Chapter 2

One Dimensional Motion

Motion in One Dimension

- Displacement, Velocity and Speed
- Acceleration
- Motion with Constant Acceleration

Introduction

- Kinematics - Concepts needed to describe motion - displacement, velocity & acceleration.
- Dynamics - Deals with the effect of forces on motion.
- Mechanics - Kinematics + Dynamics

Goals of Chapter 2

Develop an understanding of kinematics that comprehends the interrelationships among

- physical intuition
- equations
- graphical representations

When we finish this chapter you should be able to move easily among these different aspects.

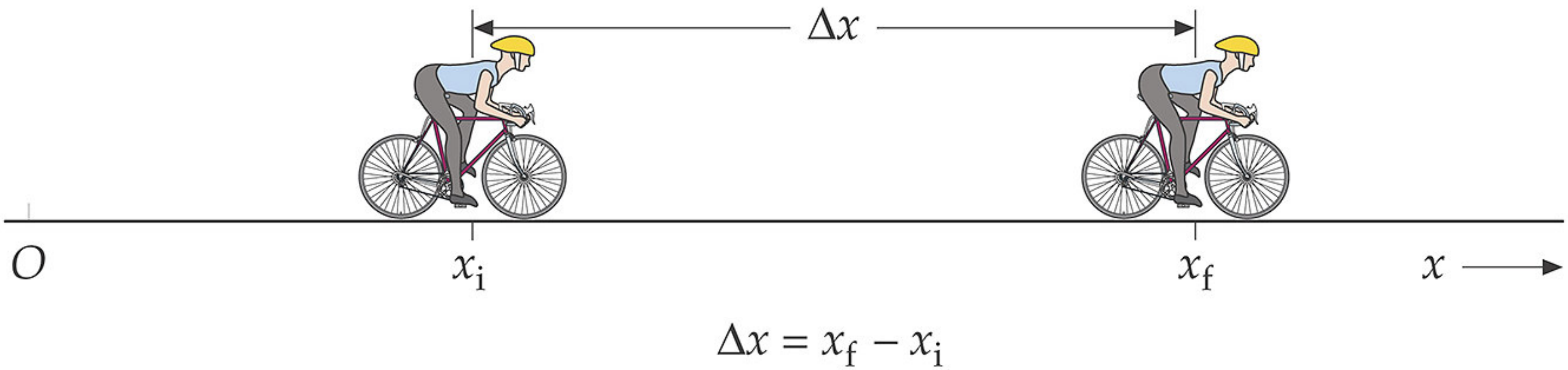
Kinematic Quantities Overview

Scalar	**Distance**	**Speed**	
Vector	**Displacement**	**Velocity**	**Acceleration**
Units	**m**	**m/s**	**m/s^2**

The words speed and velocity are used interchangably in everyday conversation but they have distinct meanings in the physics world.

Displacement

Position along a 1-dimensional coordinate axis is denoted by the coordinate value "x"

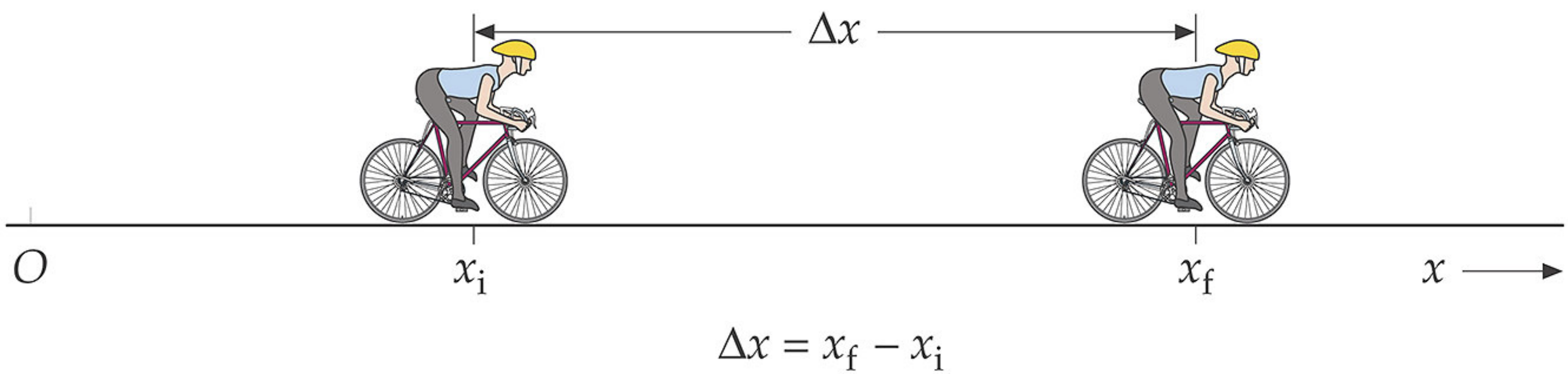


Δ represents a difference in a quantity. The "inital value" is always subtracted from the "final value"

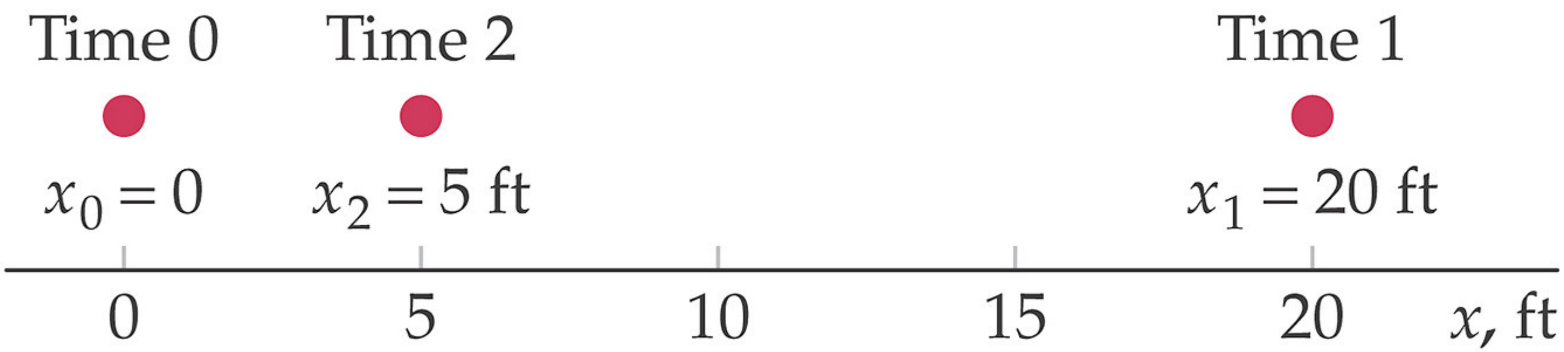
Displacement

The displacement is a vector quantity. It's specification requires two quantities: a magnitude and a direction.

The magnitude is just the size of Δx and the direction is represented by the sign of Δx



Displacement Example



$$s_{02} = s_{01} + s_{12} = 20 + 15 = 35ft$$

S is the variable designating distance, a scalar quantity. It is like the odometer reading on your car.

