Momentum

Problems & Solutions

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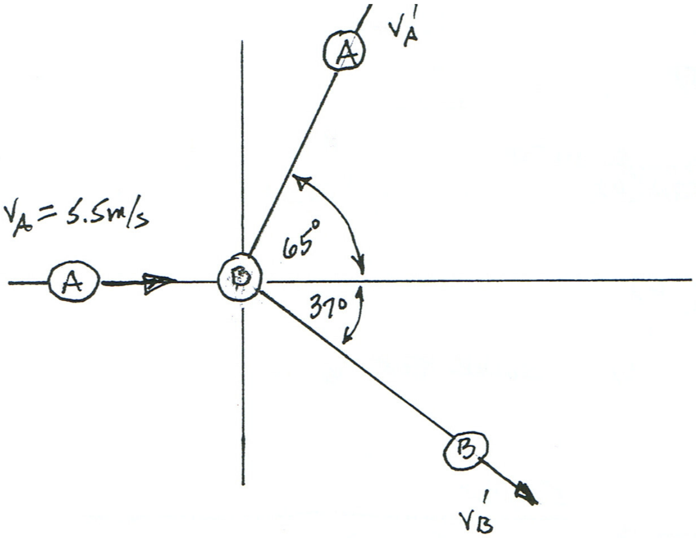


PROBLEM: MASS m_A STRIKES MASS m_B WHICH IS INITIALLY AT REST. DETERMINE V_A' AND V_B'.

GIVEN:

$$m_A = 0.025\,kg; \quad m_B = 0.050\,kg$$

$$\theta_A' = 65°; \quad \theta_B' = 37°$$

$$V_A = 5.5\,m/s; \quad V_B = 0$$

BEFORE

$$\vec{P_T} = \vec{P_A}$$

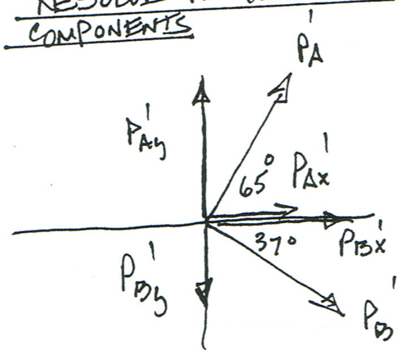
AFTER

$$\vec{P_T} = \vec{P_A'} + \vec{P_B'}$$

$$\therefore \vec{P_A} = \vec{P_A'} + \vec{P_B}$$

CONSERVATION OF LINEAR MOMENTUM.

RESOLVE VECTORS INTO COMPONENTS



$$P_{Ay}' = m_A V_{Ay}'$$
$$P_{Ax}' = m_A V_{Ax}'$$

$$P_{By}' = m_B V_{By}'$$
$$P_{Bx}' = m_B V_{Bx}'$$

$$\left.\begin{aligned} P_{Ax} &= P_{Ax}' + P_{Bx}' \\ P_{Ay} &= P_{Ay}' + P_{By}' \end{aligned}\right\} \text{STARTING EQUATIONS}$$

$$P_{Ay} = 0$$

WRITE MOMENTUM EQN IN TERMS OF MASSES AND VELOCITIES

Y-DIR $\quad m_A V_{Ay}' = m_B V_{By}' \quad (1)$

X-DIR $\quad m_A V_A = m_A V_{Ax}' + m_B V_{Bx}' \quad (2)$

SET UP MASS RATIOS

$$V_{Ay}' = \frac{m_B}{m_A} V_{By}' \qquad \frac{m_B}{m_A} = \frac{0.050}{0.025} = 2$$

$$V_A = V_{Ax}' + \frac{m_B}{m_A} V_{Bx}'$$

$$\boxed{\begin{aligned} V_{Ay}' &= 2 V_{By}' \quad (3) \\ V_A &= V_{Ax}' + 2 V_{Bx}' \quad (4) \end{aligned}}$$

NOW WE SUBSTITUTE THE TRIG.

