Heat Transfer Problems With Solutions

Physics 1401

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WBL

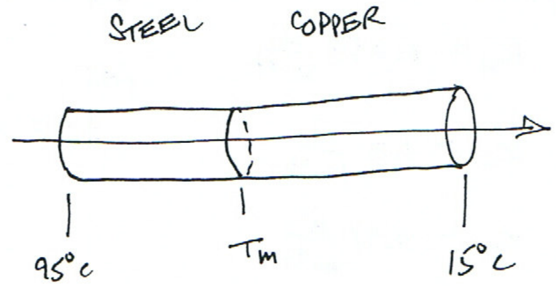
CHAP 11 #80

$K_s = 46 \frac{W}{m \cdot {}^\circ C}$

$K_c = 390 \frac{W}{m \cdot {}^\circ C}$

$R = 5.0\ cm.$

$L = 4.0\ cm.$



SAME HEAT FLOWS THROUGH BOTH CYLINDERS

$\therefore Q_s = Q_c \quad$ OR $\quad \frac{Q_s}{Q_c} = 1$

$$Q_s = K_s A \frac{\Delta T_s}{\Delta X_s} \cdot \Delta t \qquad Q_c = K_c A \frac{\Delta T_c}{\Delta X_c} \cdot \Delta t$$

$$\Delta T_s = 95 - T_m \qquad \Delta T_c = T_m - 15$$

$$\Delta X_s = \Delta X_c = 4.0\ cm$$

$$\frac{Q_s}{Q_c} = \frac{K_s A \frac{\Delta T_s}{\Delta X_s} \cdot \Delta t}{K_c A \frac{\Delta T_c}{\Delta X_c} \Delta t} = \frac{K_s}{K_c} \frac{\Delta T_s}{\Delta T_c} = \frac{K_s}{K_c} \frac{(95 - T_m)}{T_m - 15} = 1$$

$$K_s(95 - T_m) = K_c(T_m - 15)$$

$$K_s 95 - K_s T_m = K_c T_m - 15 K_c$$

$$K_s 95 + 15 K_c = (K_c + K_s) T_m$$

$$\frac{95 K_s + 15 K_c}{K_s + K_c} = T_m$$

$$\frac{95 + 15 \frac{K_c}{K_s}}{1 + \frac{K_c}{K_s}} = T_m$$

$$\frac{K_c}{K_s} = \frac{390}{46} = 8.48$$

$$T_m = \frac{95 + 15(8.48)}{1 + 8.48}$$

$$\boxed{T_m = 23.4\,^\circ C}$$

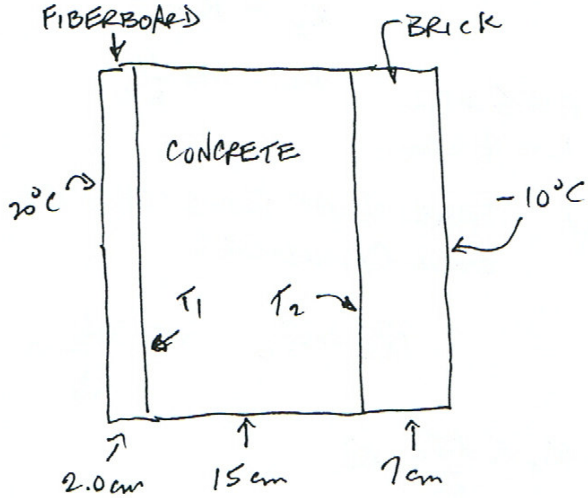
$$Q_s = K_s A \frac{\Delta T_s}{\Delta X_s} \Delta t$$

$$= 46 \cdot \pi \cdot (0.05)^2 \frac{(95 - 23.4)}{0.04} 20 \cdot 60$$

$$Q_s = \frac{46(3.14)}{0.04} 25 \times 10^{-4} (71.6)(1200)$$

$$\boxed{Q_s = 7.76 \times 10^5\ J}$$

WBL - CHAP 11 #78 - ANOTHER APPROACH



AREA = 3.5 m × 5.0 m.

$K_F = 0.059 \frac{W}{m \cdot °C}$

$K_C = 1.3 \frac{W}{m \cdot °C}$

$K_B = 0.71 \frac{W}{m °C}$

QUES: ΔQ in 1 hr = ?