$$\Delta x_{02} = x_2 - x_0 = 5 - 0 = 5ft$$

Δx is the variable designating displacement, a vector quantity

The displacement only depends on the two points

Displacement Example

Alternatively - the displacements are additive

The individual displacements are given by

$$\Delta x_{01} = x_1 - x_0 = 20 - 0 = 20ft$$

$$\Delta x_{12} = x_2 - x_1 = 5 - 20 = -15ft$$

Adding the displacements together we see a cancellation

$$\Delta x_{01} + \Delta x_{12} = (x_1 - x_0) + (x_2 - x_1) = x_2 - x_0$$

This is equivalent to the following

$$\Delta x_{02} = \Delta x_{01} + \Delta x_{12} = 20 - 15 = 5ft$$

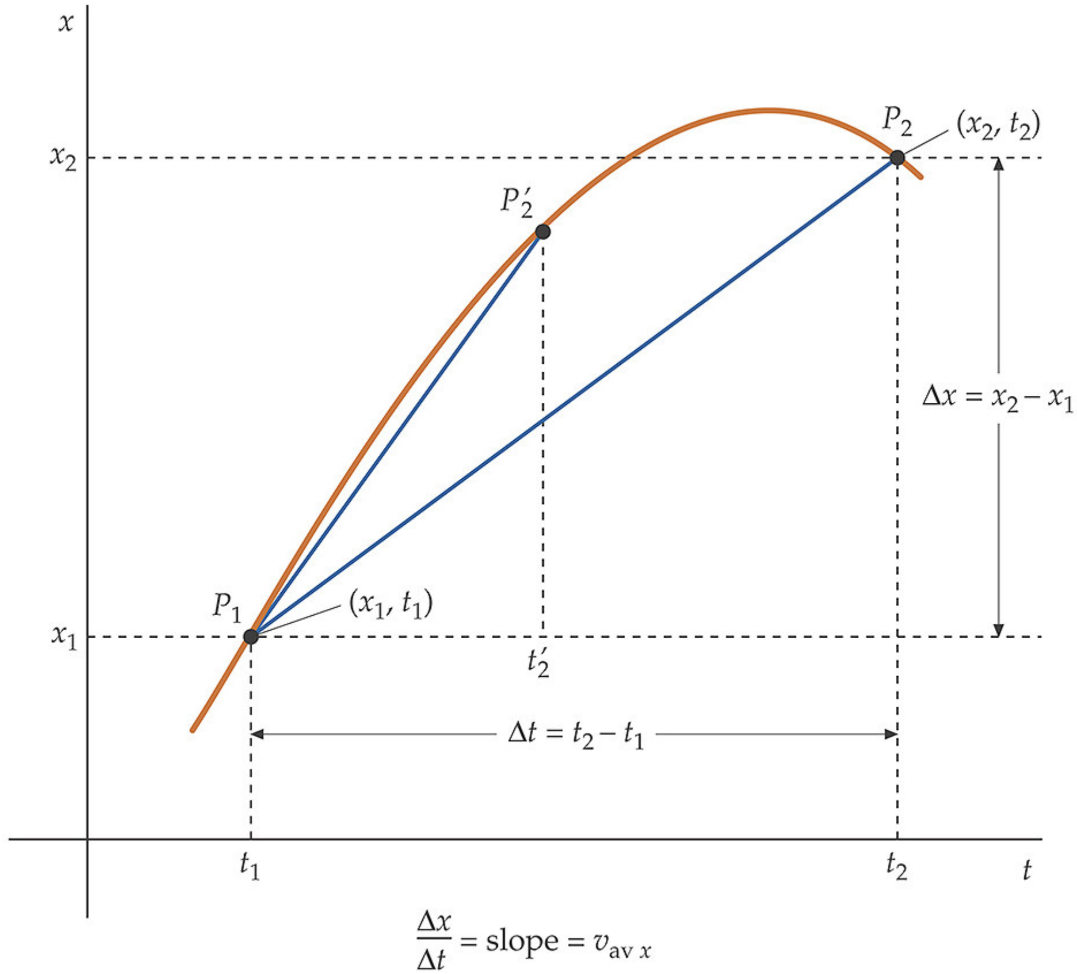
Average Velocity & Speed

The speed is a scalar quantity and it is always positive

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{s}{\Delta t}$$

Average Velocity

The average velocity for the interval between $t=t_1$ and $t=t_2$ is the slope of the straight line connecting the point P_1 (t_1,x_1) and and P_2 (t_2,x_2) on an x versus t graph.



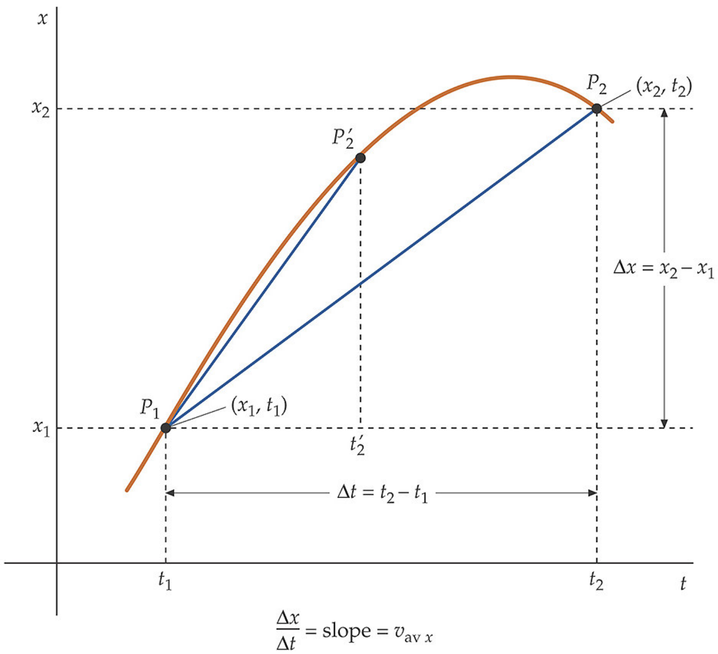
Average Velocity

$$V_{Avg,x} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Or, looking at it another way

$$\Delta x = V_{Avg,x} \times \Delta t$$

Average Velocity

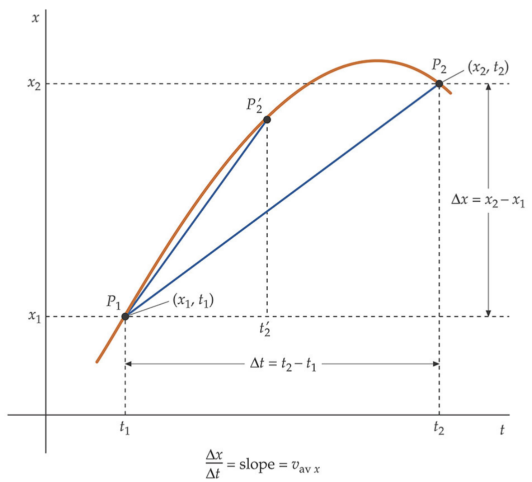


$$V_{Avg,x} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Many different motions (curves) between P_1 and P_2 will produce the same average velocity.

We only know the end points of the curve we don't know the points in between.