$$V_{Ay}' = V_A' \sin 65$$
$$V_{Ax}' = V_A' \cos 65$$
$$V_{By}' = V_B' \sin 37$$
$$V_{Bx}' = V_B' \cos 37$$

$$V_A' \sin 65 = 2V_B' \sin 37 \quad (5)$$

$$V_A' = 2V_B' \frac{\sin 37}{\sin 65} \quad (5a)$$ SUBSTITUTE INTO EQN (6)

$$V_A = V_A' \cos 65 + 2V_B' \cos 37 \quad (6)$$

$$V_A = \left[2V_B' \frac{\sin 37}{\sin 65} \right] \cos 65 + 2V_B' \cos 37$$ SOLVE FOR V_B'

$$V_B' = \frac{V_A/2}{\sin 37 \frac{\cos 65}{\sin 65} + \cos 37} = \frac{5.5/2}{0.6018\left(\frac{0.4226}{0.9063}\right) + 0.7986}$$

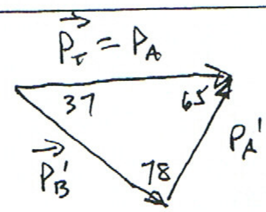
$$\boxed{V_B' = 2.55 \text{ m/s}}$$

SUBSTITUTE INTO (5a) AND SOLVE FOR V_A'

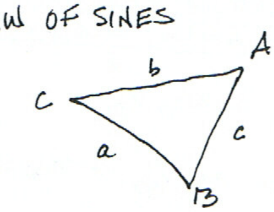
$$V_A' = 2V_B' \frac{\sin 37}{\sin 65} = 2(2.55) \cdot \frac{0.6018}{0.9663}$$

$$\boxed{V_A' = 3.39 \text{ m/s}}$$

$$\vec{P}_T = \vec{P}_A' + \vec{P}_B'$$



LAW OF SINES



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$P_T = mV = (0.025)(5.5)$$

$$P_T = 0.138 \ \frac{\text{kg-m}}{\text{s}}$$

$$\frac{P_A}{\sin 78} = \frac{0.138}{0.978} = 0.141 = \frac{P_B'}{\sin 65} = \frac{P_A'}{\sin 37}$$

$$P_B' = m_B V_B' = 0.141\,(0.906)$$

$$V_B' = \frac{0.141\,(0.906)}{0.050} = 2.55 \text{ m/s}$$

$$\boxed{V_B' = 2.55 \text{ m/s}}$$

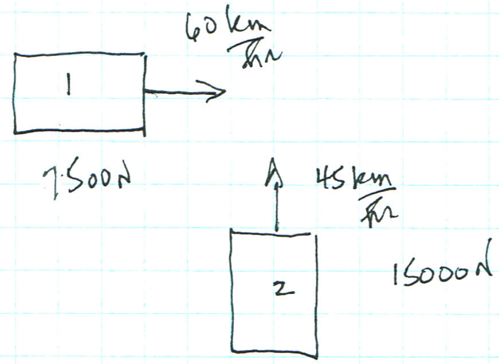
$$P_A' = m_A V_A' = 0.141\,(0.601)$$

$$V_A' = \frac{0.141\,(0.601)}{0.025}$$

$$\boxed{V_A' = 3.39 \text{ m/s}}$$

CHAPTER 6

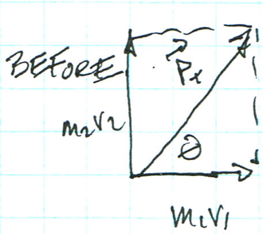
45.



a.) $v_f = ?$

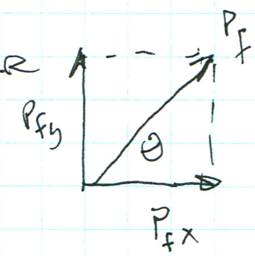
b.) $\frac{KE_f - KE_i}{KE_i} = ?$ FRACTION LOST

MOMENTUM IS CONSERVED SINCE $\Sigma \vec{F}_{ext} = 0$



SAME θ

$\vec{P}_T = \vec{P}_f$



X DIRECTION

$$P_x = P_{fx}$$

$$m_1 v_1 = (m_1 + m_2) v_f \cos\theta$$

TWO EQN BUT ONLY 1 UNKNOWN v_f

y DIRECTION

$$P_y = P_{fy}$$

$$m_2 v_2 = (m_1 + m_2) v_f \sin\theta$$

WE CAN FIND θ FROM THE GIVEN INFO – OR WE CAN FIND IT FROM THESE EQN.

