s$$

(b.)

$$\Sigma F = -N - mg = -\frac{mv^2}{R}$$

$$N = \frac{mv^2}{R} - mg = \frac{m}{D/2}\cdot 2g(h-D) - mg = \left[\left(\frac{4(h-D)}{D}\right) - 1\right] mg$$

$$N = \left[\frac{4h - 4D - D}{D}\right] mg = \left[\frac{4h - 5D}{D}\right] mg = \left[4\frac{h}{D} - 5\right] mg$$

$$N = \left[4\left(\frac{40}{20}\right) - 5\right] mg = 3mg = 3(988)(9.8) = 29.0\,kN$$

(c.) min height $\rightarrow N = 0$

$$\frac{4h}{D} - 5 = 0 \qquad 4h = 5D$$

$$h = \frac{5}{4}D = \frac{5}{4}(20) = 25\,m$$

CLEARER APPROACH

STAY AT v^2 LEVEL

BOTTOM $v_f^2 = 2gh$

TOP $\frac{mv^2}{R} - mg = 0$

$$v_{min}^2 = gR$$

MIN AT TOP

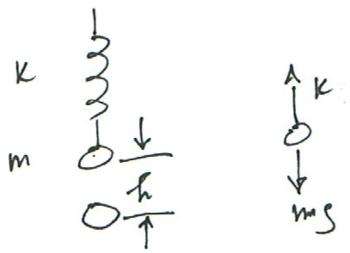
$$v_f^2 = 2gD + v_T^2$$

$$2gh = 2gD + gR$$

$$2gh = 4gR + gR = 5gR$$

$$h = \frac{5}{2}R$$

CHAP 6 # 37 p. 10



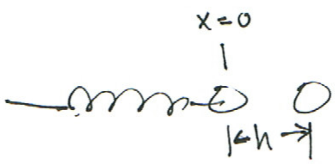
$$\Sigma F_y = k \;\; - mg = 0$$

$$kh = mg$$

$$k = \frac{mg}{h}$$

QUES:

WHAT IS THE TOTAL ENERGY OF THE SPRING-MASS SYSTEM?



HORIZONTAL

$$E_T = \frac{1}{2}kh^2$$

AFTER RELEASE

$$E_T = \frac{1}{2}kh^2 = \frac{1}{2}mv^2 + \frac{1}{2}k\Delta x^2$$

SUB $k = \frac{mg}{h}$

$$E_T = \frac{1}{2}\left(\frac{mg}{h}\right)h^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mg}{h}\right)\Delta x^2$$

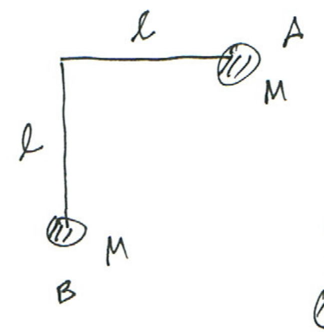
$$E_T = \frac{1}{2}mgh = \frac{1}{2}mv^2 + \frac{1}{2}mgh\left(\frac{\Delta x}{h}\right)^2$$

$\Delta x = 0$ $\quad \frac{1}{2}mgh = \frac{1}{2}mv^2$

$\Delta x = h$ $\quad \frac{1}{2}mgh = \frac{1}{2}mv^2 + \frac{1}{2}mgh$

$$\rightarrow \frac{1}{2}mv^2 = 0$$

CHAP 7 #32 p. 11



QUES: WHAT IS THE TENSION IN THE STRING AT POSITION B?

THE MASS IS TRAVELING IN AN ARC
→ CENTRIPETAL MOTION

$$\Sigma F = T - mg = ma_c = \frac{mv^2}{l}$$

$$T = mg + \frac{mv^2}{l}$$

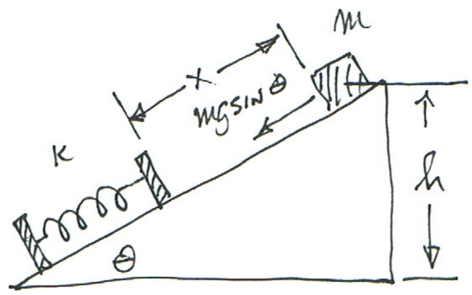
FROM CONSERVATION OF TOTAL MECHANICAL ENERGY

PE_{max} AT TOP $= KE_{max}$ AT BOTTOM

$$mgl = \frac{1}{2}mv^2$$

$$\sqrt{2gl} = v$$

CHAPTER 7 #39 p. 14



FRICTIONLESS

QUES: FIND COMPRESSION OF SPRING WITH ACCEL OF MASS IS ZERO.