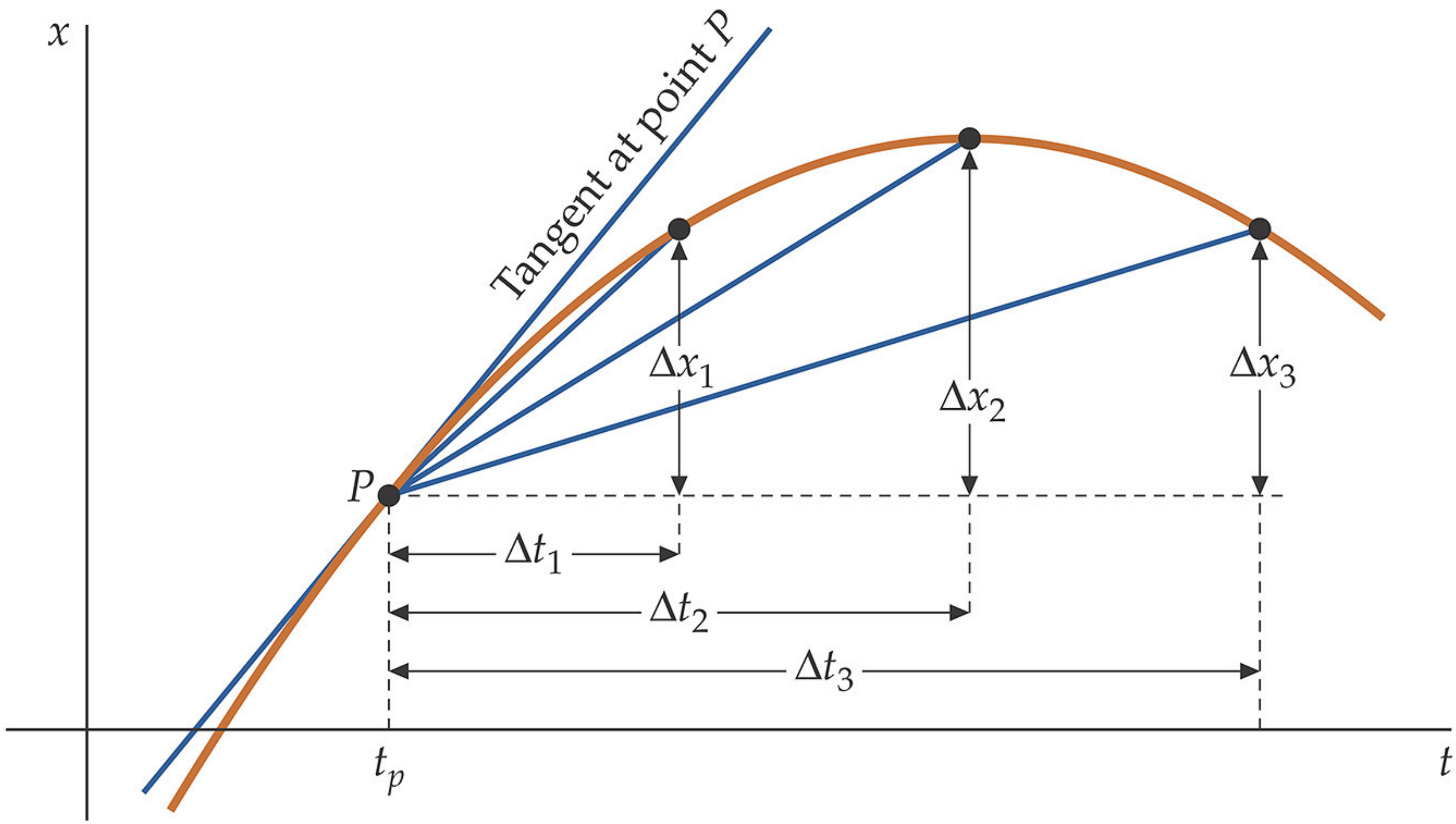
The Message of Missing Variables



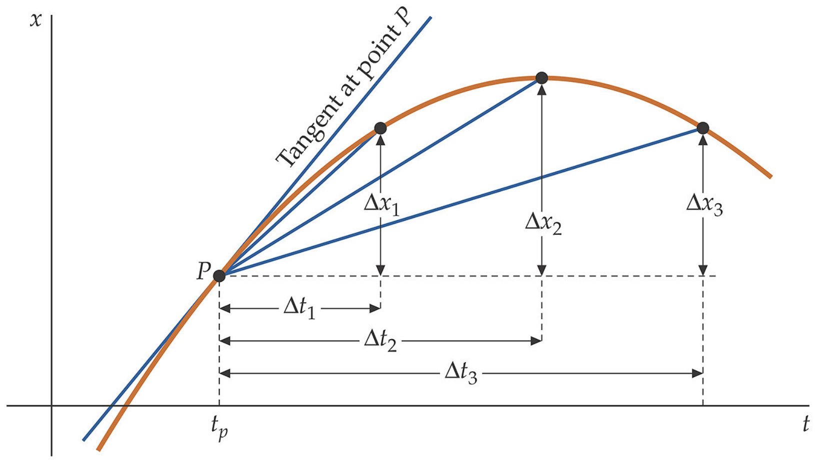
$$V_{Avg,x} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

- A benefit of using algebraic descriptions is that we can see what variables DON'T appear in the equation.
- If the variable is not in the equation then the result of the calculation will not depend on the values of that variable.
- The average velocity doesn't depend on the x-value of the points between P_1 and P_2

Instantaneous Velocity

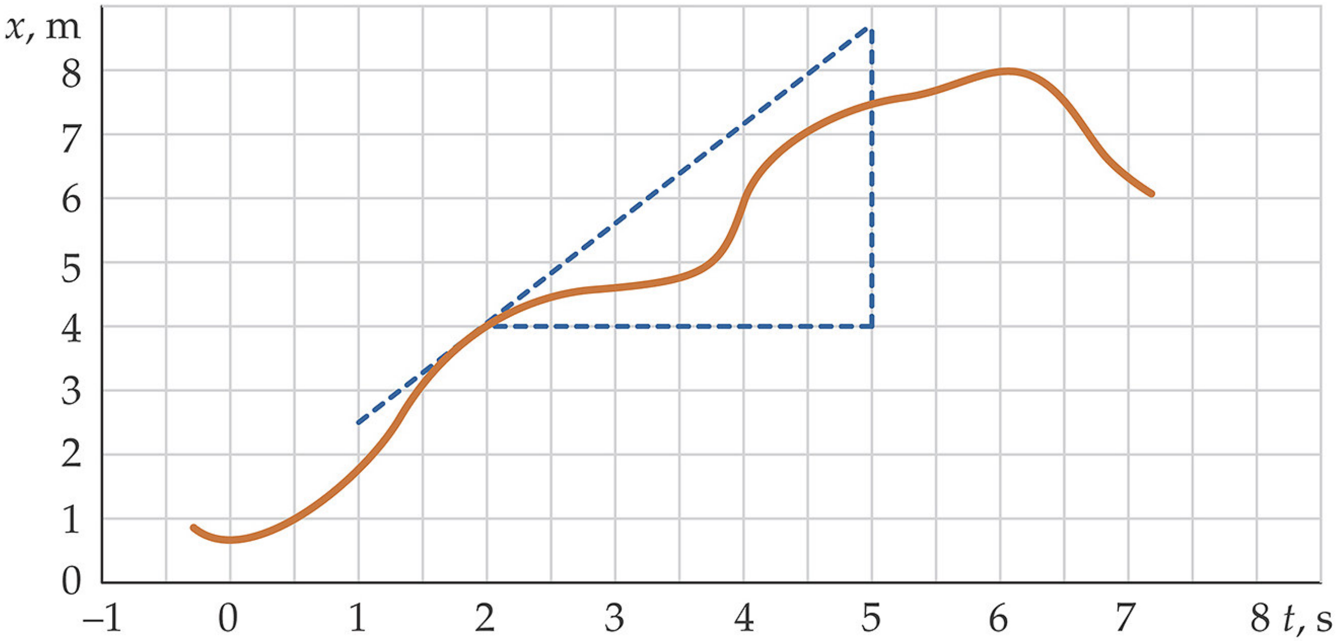


Instantaneous Velocity



$$v_x(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous Velocity



Instantaneous velocity at t = 1.8s?

Greatest velocity?

Zero velocity?

Negative velocity?

Average Acceleration

$$a_{Avg,x} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

$$\Delta v = a_{Avg,x} \times \Delta t$$

Instantaneous Acceleration

$$a_x(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

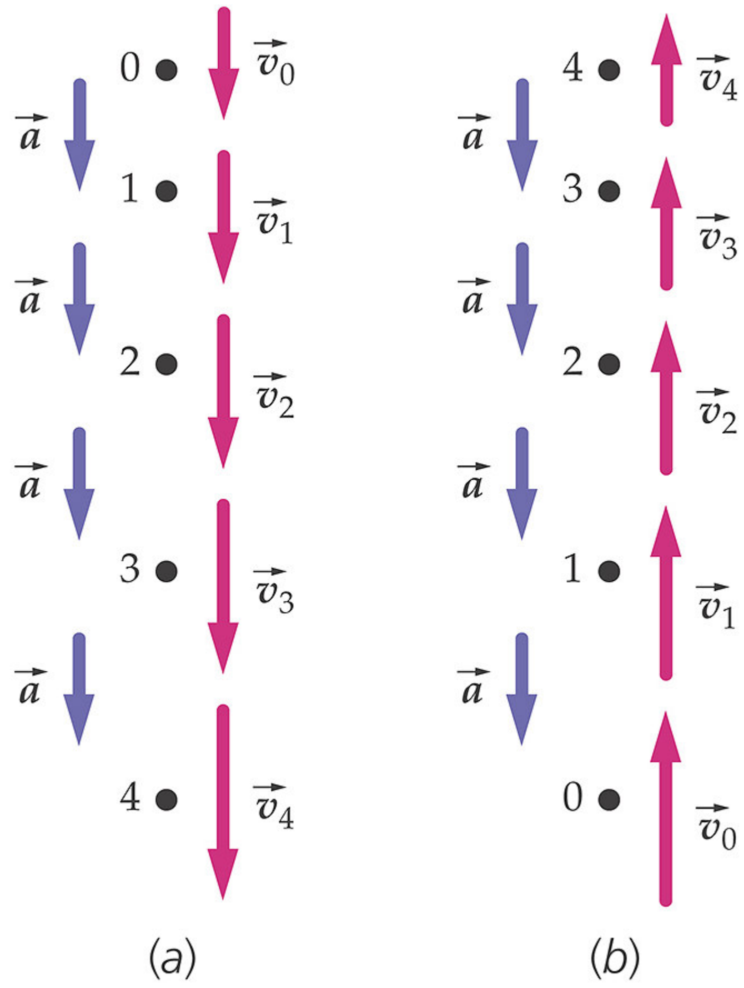
$$a_x(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2 x}{dt^2}$$

Constant Acceleration

A constant acceleration acting on a body changes the value of its velocity by a constant amount every second.

