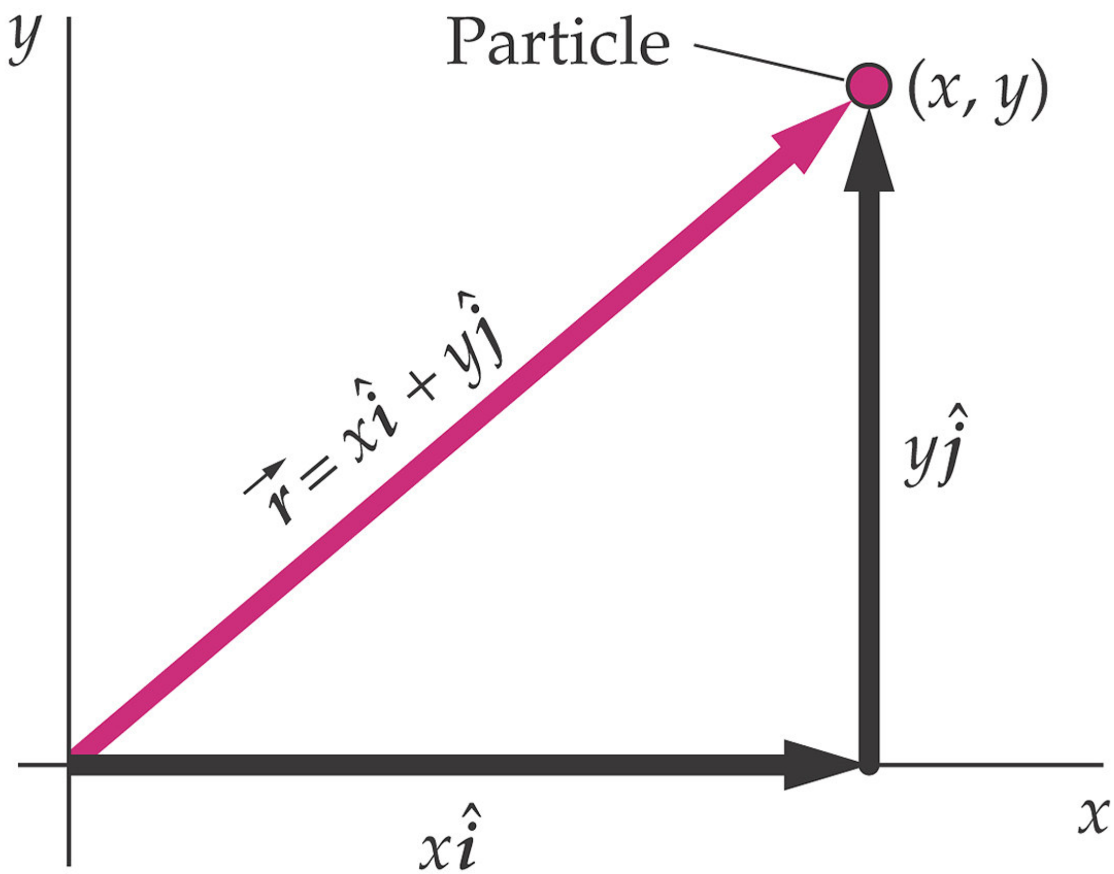
Chapter 4

Motion in Two and Three Dimensions

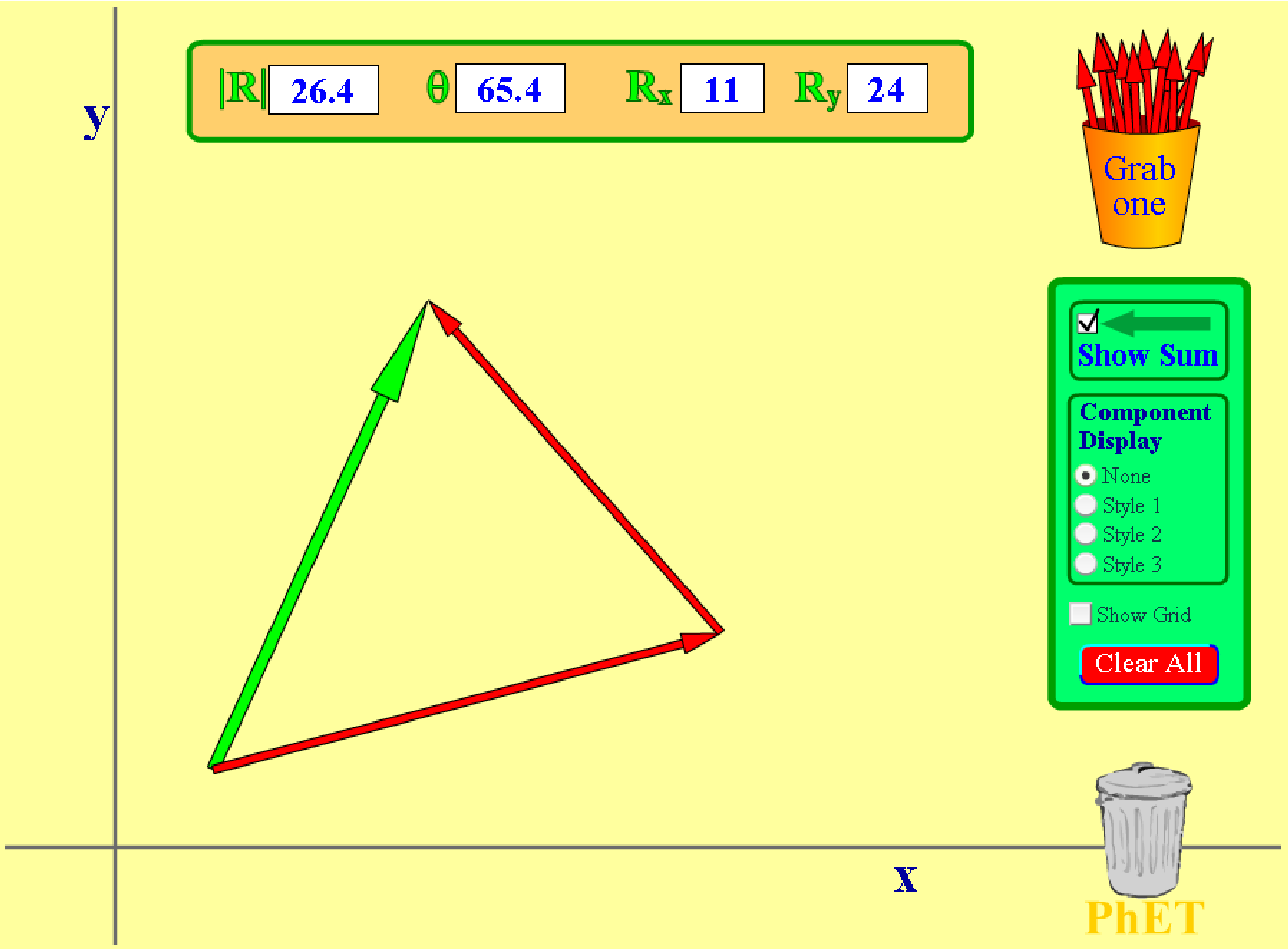
Motion in Two and Three Dimensions

- Displacement, Velocity and Speed
- Relative Motion
- Projectile Motion
- Circular Motion