TAKE

$$\frac{m_2 v_2}{m_1 v_1} = \frac{(m_1 + m_2) v_f \sin\theta}{(m_1 + m_2) v_f \cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\frac{m_2 g}{m_1 g} \frac{v_2}{v_1} = \tan\theta$$

$$\frac{(150\cancel{00})}{75\cancel{00}} \frac{(45)}{60} = \frac{2 \cdot 9}{1 \cdot 12} = \frac{3}{2} = 1.5 = \tan\theta \, ; \quad \boxed{\theta = \tan^{-1}(1.5) = 56.5^\circ \text{ TO THE N OF E}}$$

GOING BACK TO EITHER X OR Y EQN.

$$v_f = \frac{m_1 v_1}{(m_1 + m_2)\cos\theta} = \frac{v_1}{\left(1 + \frac{m_2}{m_1}\right)\cos\theta} = \frac{v_1}{\left(1 + \frac{m_2 g}{m_1 g}\right)\cos\theta} = \frac{60}{(1+2)\cos(56.5)}$$

$$v_f = 20/\cos(56.5) = \boxed{36.2 \text{ km/hr}}$$

CHAPTER 6

#45 CONTINUED

b.) FRACTION OF KE LOST $= \dfrac{KE_f - KE_i}{KE_i} = \dfrac{KE_f}{KE_i} - 1$

$$\frac{KE_f}{KE_i} = \frac{\frac{1}{2}(m_1+m_2)v_f^2}{\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2} = \frac{\left(1+\frac{m_2}{m_1}\right)v_f^2}{v_1^2 + \frac{m_2}{m_1}v_2^2}$$

REMEMBER : $\dfrac{m_2}{m_1} = \dfrac{m_2 g}{m_1 g} = \dfrac{15000}{7500} = 2$

$$\frac{KE_f}{KE_i} = \frac{(1+2)v_f^2}{v_1^2 + 2v_2^2} = \frac{3v_f^2}{v_1^2+2v_2^2} = \frac{3(36.2)^2}{(60)^2 + 2(45)^2}$$

$$\frac{KE_f}{KE_i} = \frac{3931}{7650} = 0.514$$

FRACTION OF KE LOST $= \dfrac{KE_f}{KE_i} - 1 = 0.514 - 1 = -0.486$

$$\boxed{= -49\%}$$

(5A)

PHYSICS ON ICE

QUES: FOR MAX V SHOULD HE THROW BOTH MITTENS AT ONCE OR ONE AT A TIME

MAN ON ICE THROWING MITTENS

IF HE THROWS A GLOVE HE WILL START MOVING IN THE OPPOSITE DIRECTION

CONS. OF MOMENTUM IS PRINCIPLE

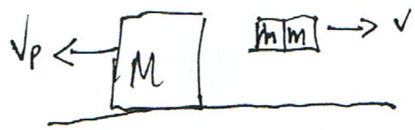
MOMENTUM BEFORE = MOMENTUM AFTER

0 = 0

A. BOTH AT ONCE

$$\Sigma P = 0 = 2mV + MV_p = 0$$

NO DIRECTIONS ASSUMED



$$V_p = -\frac{2m}{M} V$$

COMPARE

EQN TELLS US THE MOMENTUM VECTORS ARE EQUAL AND OPPOSITE

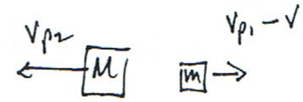
B. THROWN SEPARATELY TOTAL MOMENTUM IS ZERO

1st $\quad \Sigma P = 0 = mV + (M+m)V_{p_1} = 0 \qquad V_{p_1} = -\frac{m}{M+m} V$