Case (a) shows a falling object with an initial downward velocity v_0.

Case (b) shows a rising object with an initial upward velocity v_0.



Is He Correct?

“It goes from zero to 60 in about 3 seconds.” (© Sydney Harris.)

Some Useful Conversions

$$\frac{km}{hr} = \frac{km}{hr} \times \frac{10^3 m}{km} \times \frac{hr}{3600 s}$$

$$\frac{km}{hr} = \frac{1}{3.600} \times \frac{m}{s} = 0.2778 \frac{m}{s} = 27.8 \frac{cm}{s}$$

$$\frac{km}{hr} = \frac{km}{hr} \times \frac{10^3 m}{km} \times \frac{3.281}{1} \frac{ft}{m} \times \frac{mi}{5280 ft}$$

$$\frac{km}{hr} = \frac{3281}{5280} \frac{mi}{hr} = 0.6214 \frac{mi}{hr}$$

Some Useful Conversions

$$32\frac{ft}{s^2} = 32\frac{ft}{s^2} \times \frac{mi}{5280ft} \times \frac{3600s}{hr}$$

$$32\frac{ft}{s^2} = 32 \times \frac{3600}{5280}\frac{mi}{hr \bullet s} = 21.8\frac{mi}{hr \bullet s} \approx 22\frac{mi}{hr \bullet s}$$

A constant acceleration of 32 ft/s^2 changes the velocity of an object by about 22 mi/hr every second.

He Was Correct!

An object gaining 22 mi/hr in speed every second will be going 66 mi/hr after 3 seconds.

“It goes from zero to 60 in about 3 seconds.” (© Sydney Harris.)

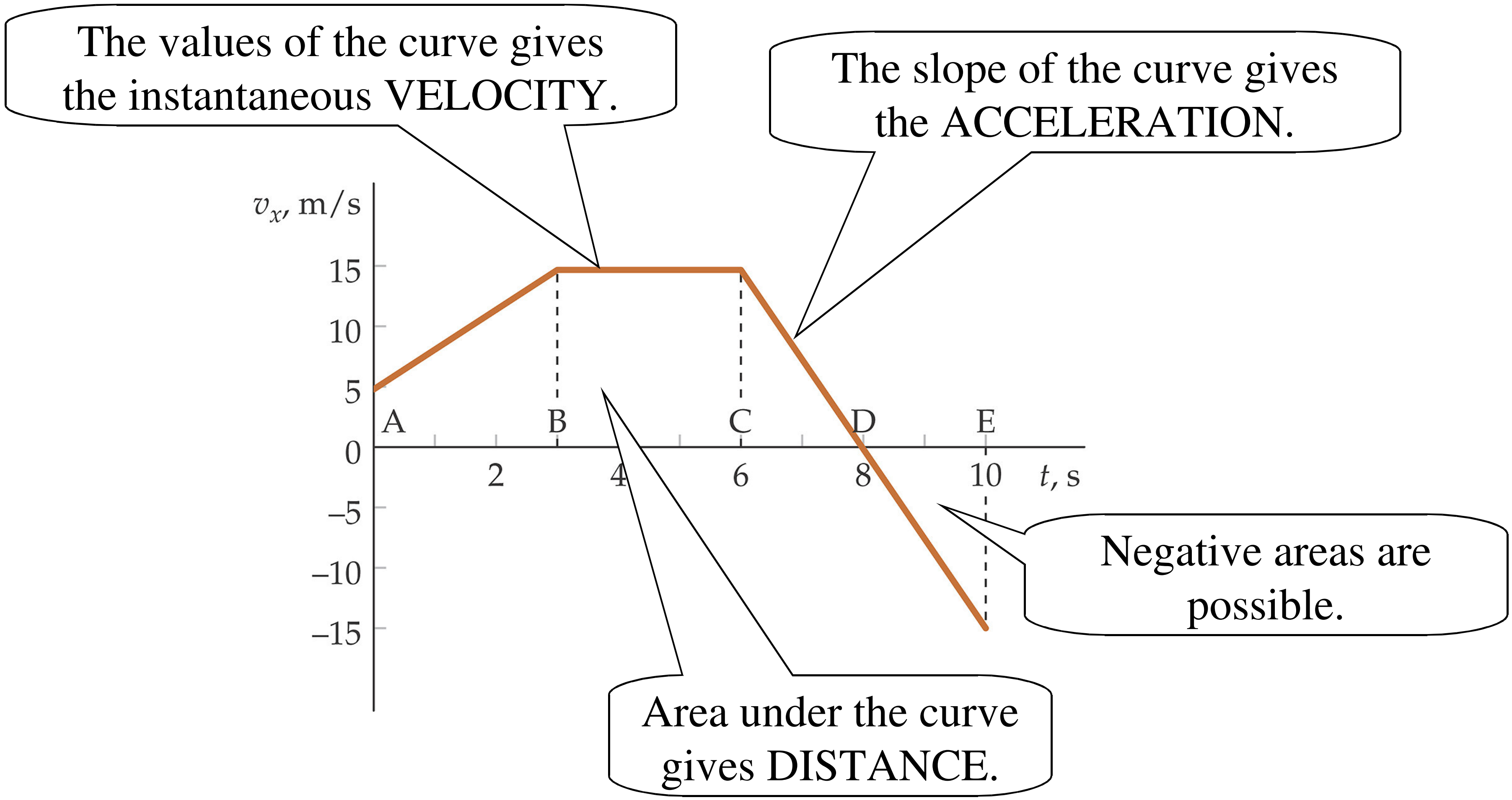
Free Fall

Assumption: acceleration due to gravity is g

$$g = 9.8 \text{ m/s}^2 \approx 10 \text{ m/s}^2$$

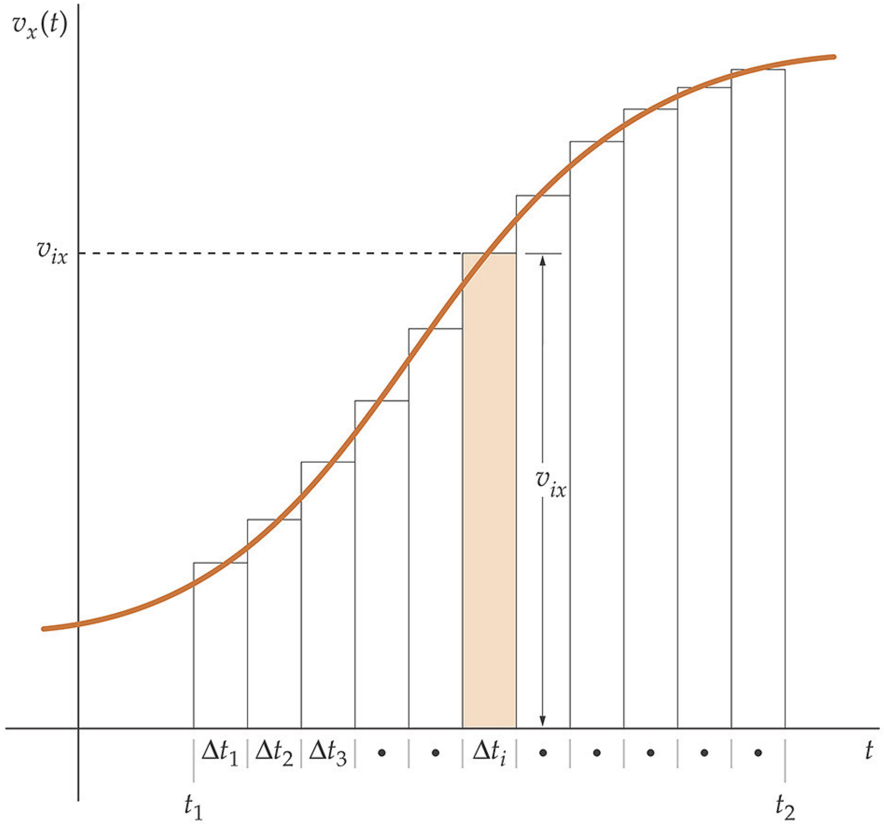
Time (s)	Velocity (m/s) V = 10t	Distance (m) $D = 5t^2$
0	0	0
1	10	5
2	20	20
3	30	45
4	40	80
5	50	125