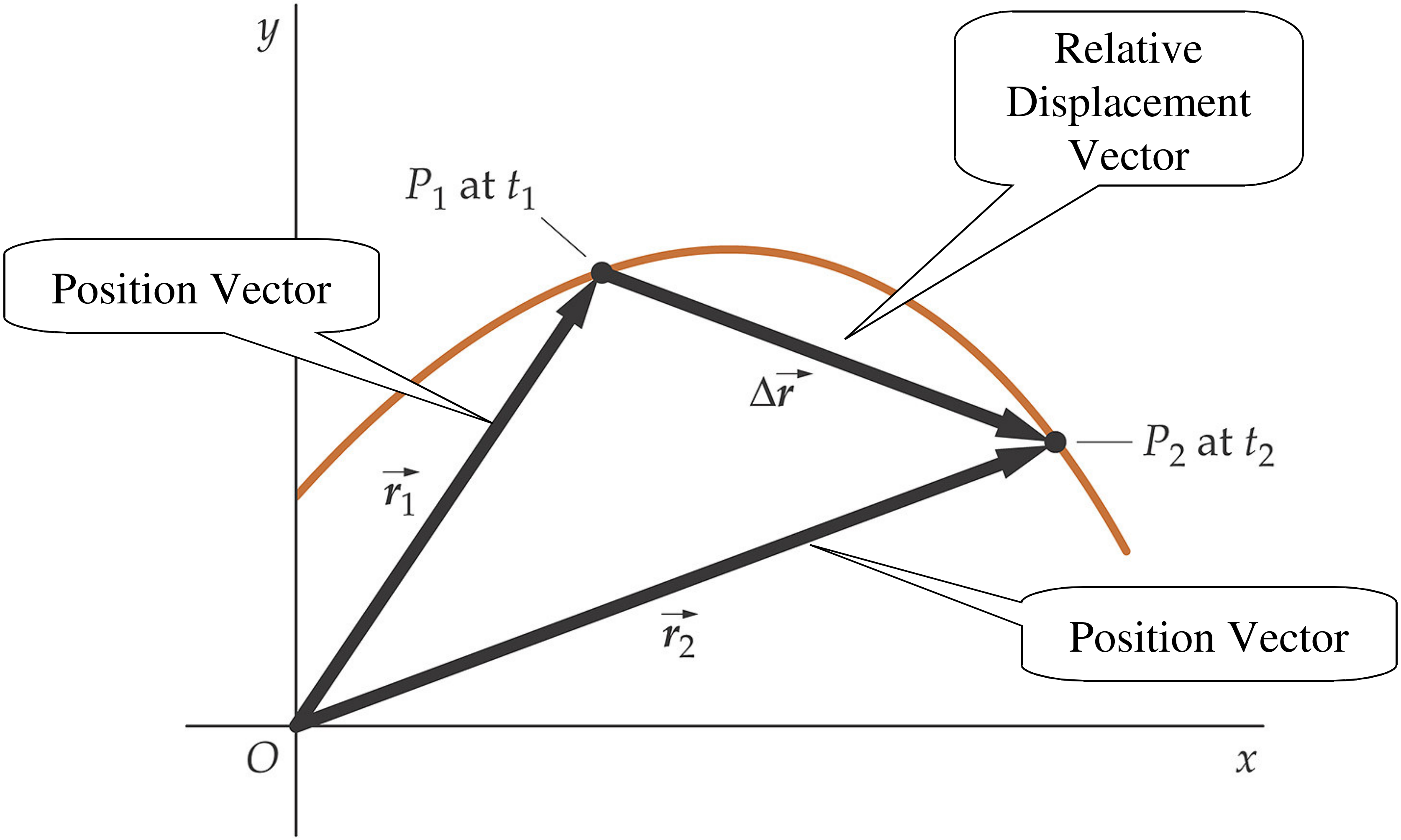
Particle Displacement



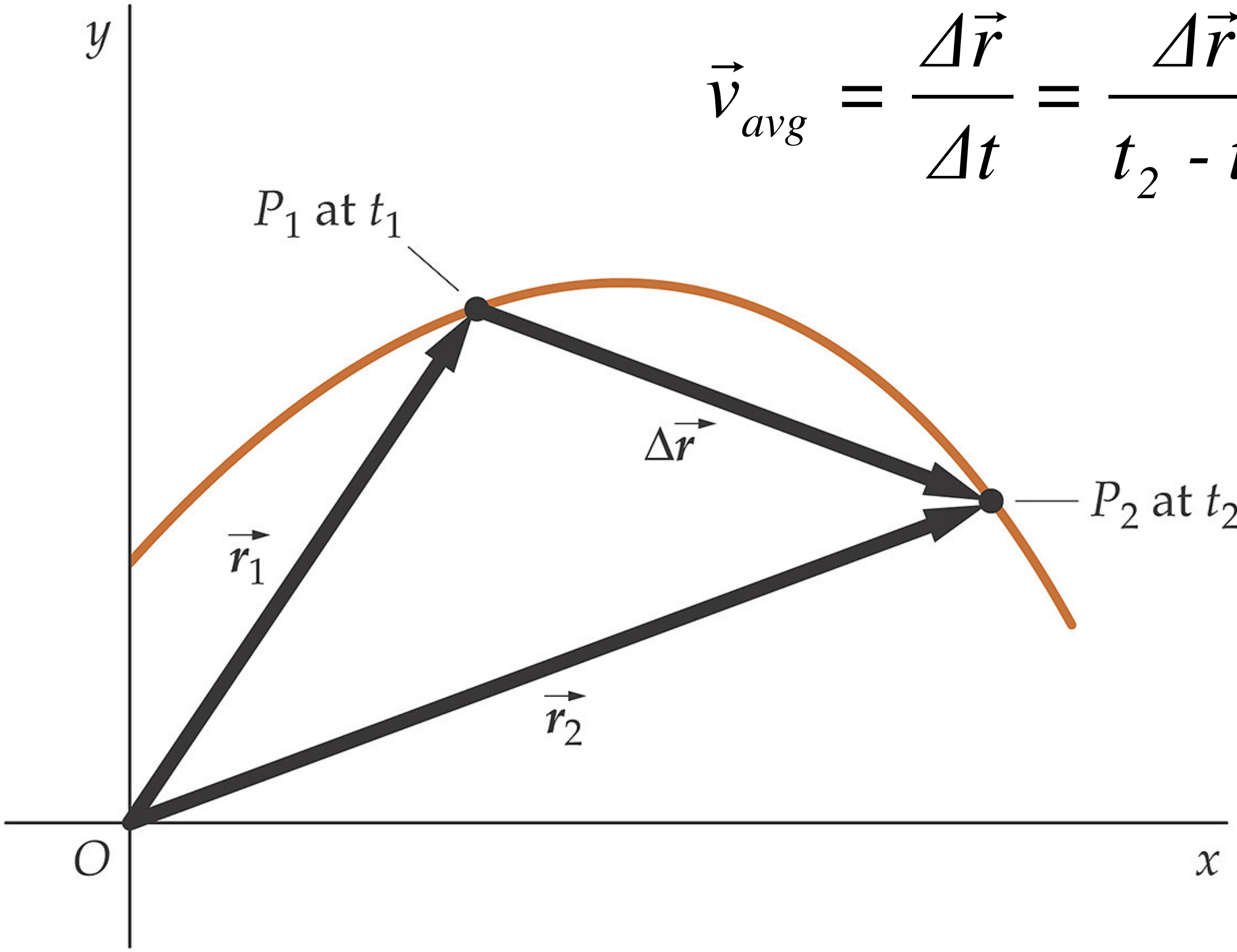
Vector Simulation



Relative Displacement



Average Velocity

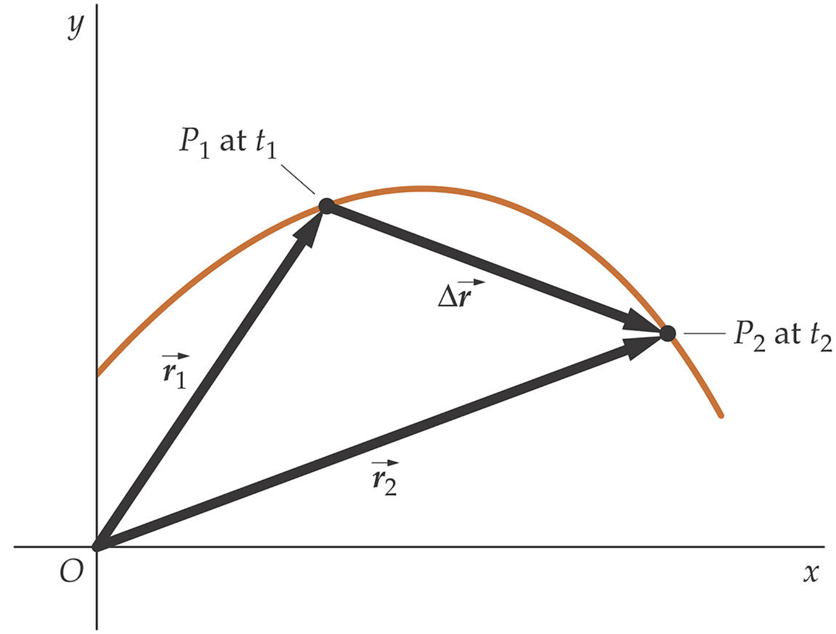


$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta \vec{r}}{t_2 - t_1}$$

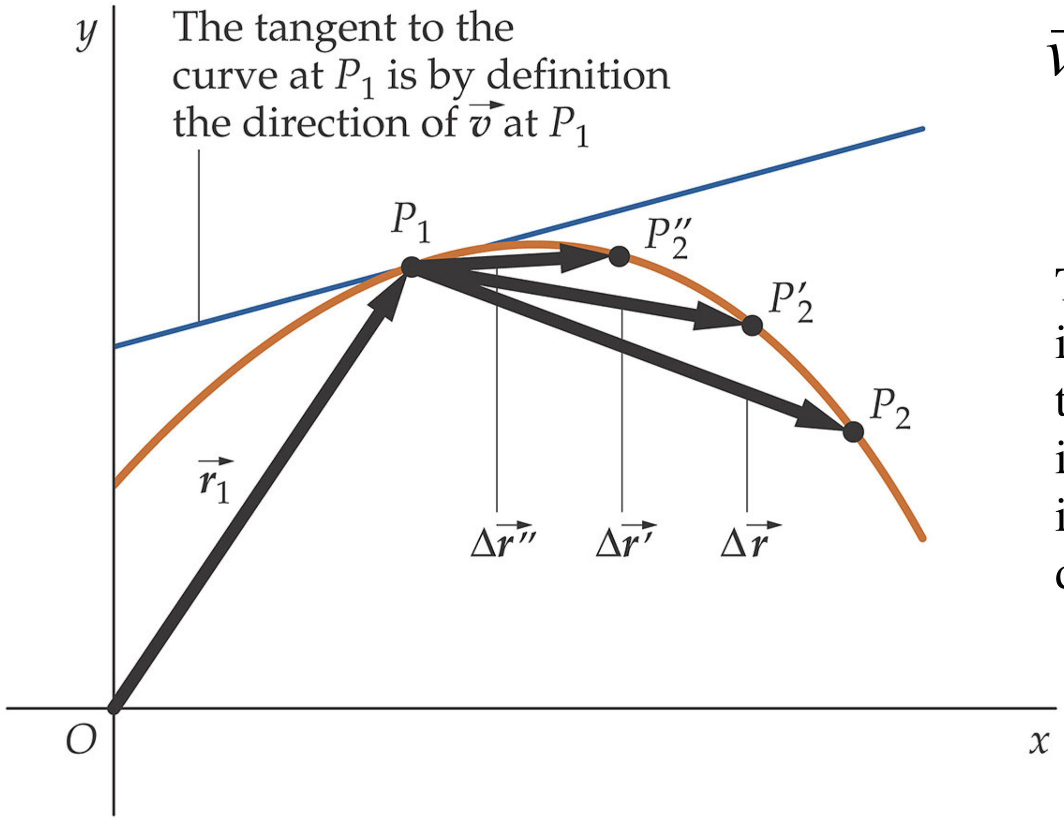
Average Velocity

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta\vec{r}}{t_2 - t_1} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{(x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})}{t_2 - t_1}$$

$$\vec{v}_{avg} = \frac{(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}}{t_2 - t_1}$$



Instantaneous Velocity Vector



$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

There is not enough information presented here to actually calculate the instantaneous velocity. This is meant only to demonstrate the process.

Instantaneous Velocity Vector

Instantaneous velocity

$$\vec{v}(t) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

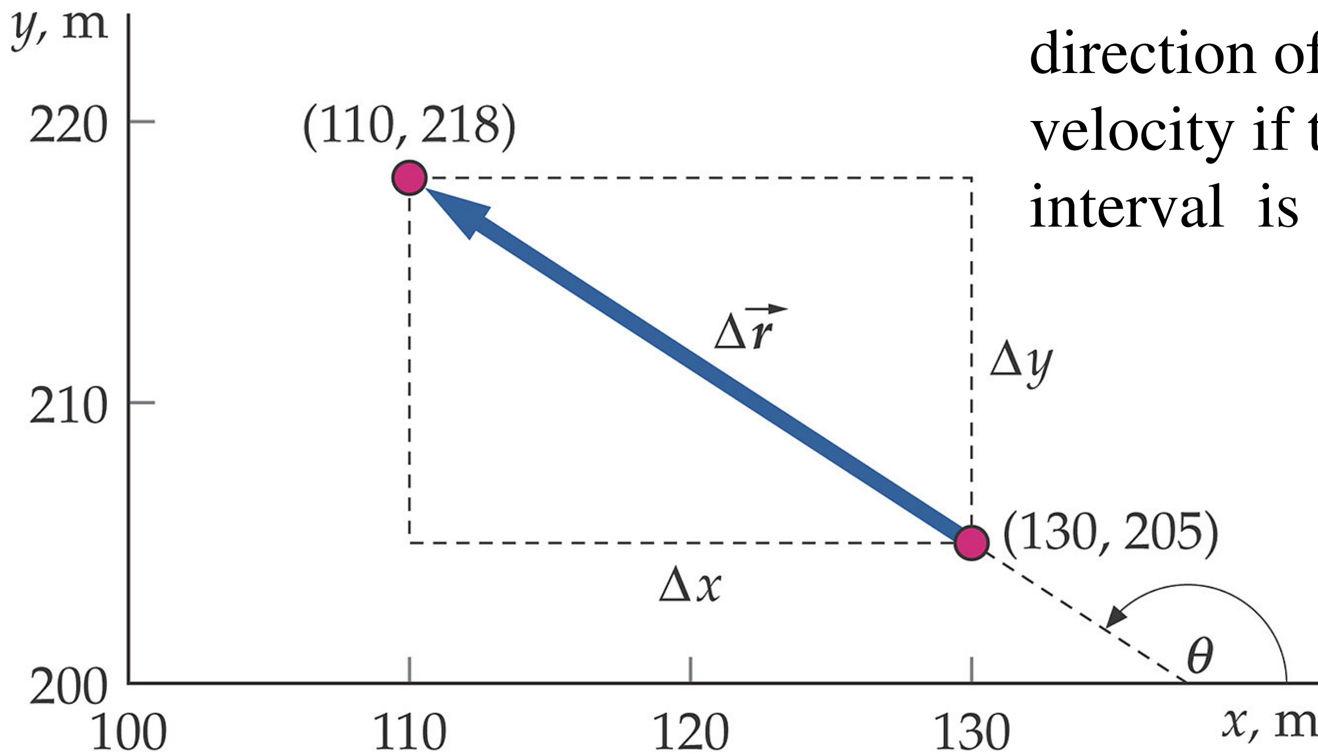
Magnitude of the velocity

$$\left|\vec{v}(t)\right| = v(t) = \sqrt{v_x{}^2 + v_y{}^2}$$

Direction of the velocity

$$\Theta = tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Sailboat Velocity



Ques:

Find the magnitude and direction of the average velocity if the time interval is 120s?

$$\vec{v}_{av} = v_{x\,av}\hat{i} + v_{y\,av}\hat{j}$$

where

$$v_{x\,av} = \frac{\Delta x}{\Delta t} = \frac{110\text{ m} - 130\text{ m}}{120\text{ s}} = -0.167\text{ m/s}$$

$$v_{y\,av} = \frac{\Delta y}{\Delta t} = \frac{218\text{ m} - 205\text{ m}}{120\text{ s}} = 0.108\text{ m/s}$$

so

$$\vec{v}_{av} = \boxed{-(0.167\text{ m/s})\hat{i} + (0.108\text{ m/s})\hat{j}}$$

$$v_{av} = \sqrt{(v_{x\,av})^2 + (v_{y\,av})^2} = \boxed{0.199\text{ m/s}}$$

$$\tan\theta = \frac{v_{y\,av}}{v_{x\,av}}$$

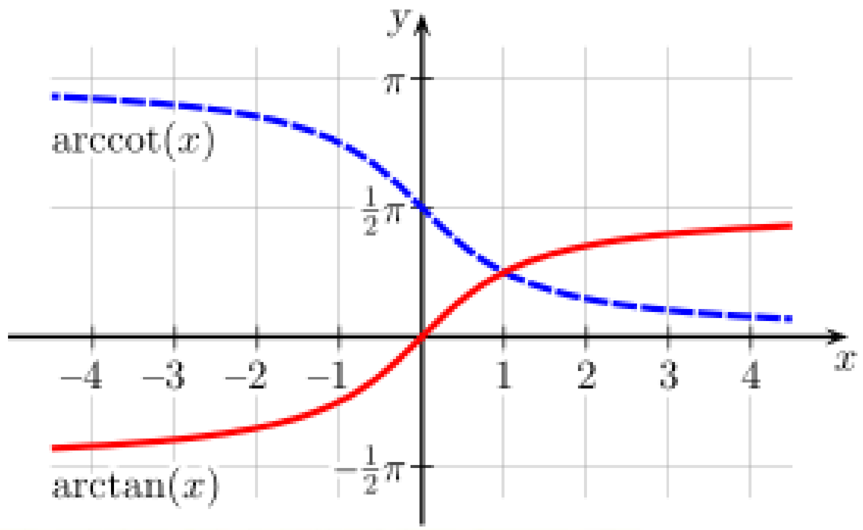
so

$$\theta = \tan^{-1}\frac{v_{y\,av}}{v_{x\,av}} = \tan^{-1}\frac{0.108\text{ m/s}}{-0.167\text{ m/s}} = -33.0^\circ + 180^\circ = \boxed{147^\circ}$$

Inverse Tangent Problem

The principle branch inverse tangent has a range of -90° to +90°

Blindly using a calculator to compute a $\tan^{-1}$ will lead to errors.