2nd INITIAL CONDITIONS – TOTAL MOMENTUM IS NON-ZERO

THROWING SPEED IS THE SAME IN ALL CASES

NOTE: 1st MITTEN ISN'T PART OF THE SYSTEM FOR THROWING THE 2ND MITTEN



MOMENTUM BEFORE = AFTER RELATIVE TO ICE

$$(M+m)V_{p_1} = m(V_{p_1} - V) + MV_{p_2}$$

$$MV_{p_1} + mV_{p_1} = mV_{p_1} - mV + MV_{p_2}$$

$$MV_{p_1} + mV = MV_{p_2}$$

$$V_{p_1} + \frac{m}{M} V = V_{p_2}$$

"SHARING RATIO"

SUB MAGNITUDE OF V_{p_1}

$$\frac{m}{M+m} V + \frac{m}{M} V = V_{p_2}$$

$$\left[\frac{m}{M+m} + \frac{m}{M}\right] V = V_{p_2}$$

COMPARISON $V_{p_2} : V_p$ SEE OTHER SIDE FOR DETAILS

$$\left[\frac{m}{M+m} + \frac{m}{M}\right] = \frac{2m}{M} \frac{\left(1 + \frac{1}{2}\frac{m}{M}\right)}{1 + \frac{m}{M}}$$

$$\therefore \frac{V_{p_2}}{V_p} = \frac{\frac{2m}{M} \frac{\left(1 + \frac{1}{2}\left(\frac{m}{M}\right)\right)}{1 + \frac{m}{M}} V}{\frac{2m}{M} V}$$

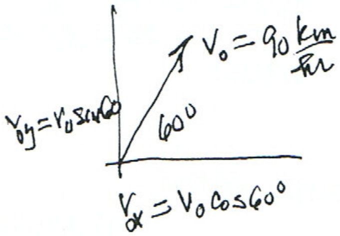
$$\frac{V_{p_2}}{V_p} = \frac{1 + \frac{1}{2}\left(\frac{m}{M}\right)}{1 + \frac{m}{M}} < 1$$

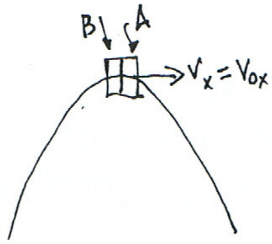
ALWAYS

$\therefore V_{p_2} < V_p$

HENCE BEST STRATEGY: THROW 2 AT ONCE

PROJECTILE MOTION





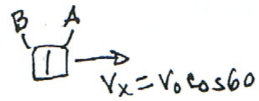
"B" FALLS STRAIGHT DOWN AS IF IT HAD BEEN RELEASED FROM REST

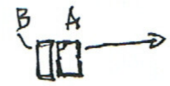
QUES: HOW FAR FROM THE GUN DOES FRAGMENTS "A" LAND?

82.8 m

AT THE TOP OF THE ARC
- BEFORE THE EXPLOSION

$V_y = 0$; $V_x = V_0 \cos 60^\circ$





MOMENTUM CONSERVATION

X-DIRECTION: $m V_{0x} = \frac{m}{2} V'_{Ax} + \frac{m}{2} V'_{Bx}$

Y-DIRECTION $m V_y = 0 = \frac{m}{2} V'_{Ay} + \frac{m}{2} V'_{By}$ } IN GENERAL

BUT $V'_{By} = 0 \quad \therefore \quad V'_{Ay} = 0$

ALSO $V'_{Bx} = 0 \quad \therefore \quad V'_{Ax} = 2V_{0x}$ } IN THIS SPECIFIC CASE

EXPLOSION IS JUST ENOUGH TO REDUCE ITS X VEL TO ZERO

RANGE IS DIVIDED INTO TWO VELOCITY PORTIONS

A PORTION AT V_{0x} AND A PORTION AT $2V_{0x}$

THE TIME FOR THE 1ST PORTION IS THE TIME TO REACH THE TOP OF THE TRAJECTORY.