The Most Important Graph- V vs T

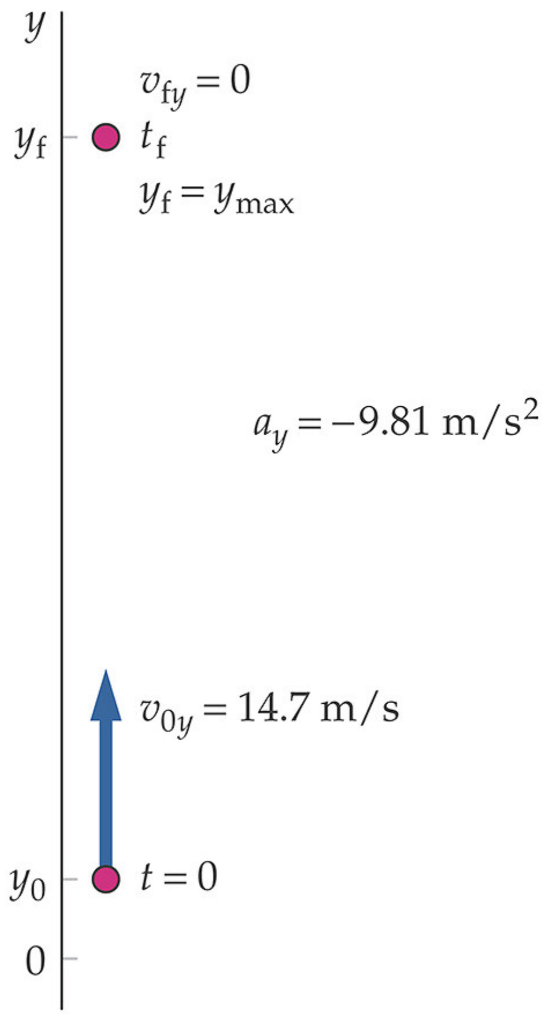


Non-Segmented V vs T Graphs

For non-segmented velocity curves we would rely on taking the integral to find the area under the curve - if we know its functional form.



1-D Projectile Problem



Given: $v_{0y} = 14.7\ m/s$

$$v_y = v_{0y} - gt$$

$$v_y = 0 = v_{0y} - gt_{1/2}$$

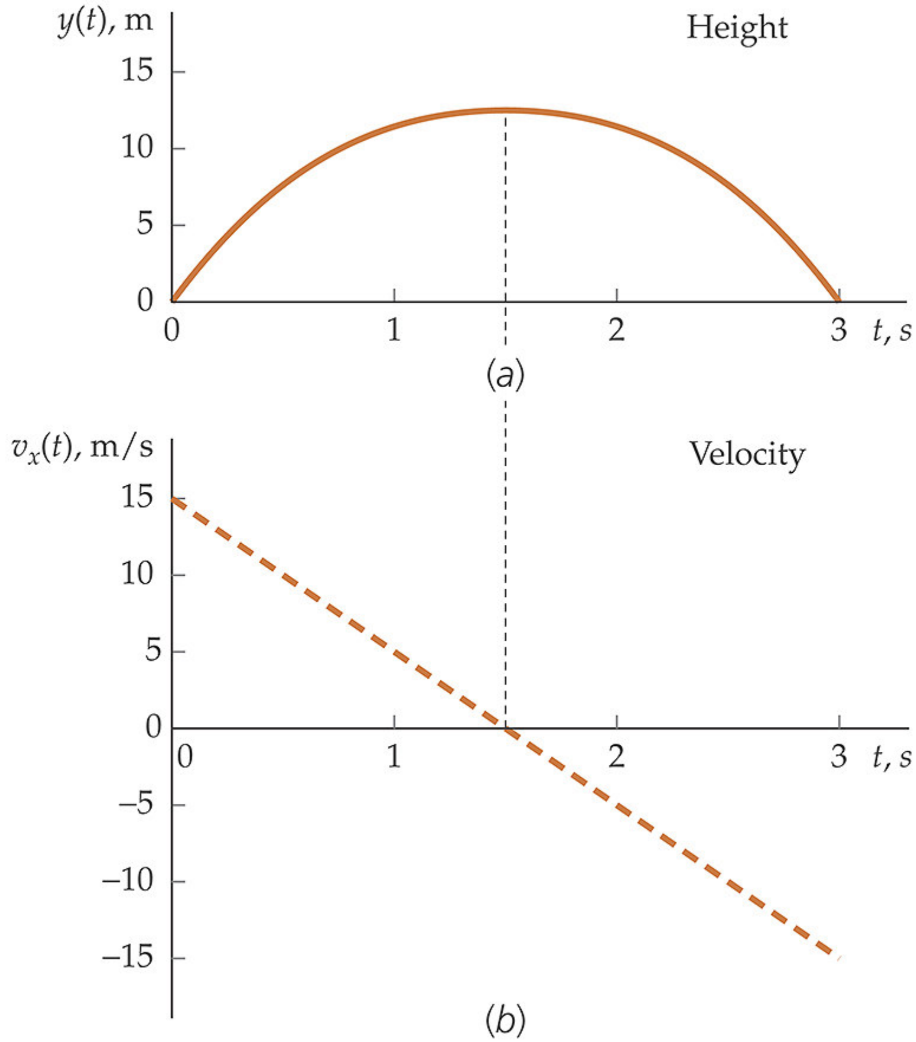
$$t_{1/2} = \frac{v_{0y}}{g}$$

$$\text{Total Time} = T = 2 \times t_{1/2} = \frac{2v_{0y}}{g}$$

$$\text{Height} = v_{avg,y} t_{1/2} = \left(\frac{v_{oy} + 0}{2}\right) t_{1/2} = \frac{v_{oy}}{2} t_{1/2}$$

1-D Projectile Problem

The linear nature of the v versus t graph indicates the constant value of the acceleration.



Total area under the velocity curve is zero showing that the displacement was zero. The cap wound up back at its starting point

Car Chase Problem

A speeder traveling at a constant 25 m/s passes a stationary police car at t = 0. At that instant the police car starts accelerating from rest at $a_p = +5\ m/s^2$

Initial conditions - all motion is in one dimension, traveling to the right, which we take to be the positive x-axis.