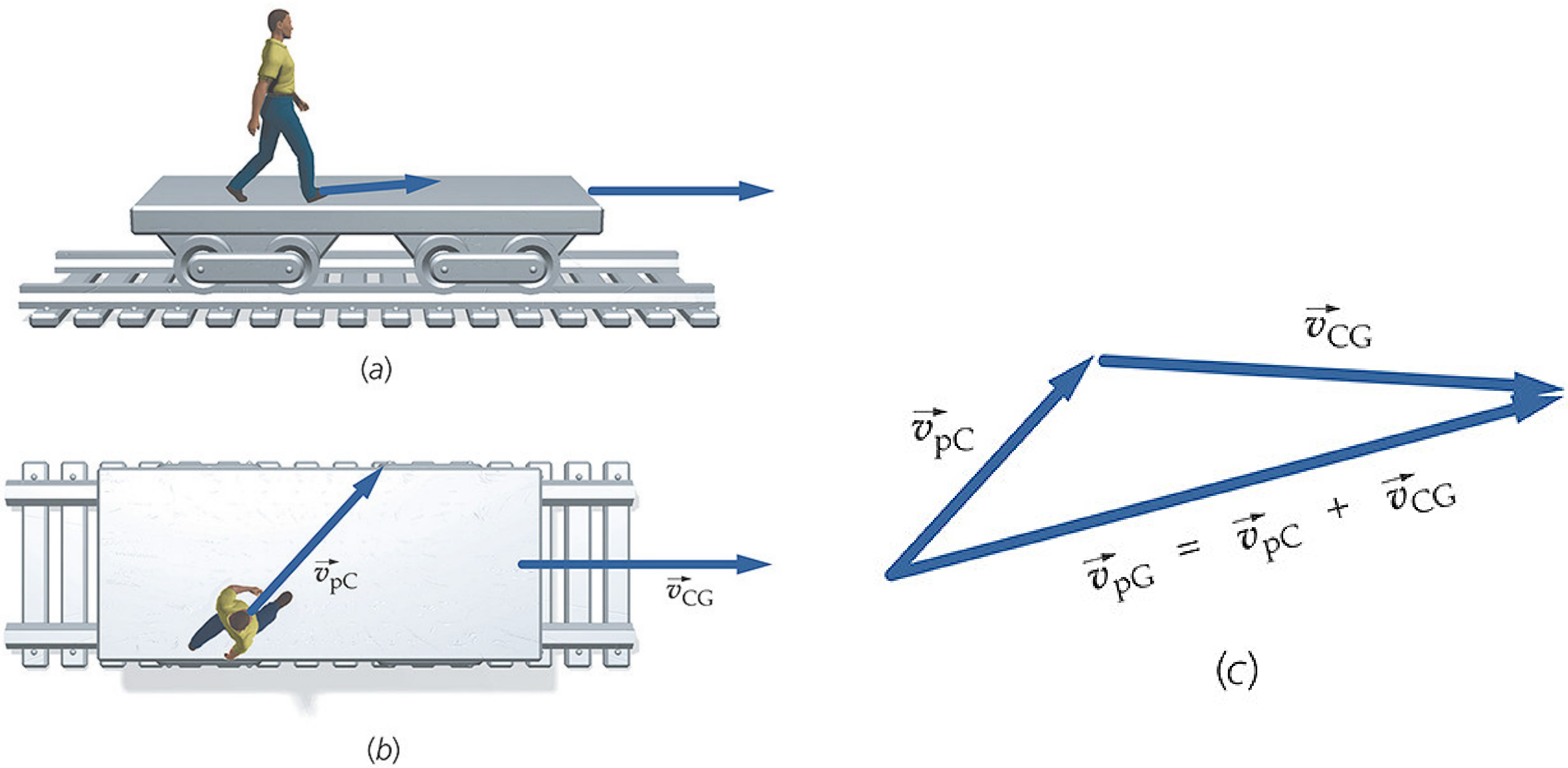


$$\tan\theta = \frac{v_{y\,av}}{v_{x\,av}}$$

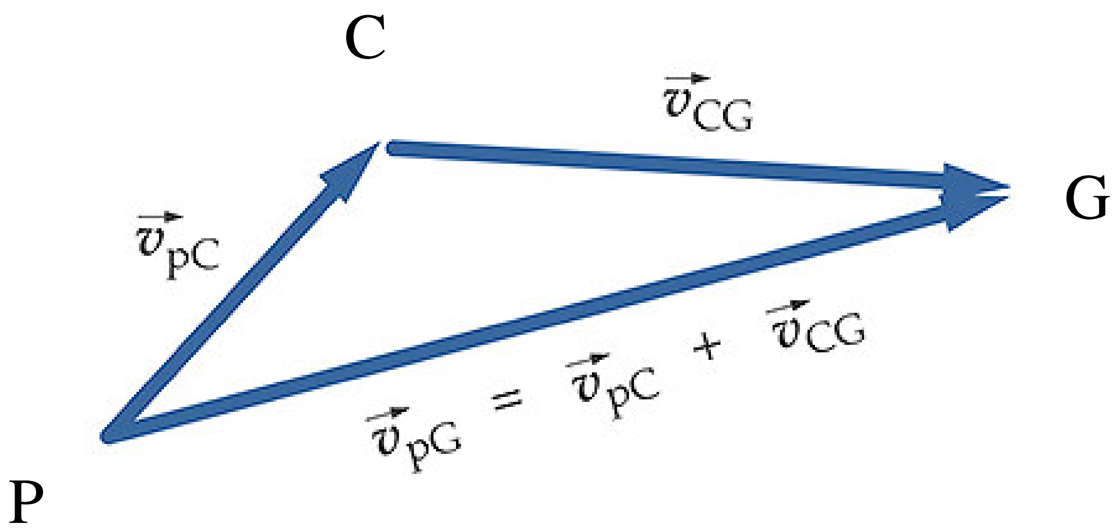
so

$$\theta = \tan^{-1}\frac{v_{y\,av}}{v_{x\,av}} = \tan^{-1}\frac{0.108\ \text{m/s}}{-0.167\ \text{m/s}} = -33.0° + 180° = \boxed{147°}$$

Relative Motion



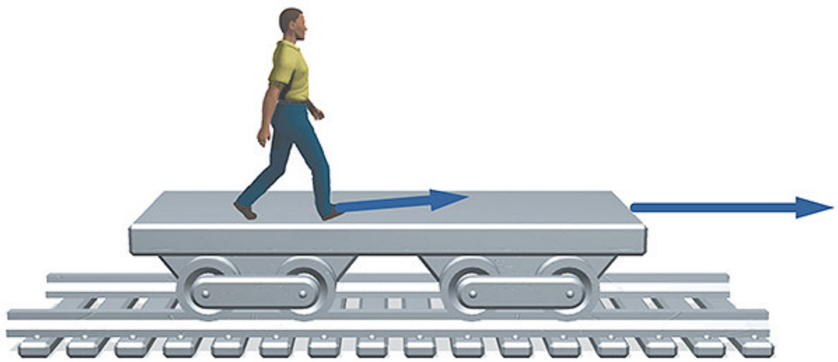
Relative Motion



$$\vec{v}_{p/G} = \vec{v}_{p/C} + \vec{v}_{C/G}$$

$$\frac{p}{G} = \frac{p}{C} \times \frac{C}{G}$$

Relative Motion



(a)

Assume the man throws a ball straight up in the air.

- What trajectory does he see?
- What trajectory does an observer on the ground see?

Acceleration Vectors

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$ Average acceleration

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$ Instantaneous acceleration

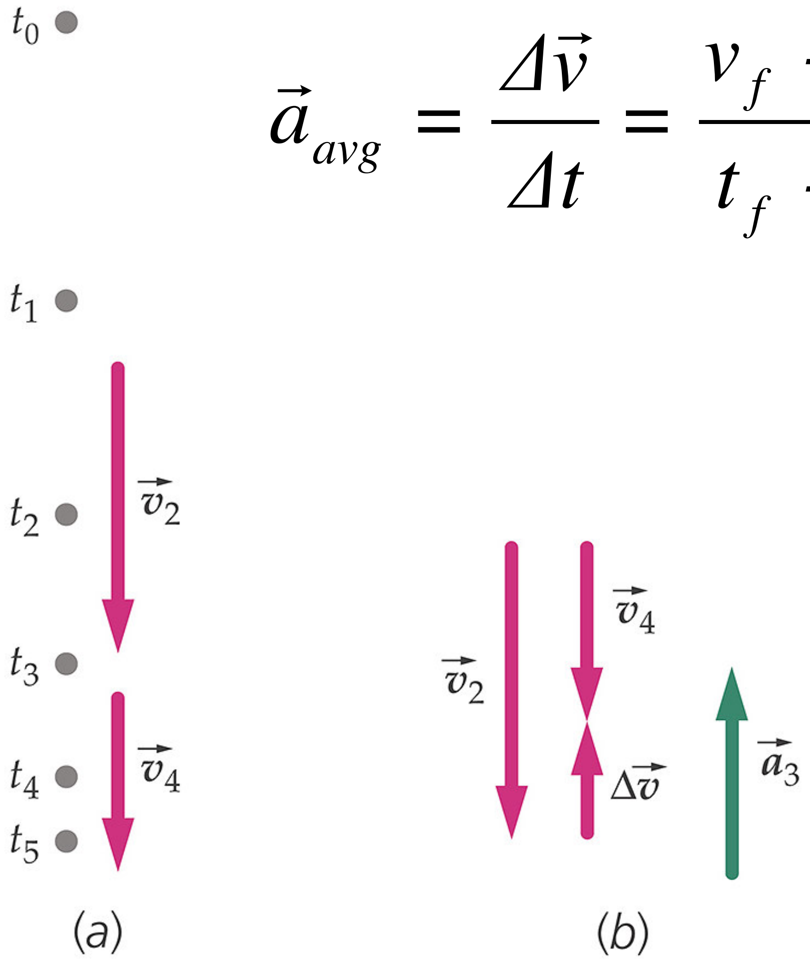
Where:

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Acceleration Vector

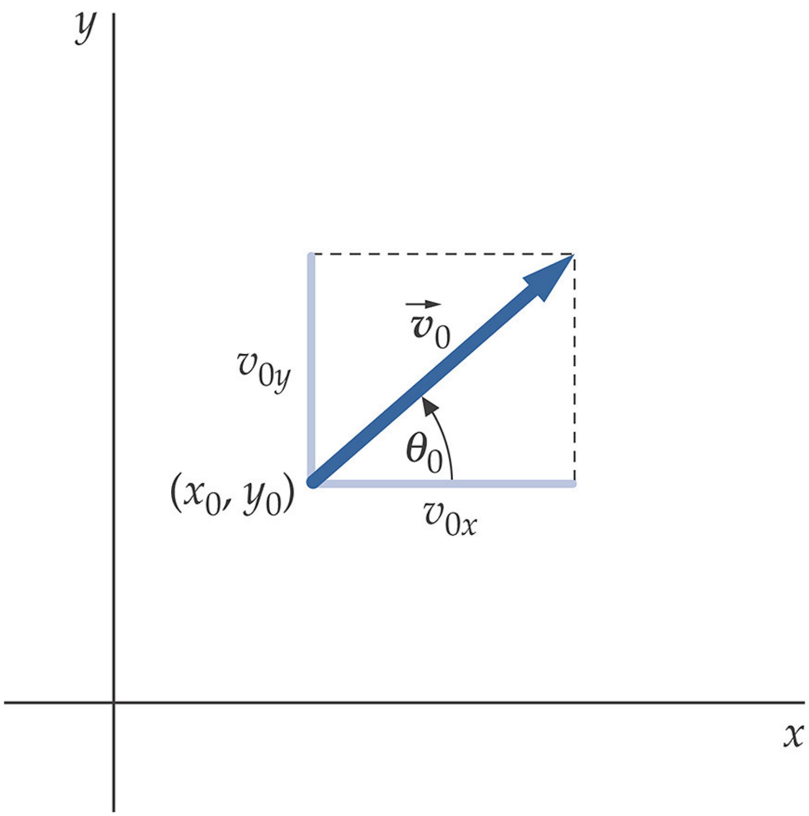
$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$



$$\vec{a}_{3avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{v_4 - v_2}{t_4 - t_2}$$

When the direction of **a** is opposite of **v**, we refer to it as a <u>deceleration</u>.

