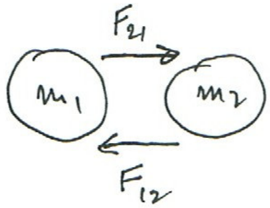
MOMENTUM FROM NEWTON'S LAWS

3/1/09

FROM NEWTON'S 3RD LAW



$$F_{12} = -F_{21}$$

$$F_{12} = |F_{21}| \equiv F$$

FORCE EXERTED OVER A TIME PERIOD Δt

$$a_1 = \frac{-F}{m_1} \qquad a_2 = +\frac{F}{m_2}$$

NEWTON'S 2ND LAW

EQUAL & OPPOSITE FORCES
BUT DIFFERENT ACCELERATIONS $\longrightarrow$ DIFFERENT VELOCITIES

$$v_1 = v_{01} + a_1 \Delta t \qquad v_2 = v_{02} + a_2 \Delta t$$

$$v_1 = a_1 \Delta t \qquad v_2 = a_2 \Delta t$$

$$v_1 = -\frac{F}{m_1} \Delta t \qquad v_2 = \frac{F}{m_2} \cdot \Delta t$$

MOVE MASS TO THE L.H.S.

$$m_1 v_1 = -F \Delta t \qquad m_2 v_2 = F \Delta t$$

THIS IS MOMENTUM

NOTICE THE "EQUAL AND OPPOSITE" IDEA AGAIN.

ADD L.H.S AND R.H.S.

$$m_1 v_1 + m_2 v_2 = -F \Delta t + F \Delta t = 0$$

INDEPENDENT OF F AND INDEPENDENT OF Δt

DEFINE THE TOTAL LINEAR MOMENTUM IS P_T

$$P_T = m_1 v_1 + m_2 v_2$$

$$\left.\begin{aligned} P_T^{BEFORE} &= m_1(0) + m_2(0) = 0 \\ P_T^{AFTER} &= m_1 v_1 + m_2 v_2 = 0 \end{aligned}\right\}$$

TOTAL LINEAR MOMENTUM IS CONSERVED IF THERE ARE NO NET EXT FORCES.

WHAT ABOUT THE DISTANCES TRAVELED: ΔX_1 AND ΔX_2?

$$m_1 v_1 + m_2 v_2 = 0$$

$$m_1 \frac{\Delta x_1}{\Delta t} + m_2 \frac{\Delta x_2}{\Delta t} = 0 \qquad \therefore \quad m_1 \Delta x_1 + m_2 \Delta x_2 = 0$$

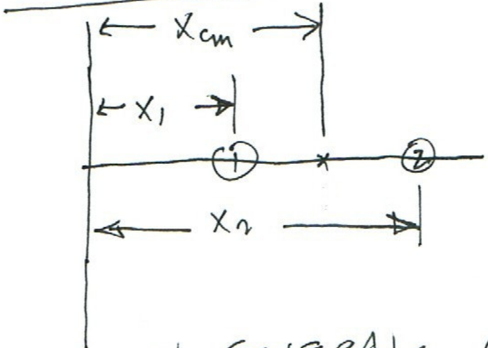
$$\Delta x_1 = -\frac{m_2}{m_1} \Delta x_2$$

NOTE: IF $m_1 = m_2$ THEN $\Delta x_1 = -\Delta x_2$

WHAT DOES THIS MEANS?

ALSO SMALLER MASS TRAVELS THE LARGER DISTANCE.

THE CONCEPT OF CENTER OF MASS



SHORT CM DEMO

IF $m_1 = m_2$ THEN THE X_{cm} IS IN THE EXACT MIDDLE.

$$X_{cm} = \frac{X_1 + X_2}{2}$$

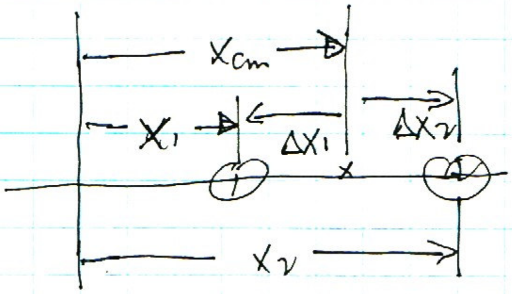
IN GENERAL $(m_1 + m_2) X_{cm} = m_1 X_1 + m_2 X_2$

$$\therefore \quad X_{cm} = \frac{m_1}{m_1 + m_2} X_1 + \frac{m_2}{m_1 + m_2} X_2$$

A MASS WEIGHTED AVERAGE

LET $X_{cm} \rightarrow$ ORIGIN.

THEN ALL DISTANCE ARE MEASURED FROM THE CM.



$$\left.\begin{aligned} \Delta X_1 &= X_1 - X_{cm} \\ \Delta X_2 &= X_2 - X_{cm} \end{aligned}\right\} \text{ MEAS REL. TO CM.}$$

$$\left.\begin{aligned} X_1 &= \Delta X_1 + X_{cm} \\ X_2 &= \Delta X_2 + X_{cm} \end{aligned}\right\} \rightarrow (m_1 + m_2) X_{cm} = m_1 X_1 + m_2 X_2$$

$$(m_1 + m_2) X_{cm} = m_1 (\Delta X_1 + X_{cm}) + m_2 (\Delta X_2 + X_{cm})$$

$$(m_1 + m_2) X_{cm} = m_1 \Delta X_1 + m_2 \Delta X_2 + (m_1 + m_2) X_{cm}$$

$$0 = m_1 \Delta X_1 + m_2 \Delta X_2$$

$\rightarrow$ IF YOU MEASURE DISPLACEMENT RELATIVE TO THE CM

THEREFORE, OVER AN ARBITRARY TIME INTERVAL Δt

$$m_1 \frac{\Delta X_1}{\Delta t} + m_2 \frac{\Delta X_2}{\Delta t} = m_1 V_1 + m_2 V_2 = 0 \quad \text{RELATIVE TO CM.}$$

EXACTLY THE CONDITIONS IN OUR 1ST EXAMPLE. THE TWO OBJECTS START AT THE CENTER OF MASS

IN OUR CASE $V_{cm} = 0$

IN GENERAL IF P_T IS CONSERVED $V_{cm} = \text{CONSTANT}$