ANS: 3.4×10^6 J

$$Q = KA \frac{\Delta T}{\Delta x} \Delta t \qquad \text{LET} \quad Q' \equiv \frac{Q}{A \cdot \Delta t} = K \frac{\Delta T}{\Delta X}$$

$$Q'_F = Q'_C = Q'_B = Q$$

$$\frac{K_F(20 - T_1)}{X_F} = \frac{K_C(T_1 - T_2)}{X_C} = \frac{K_B(T_2 + 10)}{X_B}$$

2 EQN & 2 UNKNOWNS STD SOLN.

DIFFERENT APPROACH

$$\frac{X_F}{K_F} Q'_F = 20 - T_1 \; ; \quad \frac{X_C}{K_C} Q'_C = T_1 - T_2 \; ; \quad \frac{X_B}{K_B} Q'_B = T_2 + 10$$

$$\frac{X_F}{K_F} Q' + \frac{X_C}{K_C} Q' + \frac{X_B}{K_B} Q' = (20 - T_1) + (T_1 - T_2) + (T_2 + 10) = 20 + 10 = 30$$

$$Q'\left(\frac{X_F}{K_F} + \frac{X_C}{K_C} + \frac{X_B}{K_B}\right) = 30$$

$$Q' = \frac{30}{\frac{X_F}{K_F} + \frac{X_C}{K_C} + \frac{X_B}{K_B}}$$

$$Q' = \frac{30}{\frac{0.02}{0.059} + \frac{0.15}{1.3} + \frac{0.07}{0.71}}$$

$$Q' = \frac{30}{0.339 + 0.115 + 0.099} = \frac{30}{0.533} = 54.2 = \frac{Q}{A \cdot \Delta t} = \frac{W}{m^2 \cdot s}$$

$$Q = Q' \cdot A \cdot \Delta t = 54.2\,(3.5 \times 5)\,3600$$

$$Q = 3.42 \times 10^6 \text{ J}$$

EQUIVALENT RESISTANCE (THERMAL)

$$\frac{X_{TOT}}{K_{eff}} = \frac{X_F}{K_F} + \frac{X_C}{K_C} + \frac{X_B}{K_B}$$

$$\frac{1}{K_{eff}} = \frac{1}{K_F}\left(\frac{X_F}{X_{TOT}}\right) + \frac{1}{K_C}\left(\frac{X_C}{X_{TOT}}\right) + \frac{1}{K_B}\left(\frac{X_B}{X_{TOT}}\right)$$

A WEIGHT AVE OF INVERSE THERMAL CONDUCTIVITIES

$$\text{THERMAL RESISTANCE} = \frac{1}{\text{THERMAL CONDUCTIVITY}}$$

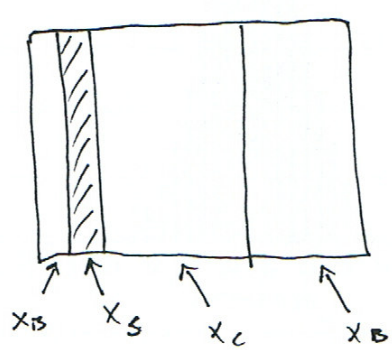
THIS IS THE WAY TO APPROACH MORE COMPLICATED STRUCTURES

WBL - CHAP 11 - #79.

ADD S BETWEEN FIBERBOARD - CONCRETE
TO DROP Q BY 1/2 - WHAT IS X_S?

OLD WAY $\longrightarrow$ 4 EQN + 4 UNKNOWNS Q, T_1, T_2, T_3

THERMAL RESISTANCE WAY



$X_B = 2.0$ cm $\quad K_B = 0.059 \frac{W}{m \cdot °C}$

$X_S = ?$ $\quad K_S = 0.042 \frac{W}{m °C}$

$X_C = 15$ cm $\quad K_C = 1.3 \frac{W}{m \cdot °C}$

$X_B = 7$ cm $\quad K_B = 0.71 \frac{W}{m °C}$

$$Q' \longrightarrow \frac{Q'}{2} = \frac{54.2}{2} = 27.1 \frac{J}{m^2 \cdot s} = \frac{\Delta T}{\frac{X_{TOT}}{K_{eff}}} = \frac{30}{\frac{X_{TOT}}{K_{eff}}} \; ; \; \frac{X_{TOT}}{K_{eff}} = \frac{30}{27.1} = 1.11$$

$$\frac{X_{TOT}}{K_{eff}} = \frac{X_F}{K_F} + \frac{X_S}{K_S} + \frac{X_C}{K_C} + \frac{X_B}{K_B}$$

$$X_S = K_S\left(\frac{X_{TOT}}{K_{eff}} - \frac{X_F}{K_F} - \frac{X_C}{K_C} - \frac{X_B}{K_B}\right) = 0.042(1.11 - 0.533) = 0.024\,m = 2.4\,cm$$