2-D Projectile Motion

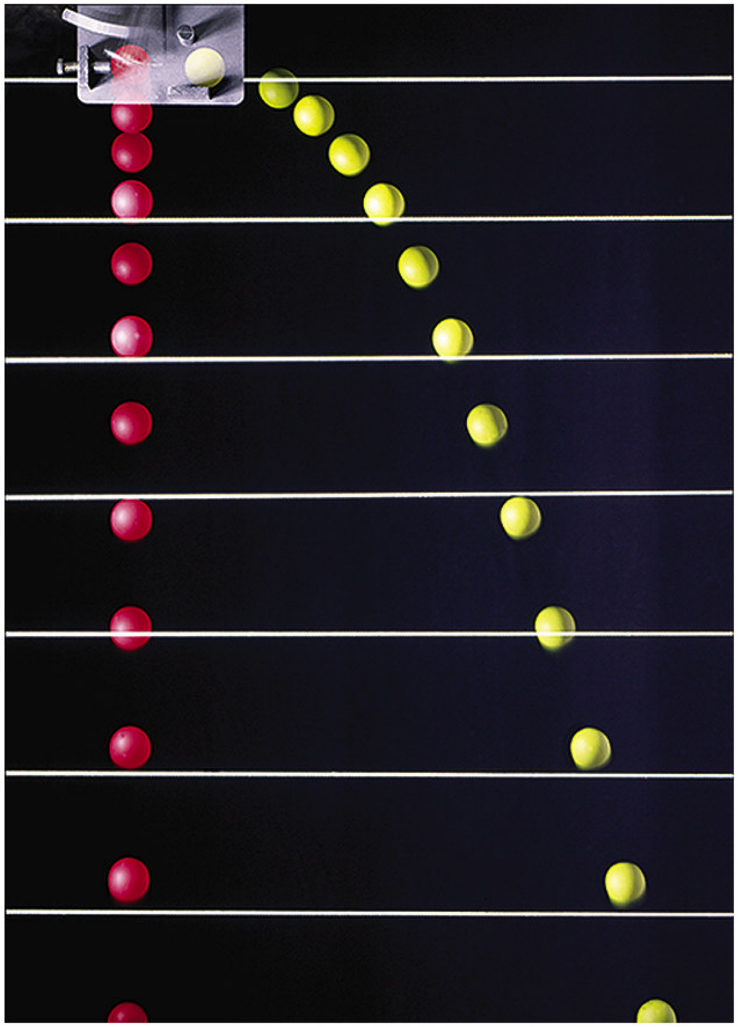


The approach to 2-D projectile problems is to resolve the velocity vector into horizontal and vertical components.

The *vertical* component is affected by gravity.

The *horizontal* component is unchanged.

2-D Projectile Motion



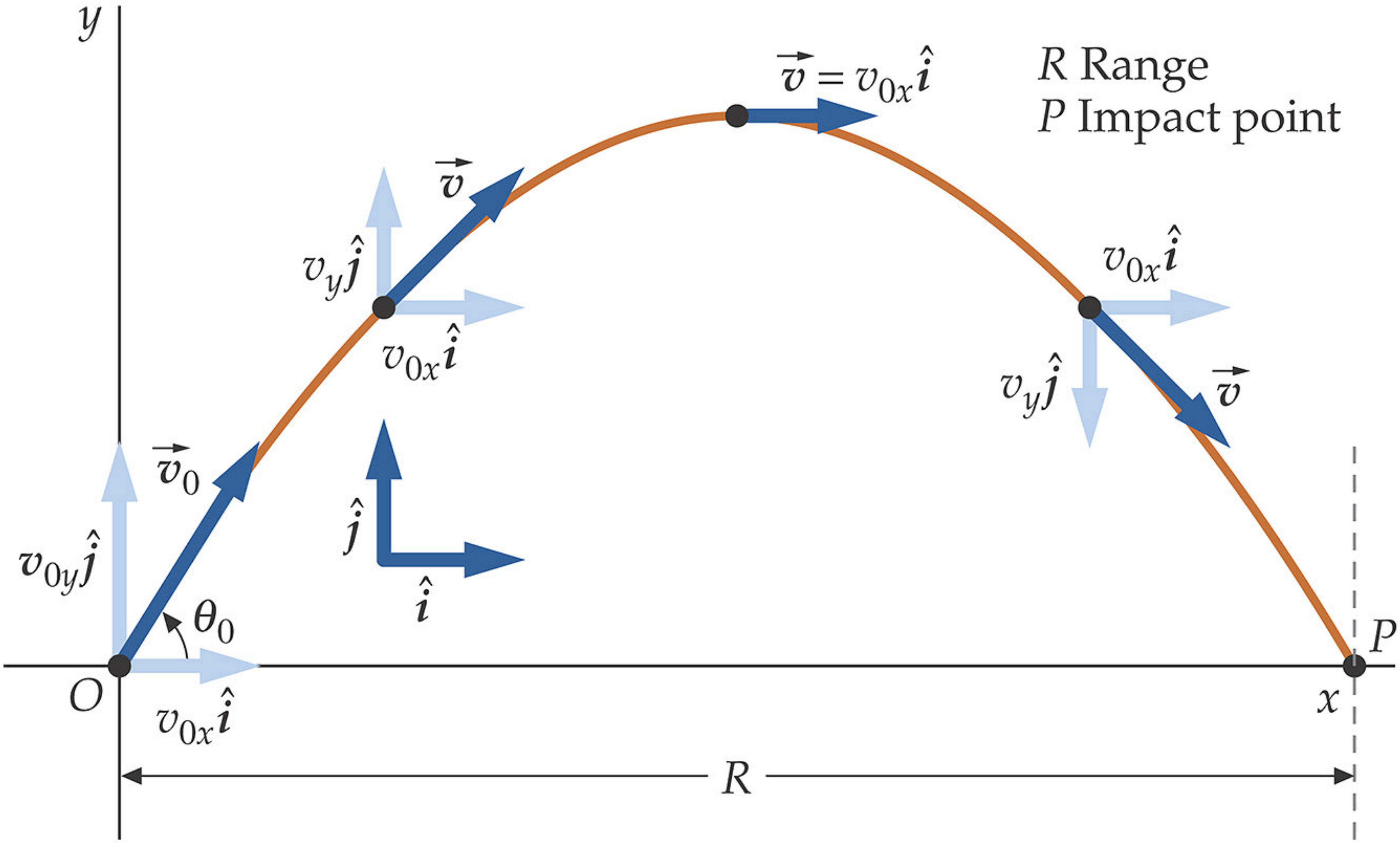
The trajectory of a 2-D projectile is a parabola.

The horizontal lines demonstrate that the vertical motion of the balls are identical in both cases.

The vertical spacing is increasing due to the acceleration of the vertical velocity.

The horizontal spacing of the yellow ball is constant.