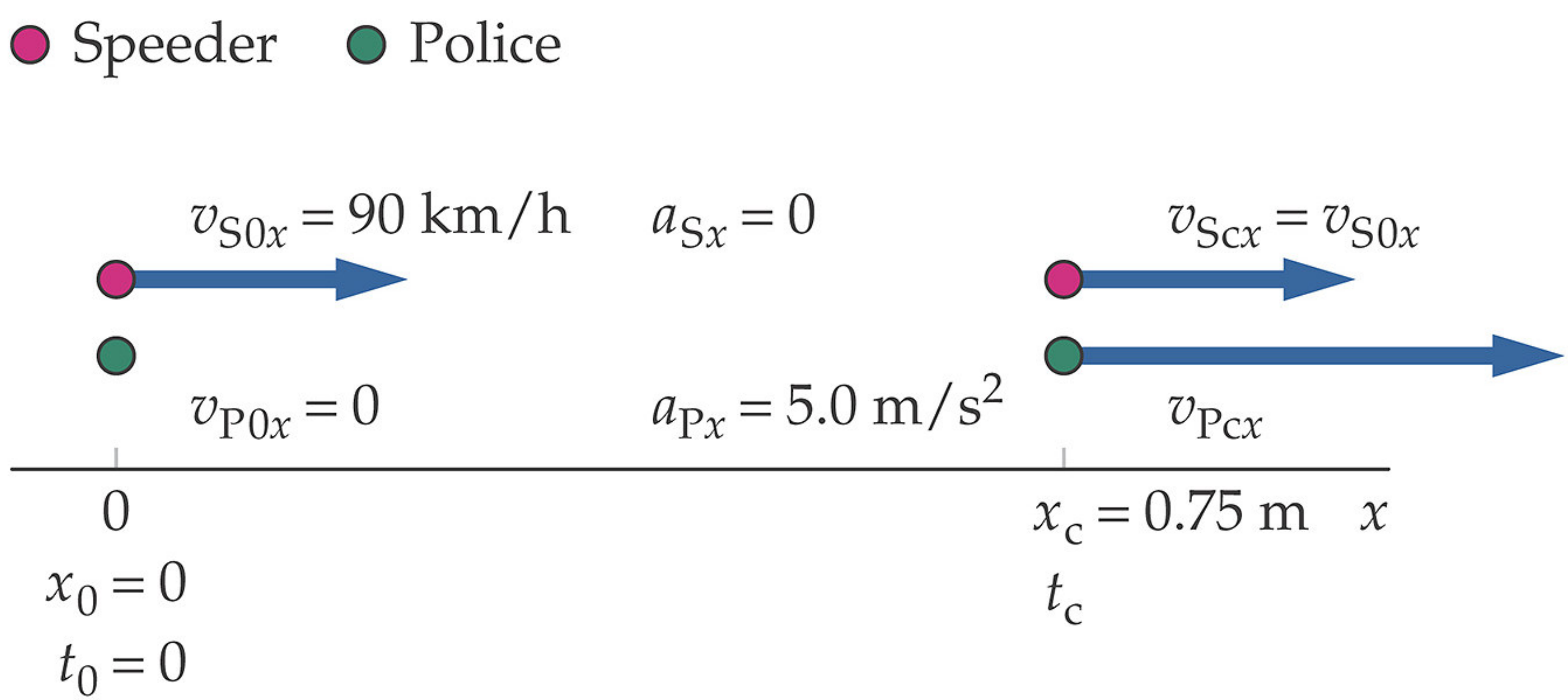
$v_s = v_{so} = 25\ m/s = constant$

$v_{po} = 0$ at $t = 0$; $a_p = +5\ m/s^2$

Questions: (a.) When does the police car catch the speeder?

(b.) What is v_p when he catches up with the speeder?

Car Chase Problem



The catch up condition is when both cars have reached the same position - they have then both traveled the same distance. The condition is $x_p = x_s$

Problem Solving Tips

- Make every attempt to minimize the amount of unnecessary notation, especially subscripts.
- If you are working on a 1-D problem you don’t need “x” as a subscript - it’s the only coordinate in the problem.
- When you finally substitute numbers into the equations don’t include the units in the equation itself - they are an unnecessary distraction.

Car Chase Problem

The catch up condition $x_s(t_c) = v_p(t_c)$

$$x_s = v_s t \; ; x_p = \tfrac{1}{2} a_p t^2$$ Position equations

$$v_s t_c = \tfrac{1}{2} a_p t_c{}^2$$ Sub into catch up condition

$$(v_s - \tfrac{1}{2} a_p t_c)t_c = 0$$ There are two solutions

$$t_c = 0$$ They are together at the beginning

$$t_c = 2v_s/a_p = 2(25)/5 = 10s$$ The catch up

Car Chase Problem

How fast is the police car going when it catches up with the speeder?

$$v_p = v_{po} + a_p t_c$$

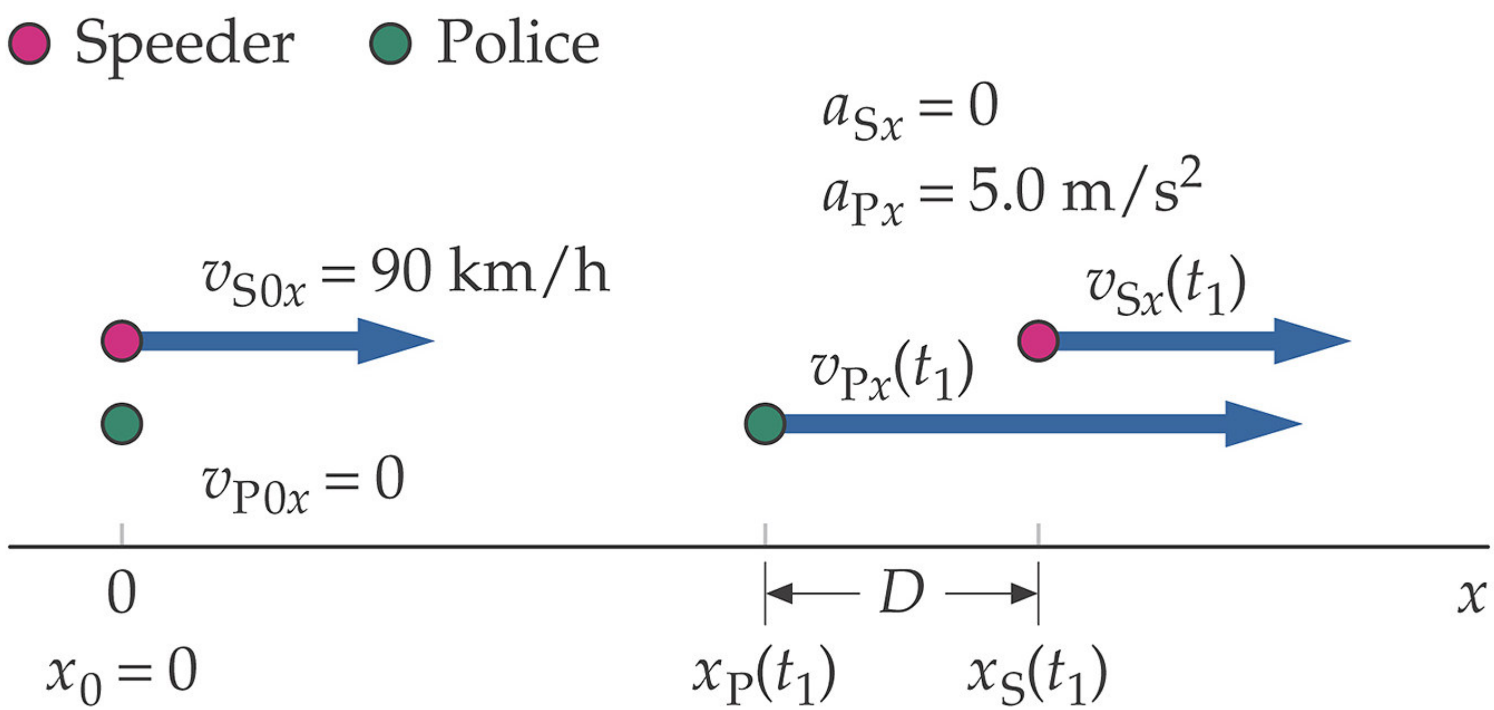
$$v_p = 0 + a_p t_c = 5(10) = 50\, m/s$$

This is double the velocity of the speeder. Is this a coincidence?

If two cars cover the same distance in the same time then they must have the same average velocity.

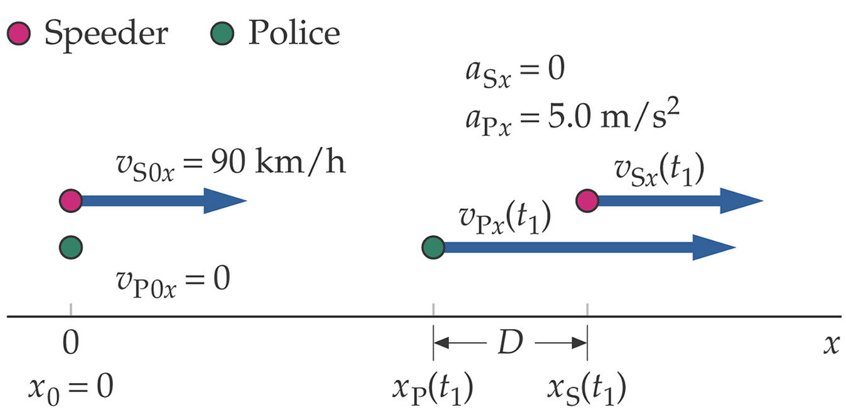
Car Chase Problem

New question - At what time are the two cars separated by a distance D?