$$V_y = V_{0y} - g t_1 = 0 \qquad t_1 = \frac{V_{0y}}{g}.$$

THE SECOND PORTION IS THE TIME IT TAKES PART "A" TO FALL TO THE GROUND FROM A HEIGHT = THE TOP OF THE TRAJECTORY

- THIS IS THE SAME TIME AS IN THE CASE OF NO EXPLOSION.
- IT IS ALSO EQUAL TO THE TIME TO REACH THE TOP
- "A" + "B" WILL HIT THE GROUND AT THE SAME TIME

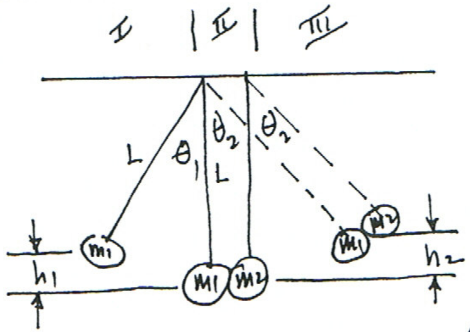
$$T = t_1 + t_2 = \frac{V_{0y}}{g} + \frac{V_{0y}}{g} = \frac{2V_{0y}}{g}$$

$$\text{RANGE} = \frac{V_{0y}}{g}\left(V_{0x} + 2V_{0x}\right) = \frac{3V_{0y}}{g} \cdot V_{0x} = \frac{3V_0^2 \sin 60 \cos 60}{g} =$$

$$\text{RANGE} = \frac{3 \cdot \left(90\times10^3 \cdot \frac{1}{3600}\right)^2 \frac{\sqrt{3}}{2} \cdot \frac{1}{2}}{9.8} = 82.8\,m$$

MOMENTUM + ENERGY

QUES: WHAT IS θ_2



$h_1 = L - L\cos\theta_1$

$h_1 = L(1-\cos\theta_1)$

$h_2 = L(1-\cos\theta_2)$

PART I (TOP → BOTTOM)

PE → KE

ENERGY CONSERVED

$PE_{max} = \frac{1}{2}m_1 v_1^2$

$m_1 g h_1 = \frac{1}{2} m_1 v_1^2$

$\sqrt{2gh_1} = v_1$

$\sqrt{2gL(1-\cos\theta_1)} = v_1$

PART II (JUST COLLISION)

COLLISION

$\vec{P}$ CONSERVED

KE NOT CONSERVED

(BECAUSE THEY STICK)

$m_1 v_1 = (m_1+m_2)v'$

$\frac{m_1}{m_1+m_2} \cdot v_1 = v'$

$\frac{m_1}{m_1+m_2}\sqrt{2gL(1-\cos\theta_1)} = v'$

PART III (BOTTOM TO TOP)

KE → PE

ENERGY CONSERVED

$\frac{1}{2}(m_1+m_2)v'^2 = (m_1+m_2)gh_2$

$\frac{1}{2}v'^2 = gh_2$

$\frac{v'^2}{2g} = h_2 = L(1-\cos\theta_2)$

$$h_2 = \frac{v'^2}{2g} = \frac{1}{2g}\left(\frac{m_1}{m_1+m_2}\right)^2 (2gL(1-\cos\theta_1))$$

$$h_2 = \cancel{L}(1-\cos\theta_2) = \frac{1}{\cancel{2g}}\left(\frac{m_1}{m_1+m_2}\right)^2(\cancel{2gL}(1-\cos\theta_1))$$

$$(1-\cos\theta_2) = \left(\frac{m_1}{m_1+m_2}\right)^2(1-\cos\theta_1)$$

Multiply both sides by L. $h_1 = L(1-\cos\theta_1)$; $h_2 = L(1-\cos\theta_2)$

$$h_2 = \left(\frac{m_1}{m_1+m_2}\right)^2 h_1$$

If $m_1 = m_2$

$$h_2 = \left(\frac{1}{2}\right)^2 h_1 = \frac{1}{4}h_1$$

$$\boxed{\text{NOTE: FOR } m_1 = m_2\text{, IF THE MASSES DON'T STICK } h_2 = h_1}$$