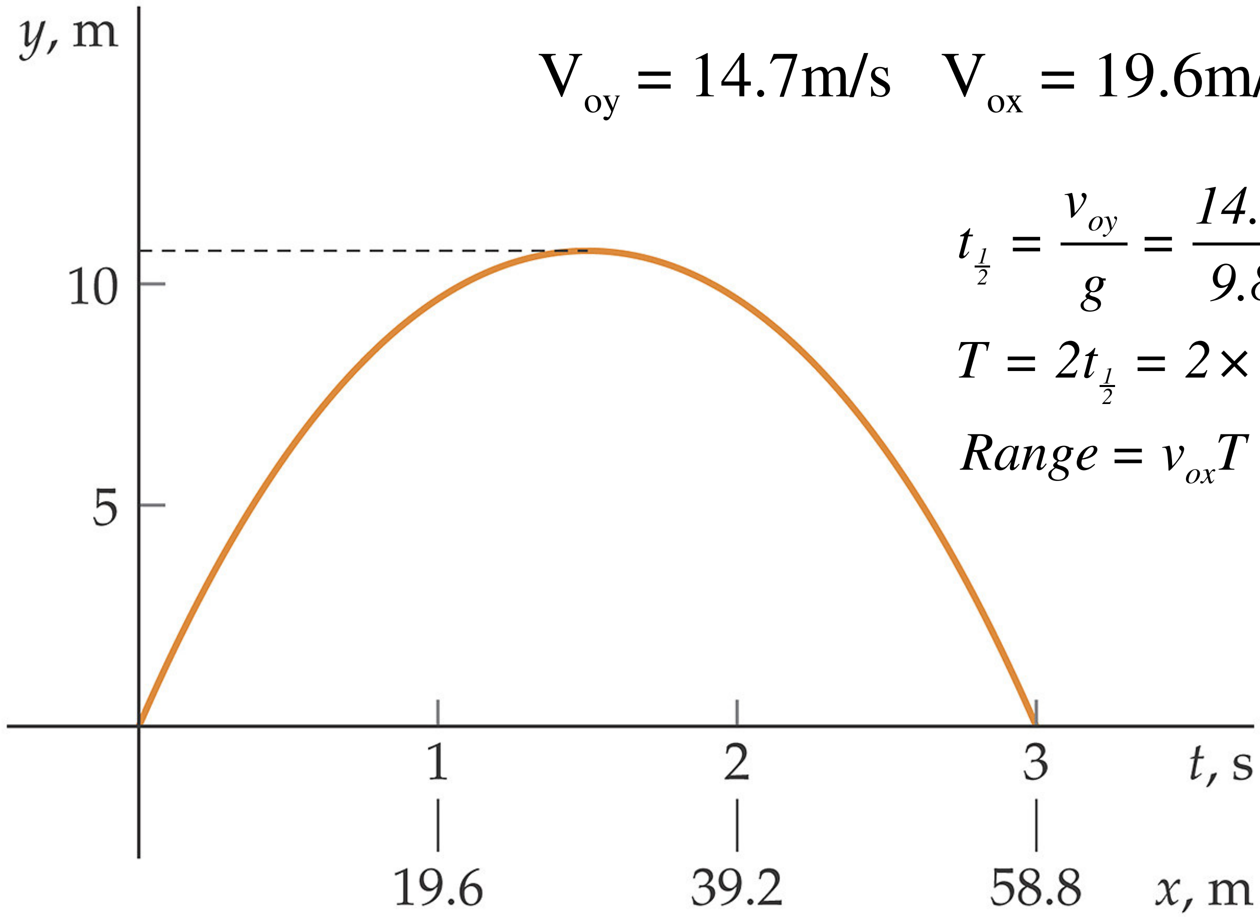
2-D Projectile Motion Y vs X



2-D Projectile Motion Y vs T

$V_o = 24.5$ m/s @ 36.9^o

$V_{oy} = 14.7$m/s $V_{ox} = 19.6$m/s

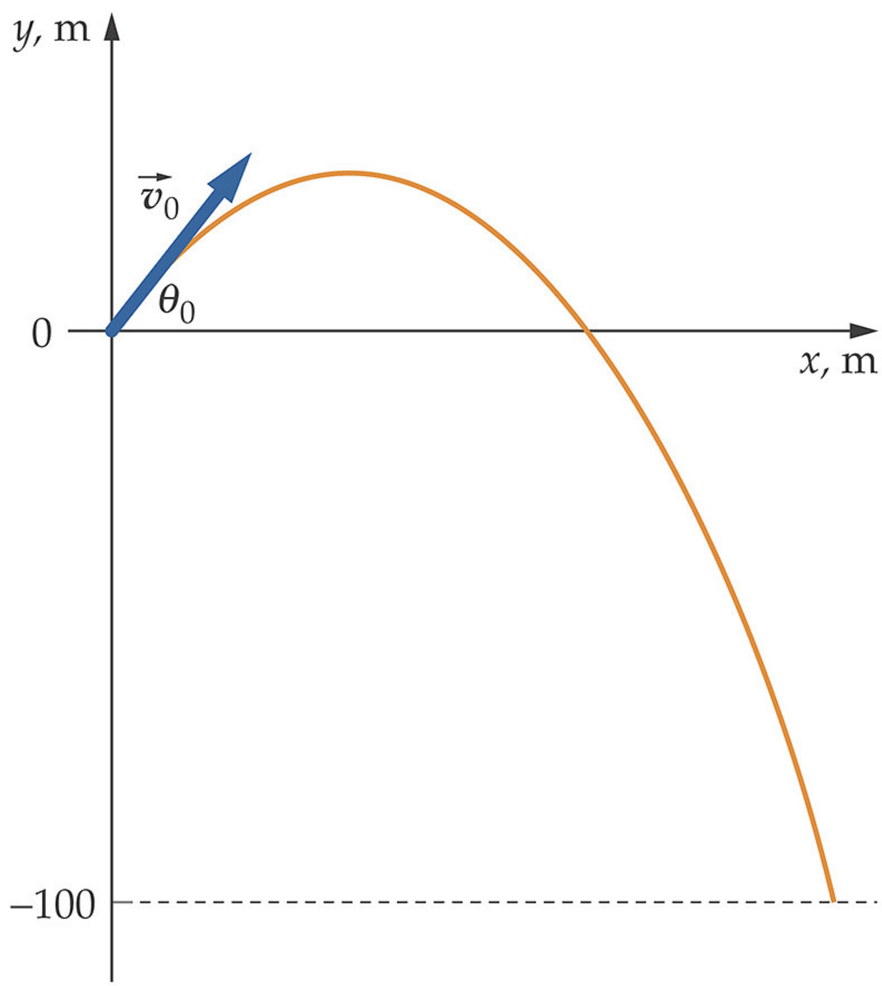


$$t_{\frac{1}{2}} = \frac{v_{oy}}{g} = \frac{14.7}{9.8} = 1.5s$$

$$T = 2t_{\frac{1}{2}} = 2 \times 1.5 = 3.0s$$

$$Range = v_{ox}T = 19.6 \times 3.0 = 58.8m$$

Launched from a Height

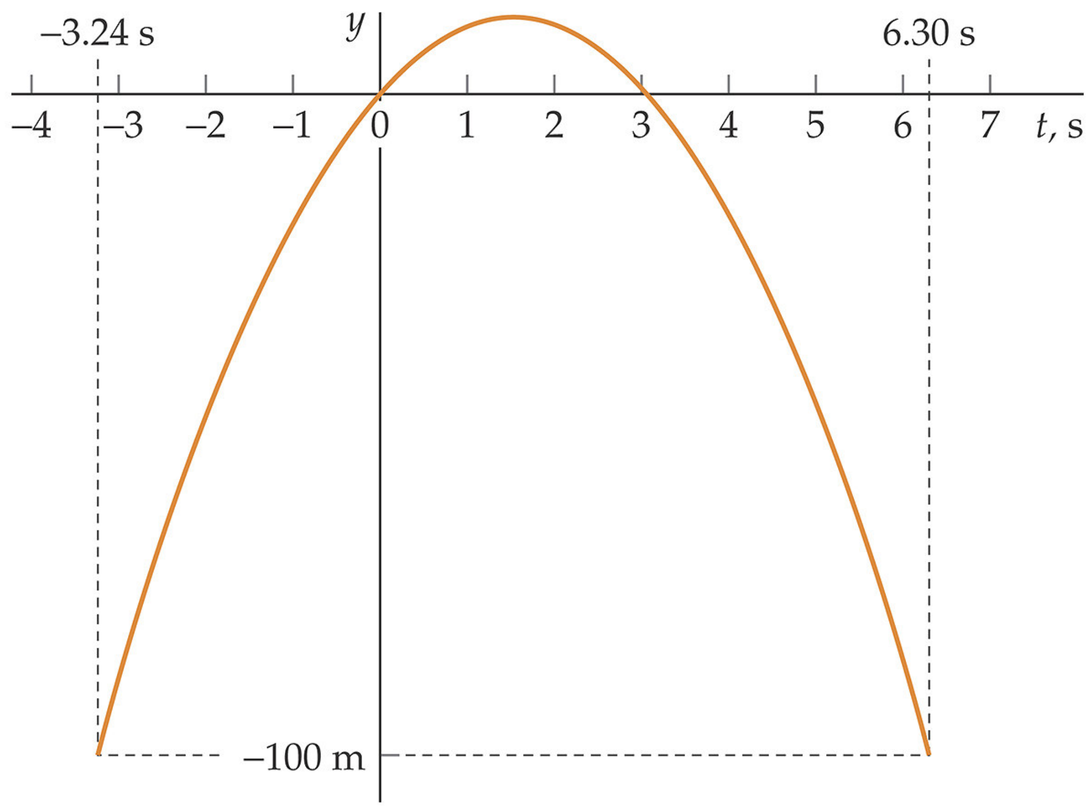


Origin at the beginning of the motion. Positive direction up.

Accel due to gravity pointed down (-y direction).

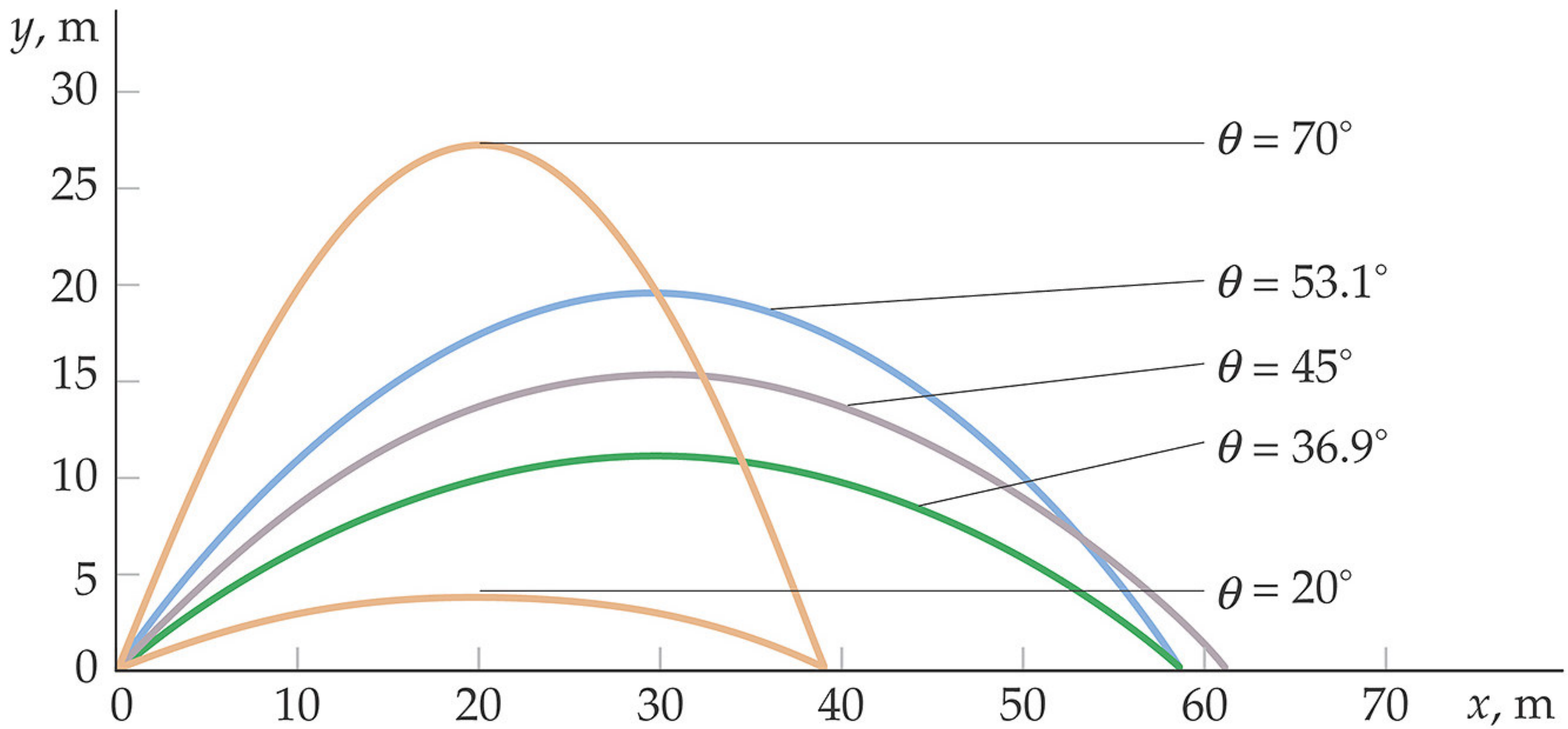
Final vertical position lower than starting position.

Tracing the Negative Solution



The mathematics describe a parabola. It extends backward and forward in time. This problem starts at t = 0. The negative solutions are not relavent to this problem.

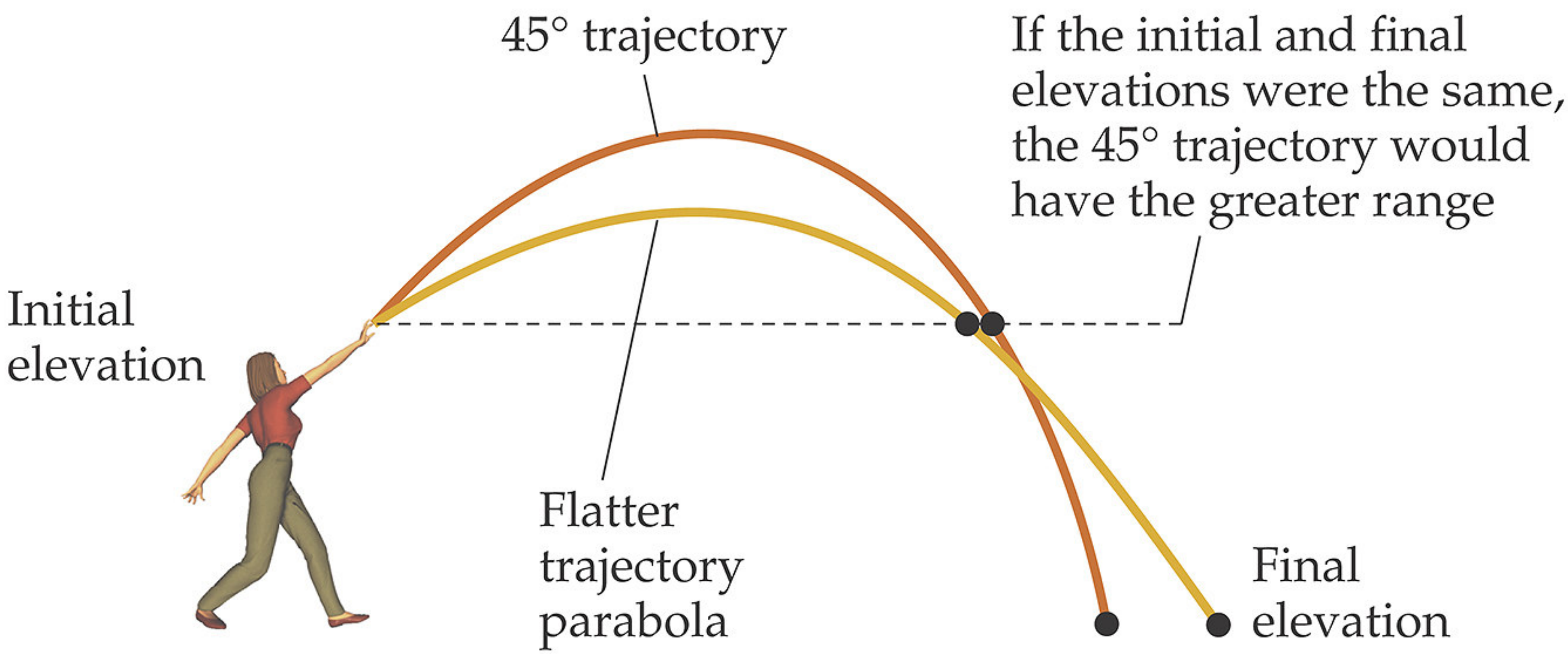
Projectile Trajectories



$$R = \frac{v_{oy}^{2}}{g} sin(2\Theta_0)$$

Trajectories with angles that are complimentary (add to 90^o) have identical ranges.