Car Chase Problem

New question - At what time are the two cars separated by a distance D?



$$At\ t = t_1 \quad x_s - x_p = D$$

$$v_s t_1 - \tfrac{1}{2} a_p t_1^2 = D$$

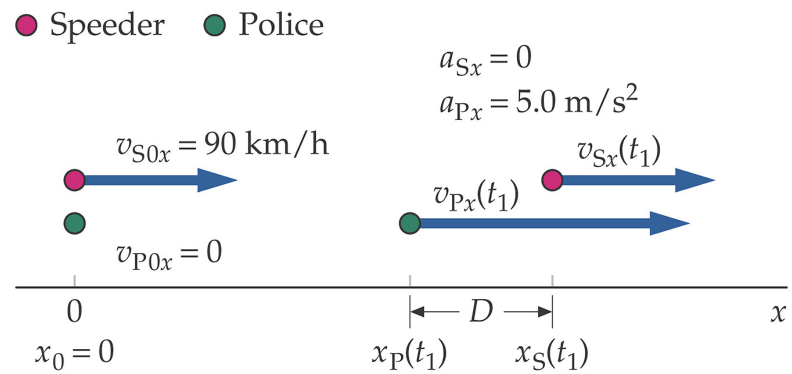
$$\tfrac{1}{2} a_p t_1^2 - v_s t_1 + D = 0$$

$$a_p t_1^2 - 2v_s t_1 + 2D = 0$$

$$t_1 = \frac{2v_s \pm \sqrt{(2v_s)^2 - 4a_p(2D)}}{2a_p}$$

Car Chase Problem

$$t_1 = \frac{2v_s \pm \sqrt{(2v_s)^2 - 4a_p(2D)}}{2a_p}$$



$$t_1 = \frac{2(25) \pm \sqrt{(2 \bullet 25)^2 - 4(5)50}}{2 \bullet 5}$$

$$t_1 = \frac{50 \pm \sqrt{(50)^2 - 1000}}{10} = \frac{50 \pm \sqrt{1500}}{10}$$

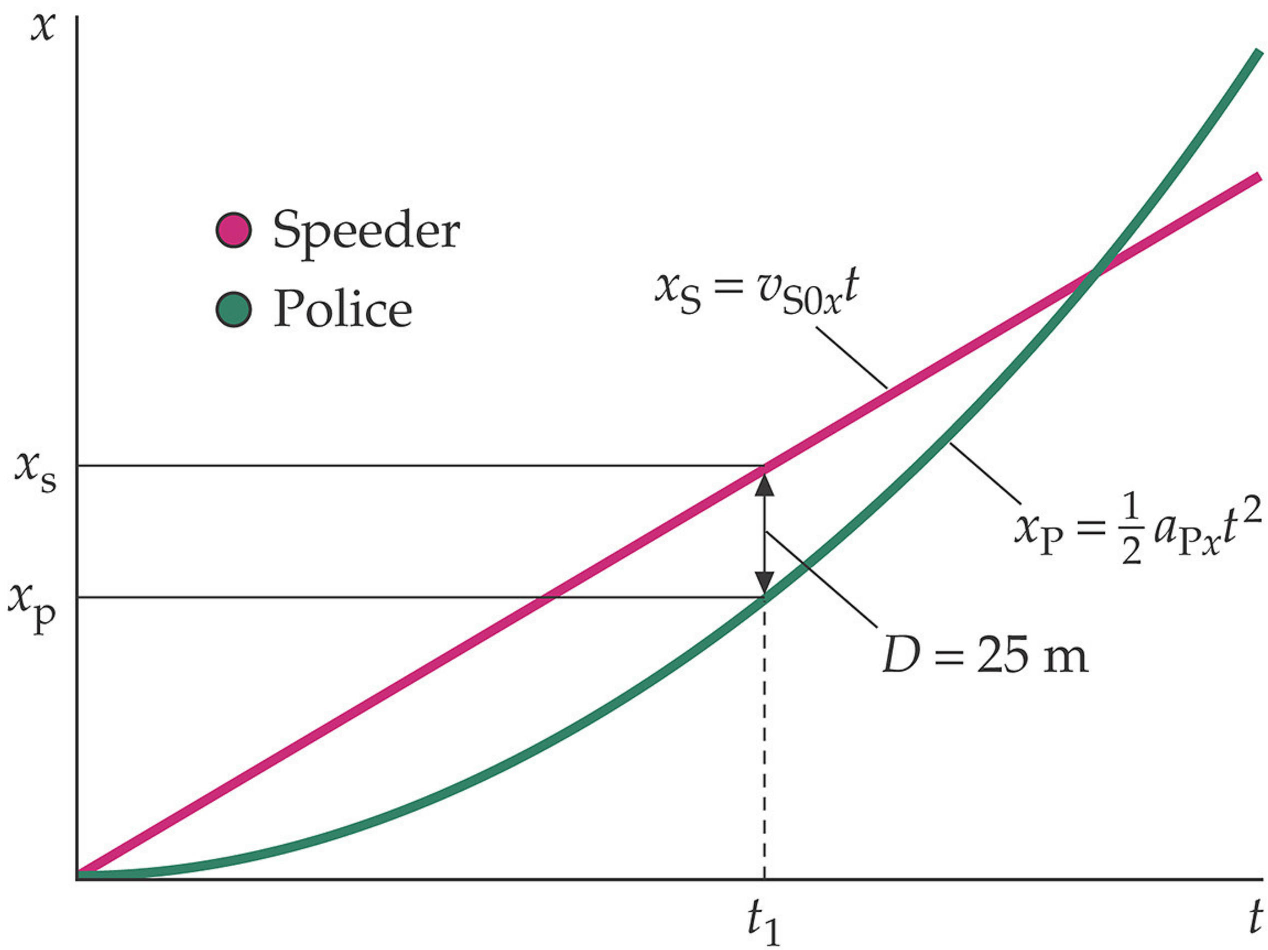
$$t_1 = 5 \pm \sqrt{15}$$

$$t_1^+ = 8.87s$$

$$t_1^- = 1.13s$$

The plus solution is when the police car is catching up to the speeder. The negative solution is when the speeder is moving away from the still accelerating police car.

Car Chase Problem



Chase Problem - Max Separation

$$t_1 = \frac{2v_s \pm \sqrt{(2v_s)^2 - 4a_p(2D)}}{2a_p}$$

The D_{max} value occurs when the radicand is zero.