FORCE ON m IS $mg\sin\theta$

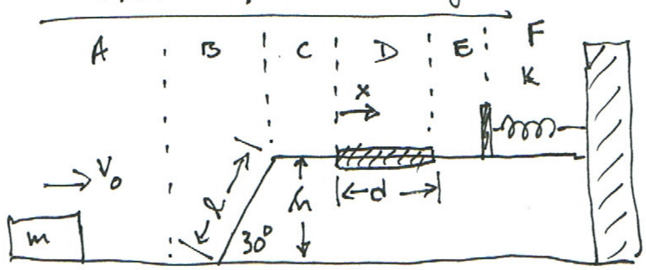
ACCEL = 0 WHEN SPRING FORCE EQUALS $mg\sin\theta$

$$\Sigma F = Kx - mg\sin\theta = 0$$

$$x = mg\sin\theta$$

$$K = \frac{mg\sin\theta}{\Delta x}$$

CHAPTER 7 # 52 p. 20



GIVEN:

$m = 5.0 kg$

$v_0 = 20 m/s$

$\mu_k = 0.40$

$d = 15 m.$

$\Delta x = 2m$ (SPRING COMPRESSION)

QUES: $k = ?$

$h = l \sin 30$

THREE INTERACTIONS: GRAVITY, FRICTION + SPRING.
ALL ARE CONSERVATIVE EXCEPT FRICTION

INITIALLY $E_T = \frac{1}{2}mv_0^2$ REGION A — TOTAL ENEGY FOR PROBLEM

REGION B

$$E_T = \frac{1}{2}mv_0^2 = \frac{1}{2}mv_{(y)}^2 + mgy; \quad 0 \le y \le h$$

REGION C

$$E_T = \frac{1}{2}mv_0^2 = \frac{1}{2}mv_c^2 + mgh$$

REGION D

$$E_T = \frac{1}{2}mv_0^2 = \frac{1}{2}mv(x)^2 + mgh + fx \quad ; \quad f = \mu_k N = \mu_k mg$$

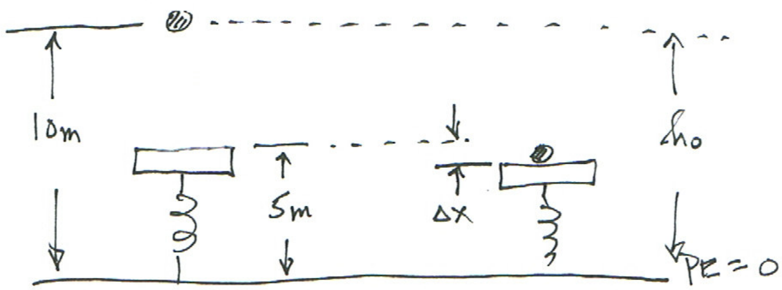
REGION E

$$E_T = \frac{1}{2}mv_0^2 = \frac{1}{2}mv_E^2 + mgh + fd$$

REGION F (AT FULL COMPRESSION $v_F = 0$)

$$E_T = \frac{1}{2}mv_0^2 = \underbrace{\frac{1}{2}mv_F^2}_{= 0} + mgh + fd + \frac{1}{2}k\Delta x^2$$

CHAPTER 7 #54 p. 20



GIVEN:

$m = 5\,kg$ $h_0 = 10\,m$

$k = 200\,N/m$

$\Delta x = 1.5\,m$

QUES: FIND ΔE LOST TO SOUND, THERMAL, ETC.

$E_T = mgh_0$

FINAL

$E_T = mgh_0 = KE + GPE + SPRING\,PE + \Delta E$

$v = 0$ SO $KE = \frac{1}{2}mv^2 = 0$

$E_T = mgh_0 = 0 + mg(5 - \Delta x) + \frac{1}{2}k\,\Delta x^2 + \Delta E$

$5(9.8)(10) = 5(9.8)(5-1.5) + \frac{1}{2}(200)(1.5)^2 + \Delta E$

$490\,J = 171.5\,J + 225\,J + \Delta E$

$\Delta E = 490 - 397$

$\Delta E = 93\,J$