Projectile Trajectories



Projectile Motion in Vector Form

$$a_x = 0; a_y = -g$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$\vec{a} = 0\hat{i} + (-g)\hat{j}$$

$$\vec{a} = -g\hat{j} = \vec{g}$$

This notation is problematic because it hides the minus sign in the notation. Never do this.

g is always a scalar and its value is 9.8 m/s^2

Associate the sign with the direction and display it explicitly.

$$\cancel{\vec{a} = -g\hat{j} = \vec{g}}$$

$$\vec{a} = -g\hat{j}$$

The Hunter-Monkey Problem

$$\Delta\vec{r}_d = \vec{v}_{do}t - \tfrac{1}{2}gt^2\hat{j}$$

$$\Delta\vec{r}_m = -\tfrac{1}{2}gt^2\hat{j}$$

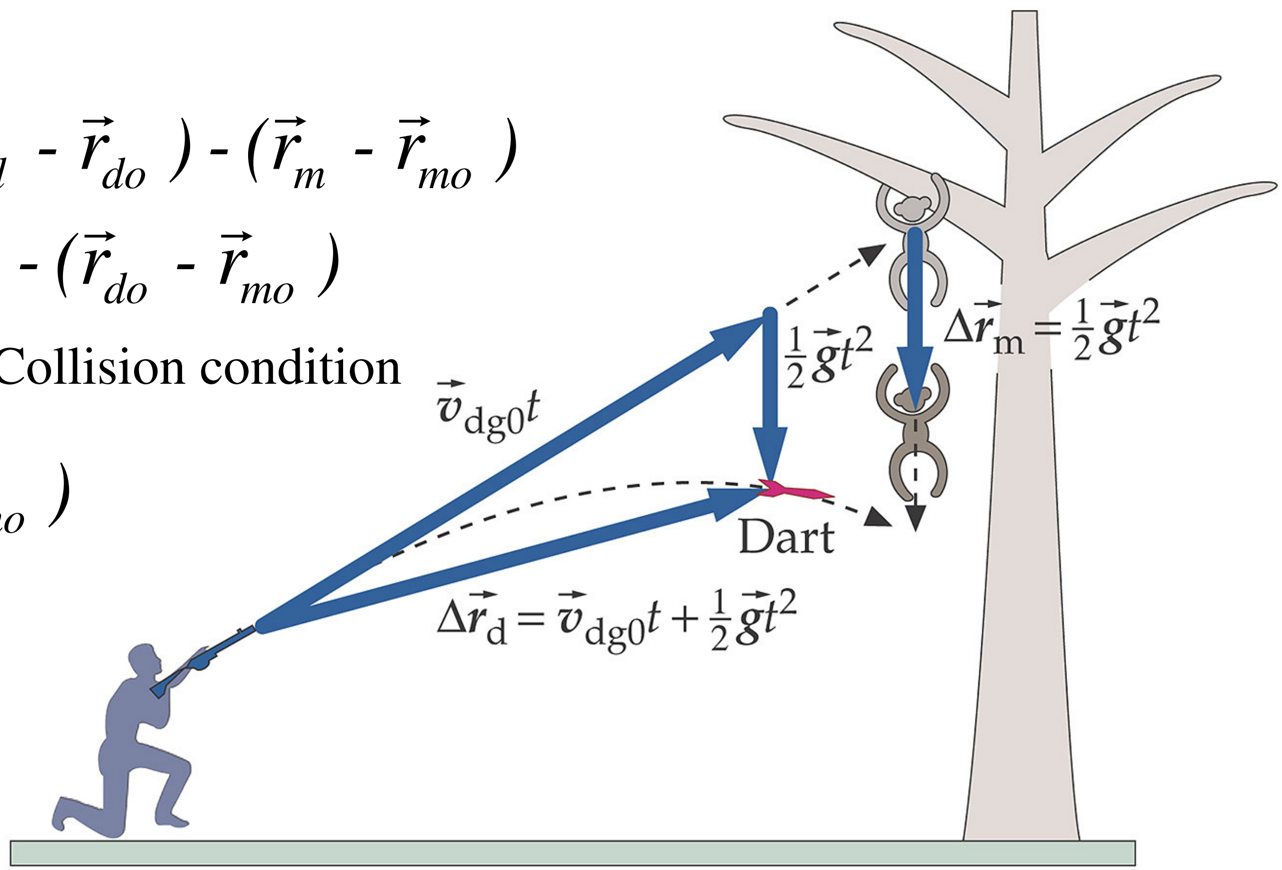
$$\Delta\vec{r}_d - \Delta\vec{r}_m = (\vec{r}_d - \vec{r}_{do}) - (\vec{r}_m - \vec{r}_{mo})$$

$$\vec{v}_{do}t = (\vec{r}_d - \vec{r}_m) - (\vec{r}_{do} - \vec{r}_{mo})$$

$$(\vec{r}_d - \vec{r}_m) = 0$$ Collision condition

$$\vec{v}_{do}t = -(\vec{r}_{do} - \vec{r}_{mo})$$

$$\vec{v}_{do}t = \vec{r}_{mo}$$

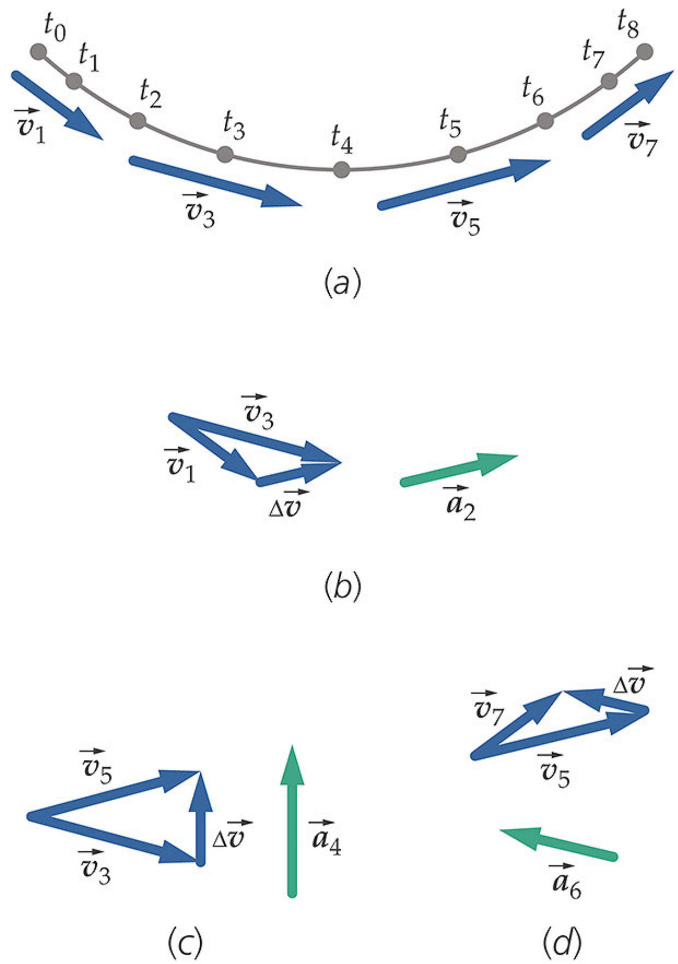


The Pendulum

A **simple pendulum** is constructed by attaching a mass to a thin rod or a light string. We will also assume that the amplitude of the oscillations is small.

Pendulum Motion

Pendulum Motion

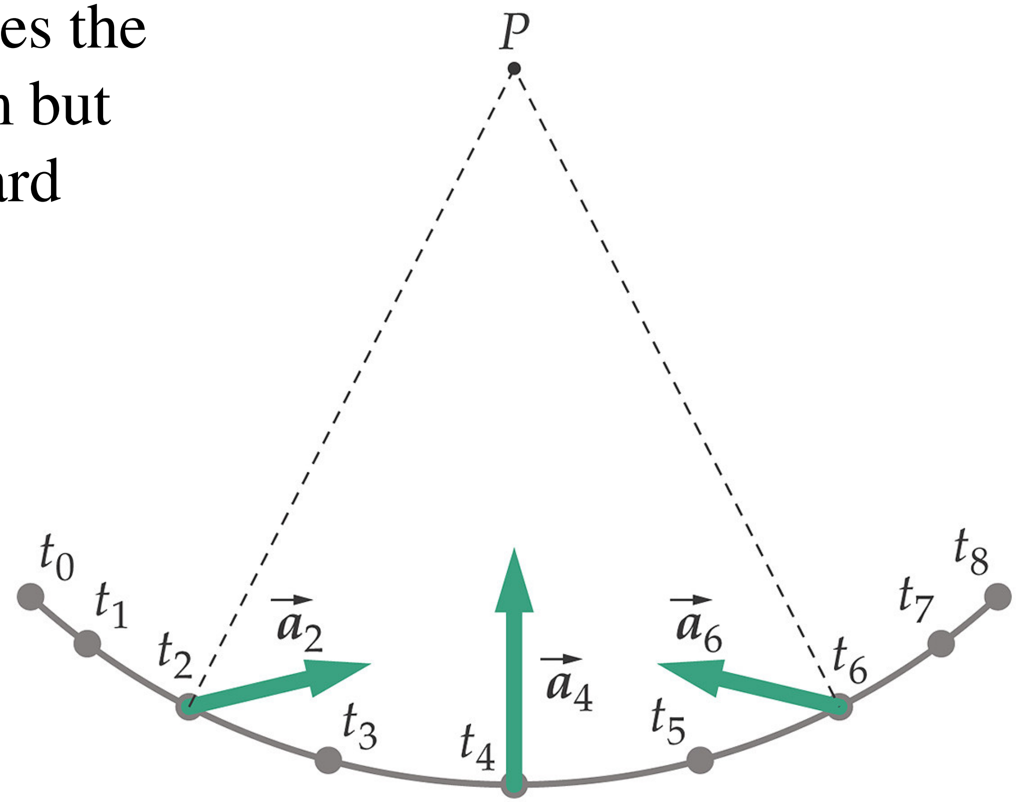


This vector analysis shows that the average acceleration is in the same direction as ΔV.

In terms of understanding the dynamics of the motion of the pendulum this is not the most productive pathway.

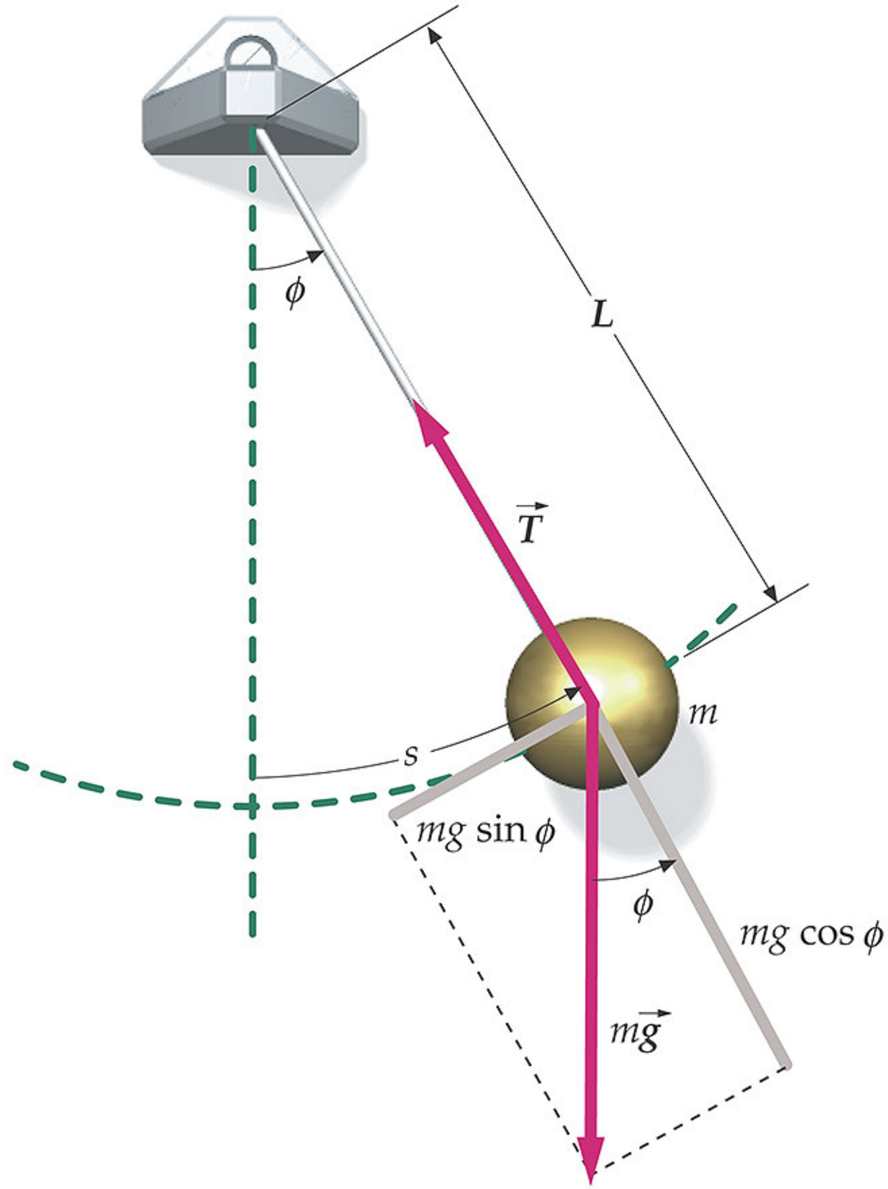
Pendulum Motion

Knowing the net acceleration vector acting on the pendulum is interesting. It describes the motion of the pendulum but this is not the path toward solving the problem of pendulum motion.