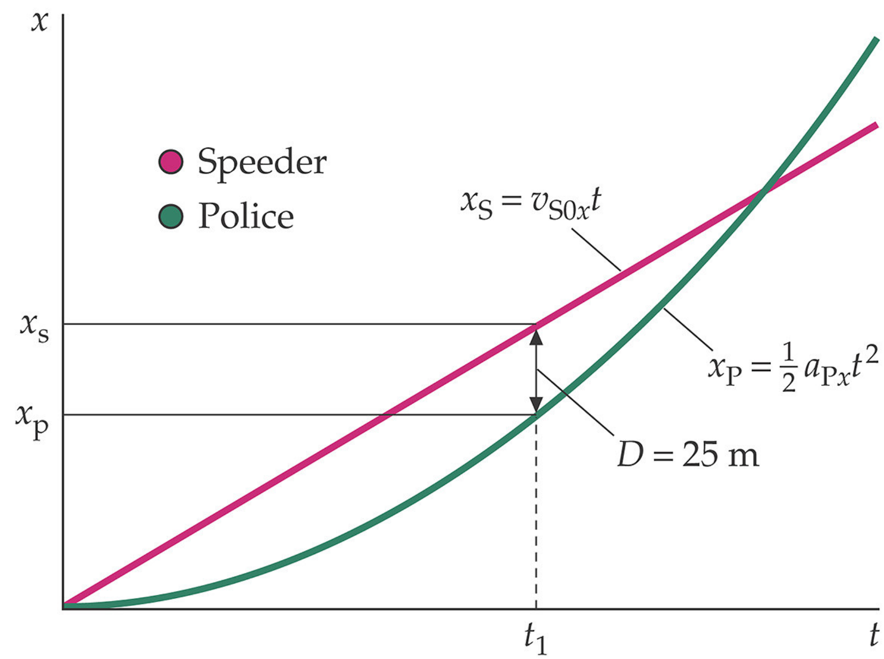
$$(2v_s)^2 - 4a_p(2D_{max}) = 0$$

$$4v_s^2 - 8a_pD_{max} = 0$$

$$v_s^2 - 4a_pD_{max} = 0$$

$$D_{max} = \frac{v_s^2}{2a_p}$$

$$D_{max} = \frac{25^2}{2 \bullet 5} = 62.5m$$



Chase Problem - Max Separation via Calculus

$$D = v_s t - \tfrac{1}{2} a_p t^2$$

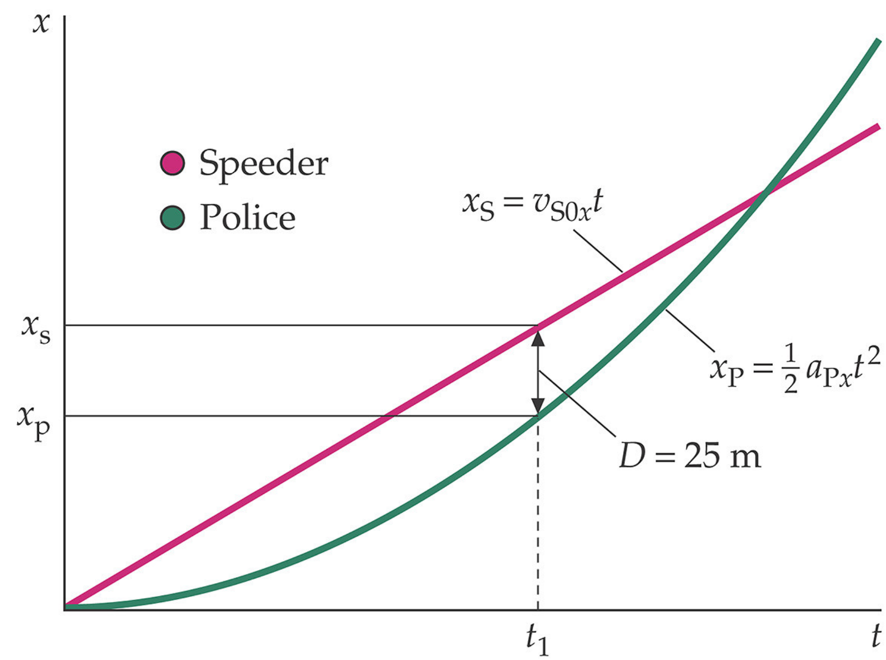
$$\frac{dD}{dt} = \frac{d}{dt}(v_s t) - \frac{d}{dt}\left(\tfrac{1}{2} a_p t^2\right) = 0$$

$$\frac{dD}{dt} = v_s - a_p t = 0$$

$$t = \frac{v_s}{a_p} = \frac{25}{5} = 5.0s$$

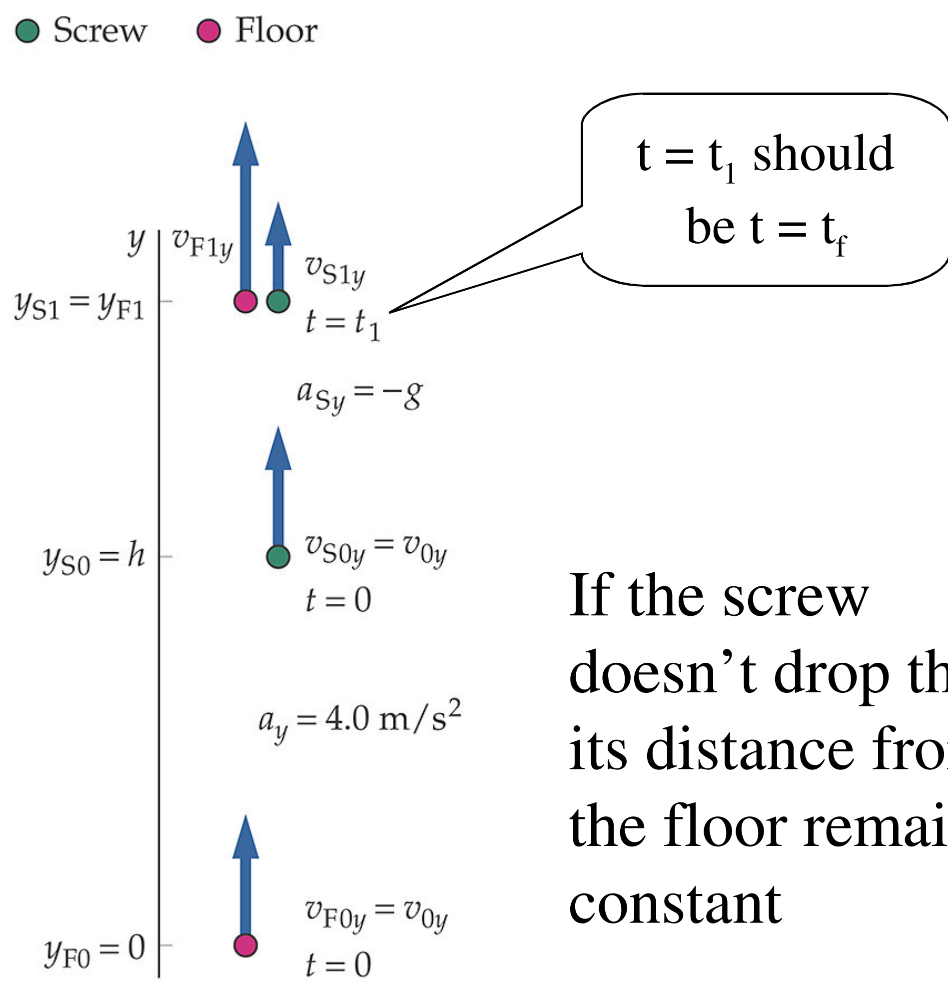
$$D = 25 \bullet 5 - \tfrac{1}{2} 5(5^2)$$

$$D = 62.5m$$



The Falling Screw Problem

We are describing the motion of both the elevator and the screw from the at rest coordinate system.



If the screw doesn't drop then its distance from the floor remains constant

The Falling Screw Problem

$$y_F - y_{F0} = v_{F0y}t + \tfrac{1}{2}a_{Fy}t^2$$
$$y_F - 0 = v_{0y}t + \tfrac{1}{2}a_{Fy}t^2$$

Floor position

$$y_S - y_{S0} = v_{S0y}t + \tfrac{1}{2}a_{Sy}t^2$$
$$y_S - h = v_{0y}t + \tfrac{1}{2}(-g)t^2$$

Screw position

Equate expressions for y_S and y_F at $t = t_f$

$$h + v_{0y}t_f - \tfrac{1}{2}g{t_f}^2 = v_{0y}t_f + \tfrac{1}{2}a_{Fy}{t_f}^2$$

$$h - \tfrac{1}{2}g{t_f}^2 = \tfrac{1}{2}a_{Fy}{t_f}^2$$

$$t_f = \sqrt{\frac{2h}{a_F + g}}$$

The Falling Screw Problem

$$t_f = \sqrt{\frac{2h}{a_F + g}}$$

The result can be understood more simply from the rest frame of the screw. That is, the coordinate frame of reference that moves with the screw.

If the elevator wasn't accelerating ($a_F = 0$) then the result would be the same as if the elevator was sitting still.

The accelerating elevator effectively changed the acceleration, as perceived by the screw, from g to $g + a_F$

Kinematic Eqns via Differentiation

$$x(t) = x_0 + v_{0x}t + ½ a_x t^2$$

$$\frac{dx}{dt} = v_x = v_{0x} + a_x t$$

$$\frac{d^2x}{dt^2} = \frac{dv_x}{dt} = a_x$$

Kinematic Equations

$$v_x = v_0 + a_x t$$

$$v_{av,x} = \tfrac{1}{2}(v_{0x} + v_x)$$

$$v_{av,x} = \frac{\Delta x}{\Delta t}$$

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$v_x{}^2 = v_0{}^2 + 2a_x\Delta x$$

Kinematic Equations

$$v_{av,x} = \frac{1}{\Delta t}\int_{t_1}^{t_2} v_x dt$$

$$\Delta v_x = \int_{t_1}^{t_2} a_x dt$$

Summary
Motion in One Dimension

- Displacement, Velocity and Speed
- Acceleration
- Motion with Constant Acceleration