The pendulum is best described using polar coordinates.

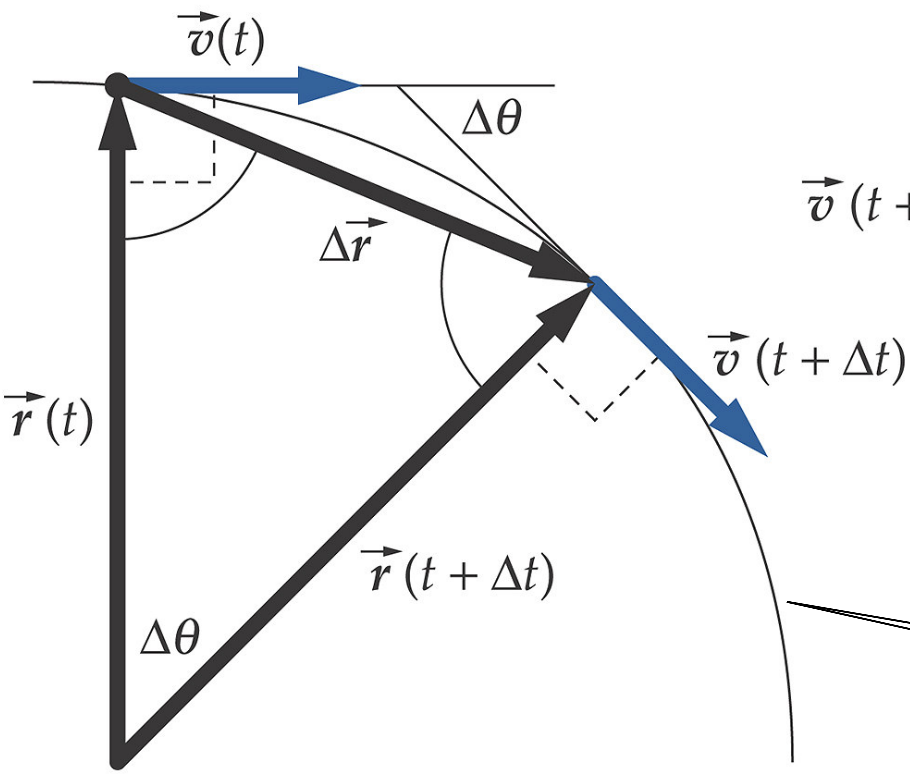
The origin is at the pivot point. The coordinates are (r, φ). The r-coordinate points from the origin along the rod. The φ-coordinate is perpendicualr to the rod and is positive in the counterclock wise direction.

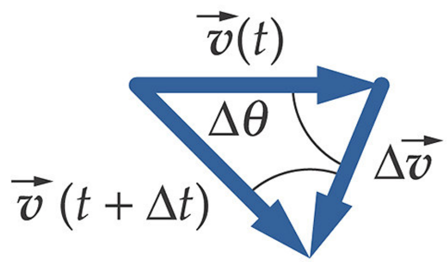


Circular Motion

$$\left|\vec{r}(t)\right| = \left|\vec{r}(t+\Delta t)\right|$$

$$\left|\vec{v}(t)\right| = \left|\vec{v}(t+\Delta t)\right|$$

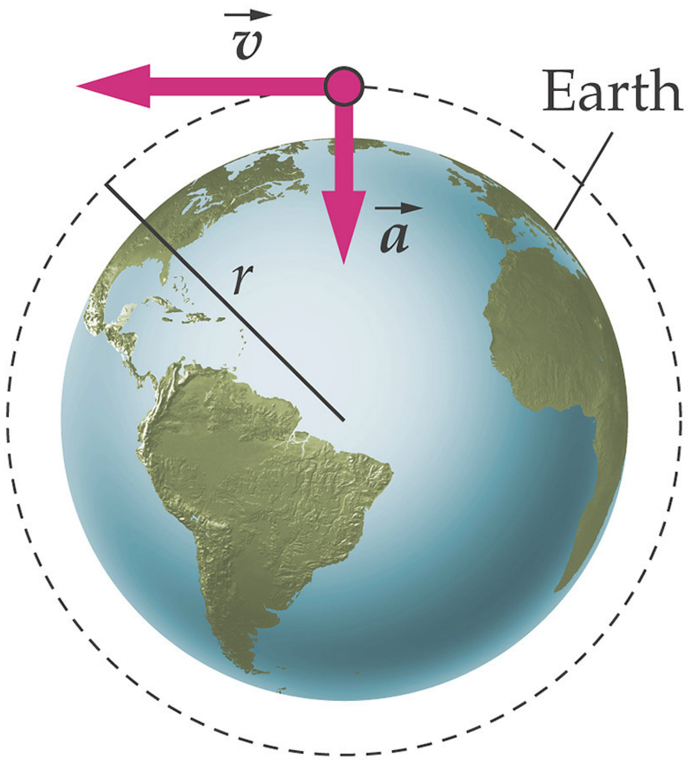




Δv points towards the center of the circular pathway.

Circular pathway

Satellite Motion

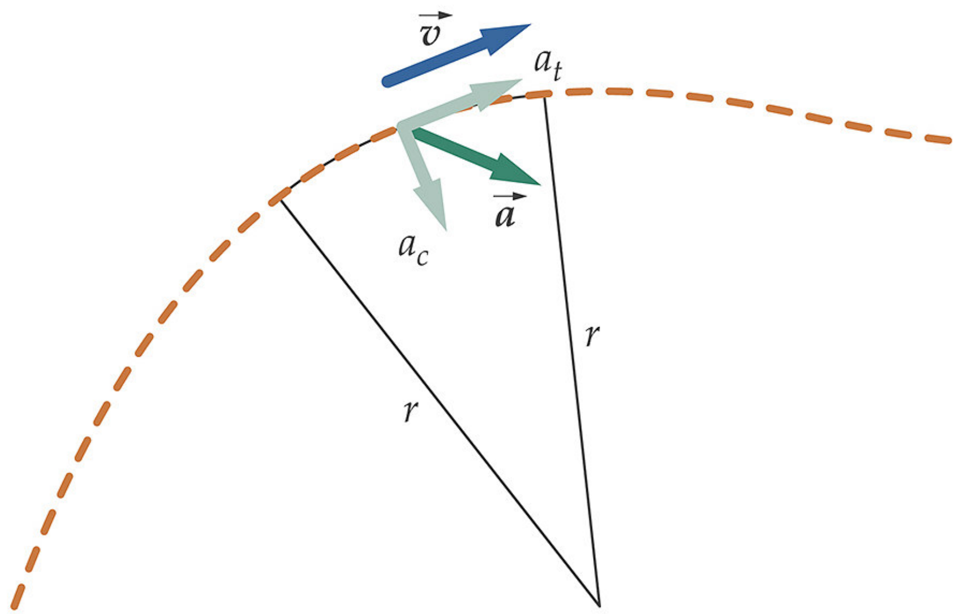


Satellite motion is an example of circular motion but it is actually free fall

Tangential Acceleration

Tangential acceleration (a_t) is parallel to V

The acceleration perpendicular to V is the *centripetal acceleration* (a_c).

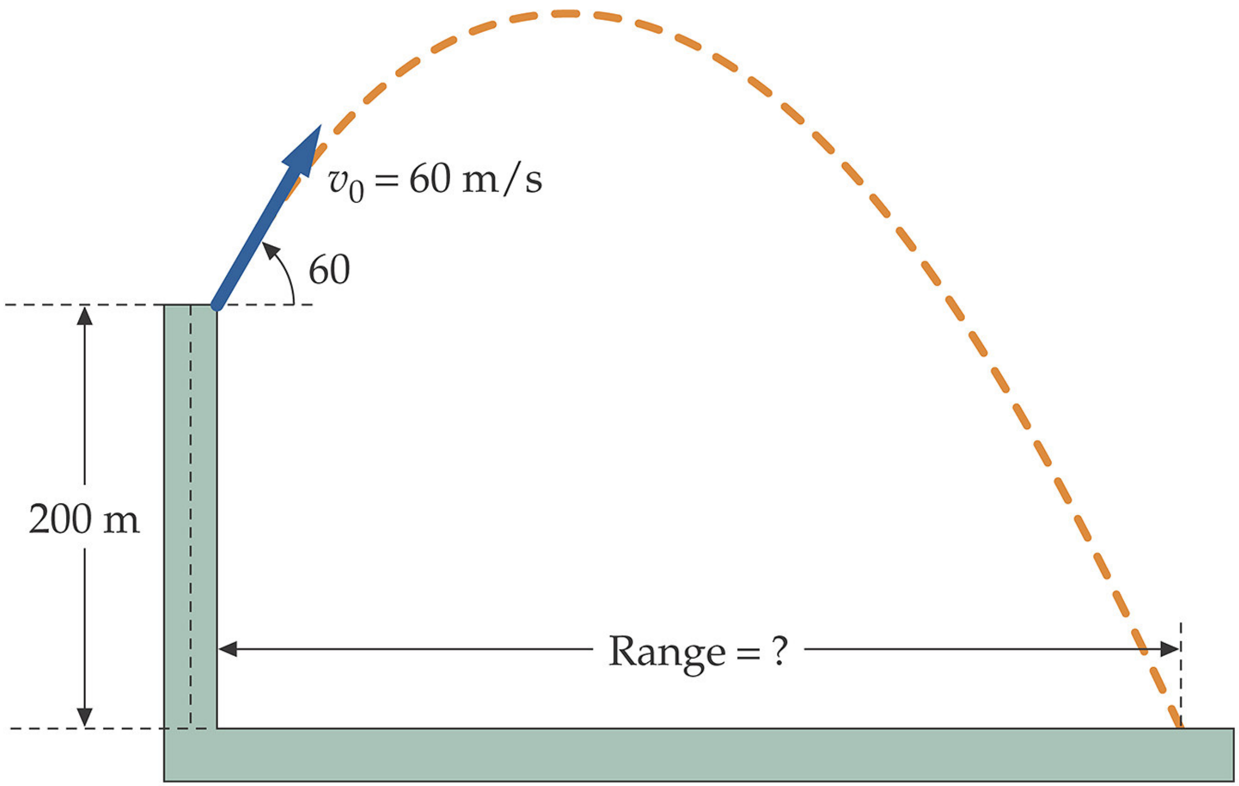


The *tangential acceleration* can only change the magnitude of V.

The *centripetal acceleration* can only change the direction of V.

Projectile Problem

Ques: Where does it land?



Projectile Problem

$$v_{oy} = v_o sin\Theta; \quad v_{ox} = v_o cos\Theta$$

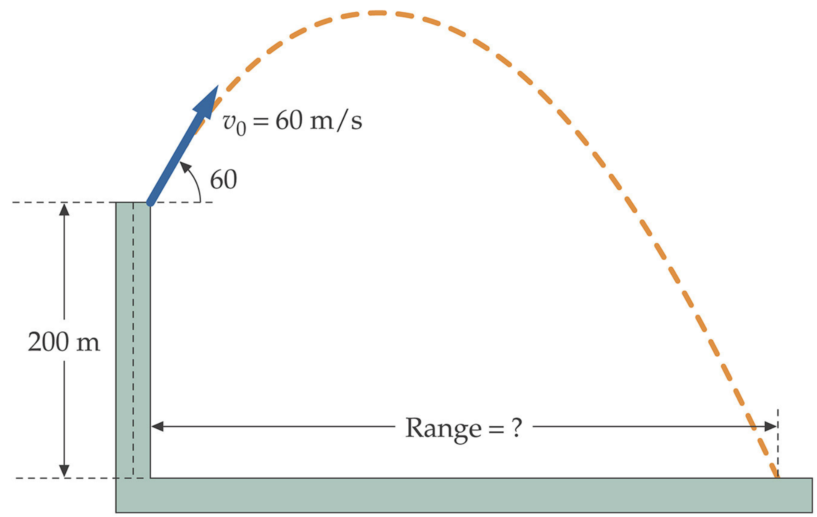
$$y = -200 = v_{oy}t - \tfrac{1}{2}gt^2$$

$$gt^2 - 2v_{oy}t - 400 = 0$$

$$t = \frac{2v_{oy} \pm \sqrt{(2v_{oy})^2 - 4g(-400)}}{2g}$$

$$t = 5.30 \pm 8.31$$

$$t^+ = 11.61s$$



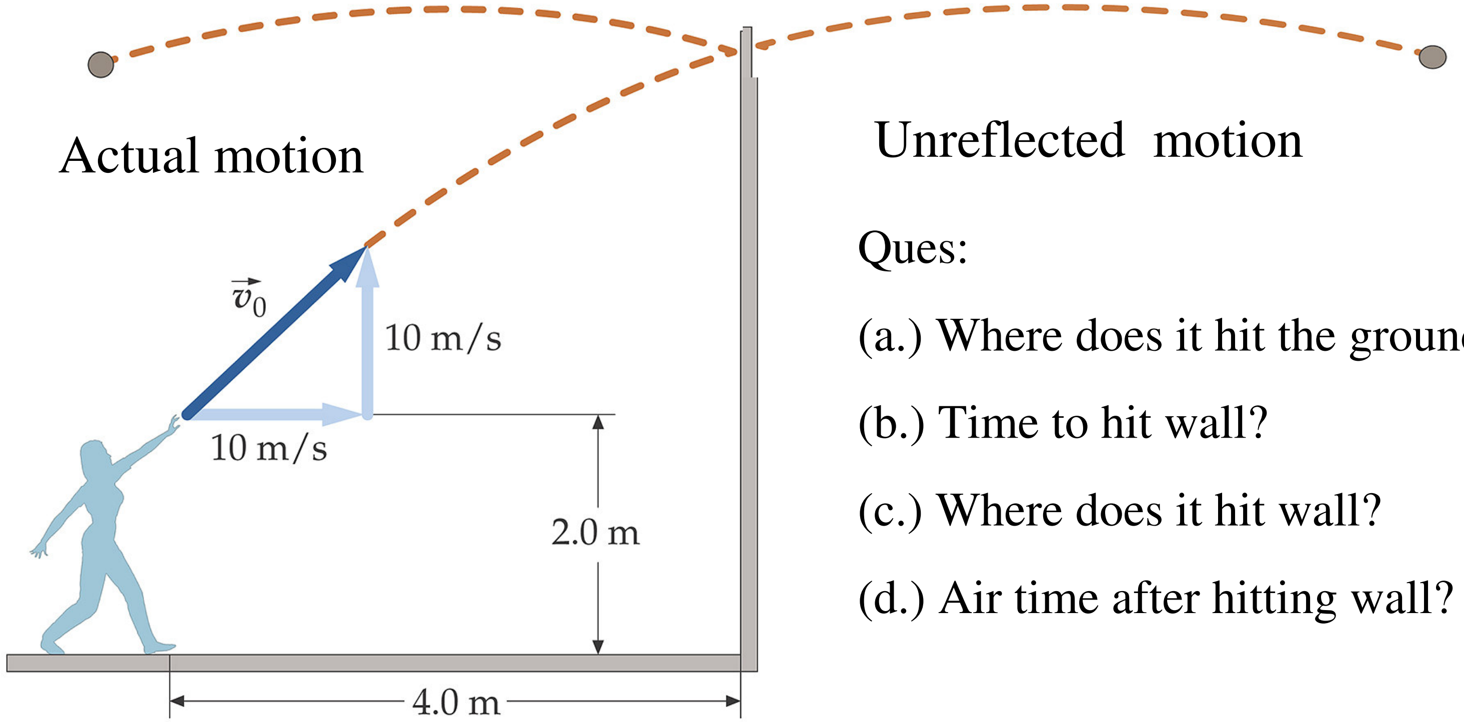
$$Range = v_{ox} \times t^+$$

$$Range = 60cos(60^0) \times 11.61$$

$$Range = 349.8m$$

A 3-Dot Projectile Problem

Problem analysis: The horizontal velocity is reversed when the ball strikes the wall. Forget the wall, solve the problem and fold the solution back, at the wall, later.



Ques:

(a.) Where does it hit the ground?

(b.) Time to hit wall?

(c.) Where does it hit wall?

(d.) Air time after hitting wall?

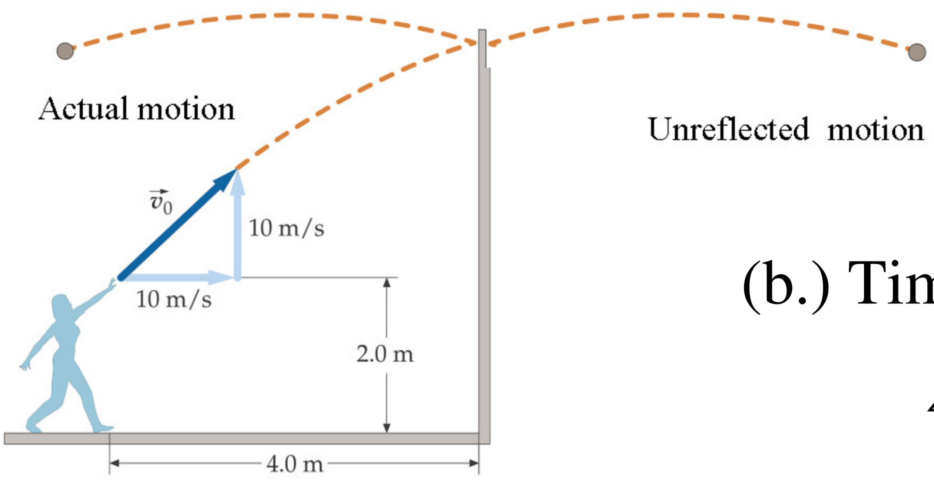
A 3-Dot Projectile Problem

$$v_o = 14.0\,{}^{m}\!/_{s} \text{ at } \Theta = 45^0$$

$$t_{1/2} = \frac{v_{oy}}{g} = \frac{10}{10} = 1.0s$$

$$y_{max} = v_{avg} t_{1/2} = \frac{10}{2} \times 1.0 = 5.0m$$

$$\Delta t_{wall} = \frac{4.0}{v_{ox}} = \frac{4.0}{10} = 0.4s$$



(b.) Time to hit the wall is

$$\Delta t_{wall} = 0.4s$$

A 3-Dot Projectile Problem

(c.) Where does it hit the wall?

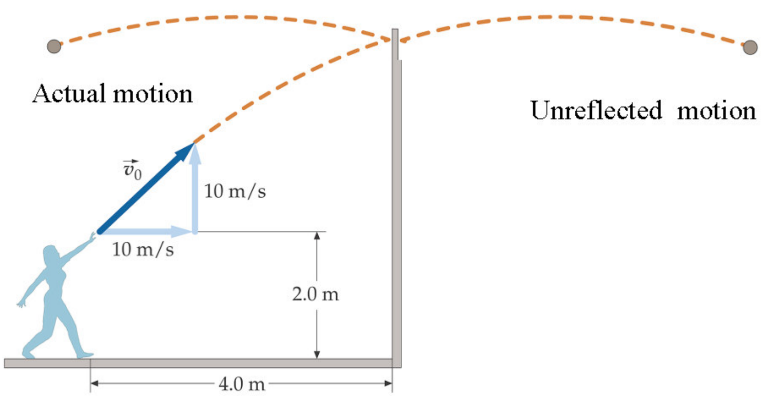
Ans: Y coordinate value at t = 0.4s

$$\Delta y(t) = v_{oy}t - \tfrac{1}{2}gt^2$$

$$y_{hit}(t = 0.4) = \Delta y(t = 0.4) + 2.0$$

$$y_{hit} = 10(0.4) - \tfrac{1}{2}10(0.4)^2 + 2.0$$

$$y_{hit} = 5.2m$$



A 3-Dot Projectile Problem

(d.) Airtime after hitting the wall?

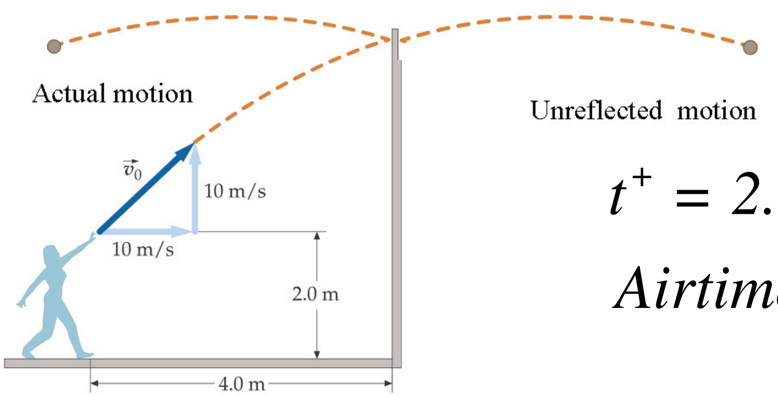
Ans: Total time of unreflected path - 0.4s

$$y = y_o + v_{oy}t - \tfrac{1}{2}gt^2$$

$$0 = 2 + 10t - 5t^2$$

$$5t^2 - 10t - 2 = 0$$

$$t = \frac{10 \pm \sqrt{10^2 - 4(5)(-2)}}{10}$$



$t^+ = 2.18s$ Total air time

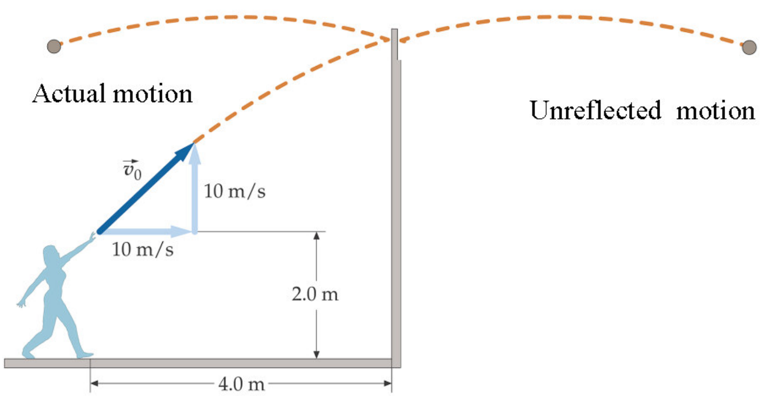
$$Airtime\ after\ wall = 2.18 - 0.4 = 1.78s$$

A 3-Dot Projectile Problem

(a.) Where does it hit the ground?

Ans: $x = v_{ox}$ * (Air time after wall hit), measured from the wall to the left.

$$x = 10 \times 1.78 = 17.8m$$



Extra Slides

Apply Newton's 2nd Law to the pendulum bob.

$$\sum F_\phi = -mg\sin\phi = ma_\phi$$

$$\sum F_r = T - mg\cos\phi = m\frac{v^2}{r} = 0$$

If we assume that φ <<1 rad, then sin φ ≈ φ and cos φ ≈1, the angular frequency of oscillations is then:

$$\sum F_\phi = -mg\sin\phi = ma_\phi = mL\alpha$$

$$-mg\sin\phi = mL\alpha$$

$$\alpha = -(g/L)\sin\phi$$

$$\alpha = -(g/L)\phi$$

$$\omega = \sqrt{\frac{g}{L}}$$

The period of oscillations is $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

Example (text problem 10.60): A clock has a pendulum that performs one full swing every 1.0 sec. The object at the end of the string weighs 10.0 N.

What is the length of the pendulum?

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Solving for L:

$$\mathrm{L} = \frac{gT^2}{4\pi^2} = \frac{\left(9.8\ \mathrm{m/s^2}\right)\left(1.0\ \mathrm{s}\right)^2}{4\pi^2} = 0.25\ \mathrm{m}$$

The gravitational potential energy of a pendulum is

$$U = mgy.$$

Taking y = 0 at the lowest point of the swing, show that $y = L(1-\cos\theta)$.