Chapter 5

Newton's Laws

Newton's Laws

- Newton's First Law
- Force & Mass
- Newton's 2nd Law
- Contact force: solids, spring & string
- Force due to gravity: Weight
- Problem solving: Free body diagrams
- Newton's 3rd Law
- Problem solving: Two or more objects

Newton's First Law of Motion:
Inertia and Equilibrium

Newton's 1st Law (The Law of Inertia):

If no force acts on an object, then the speed and direction of its motion do not change.

Inertia is a measure of an object's resistance to changes in its motion.

It is represented by the **inertial mass**.

Newton's First Law of Motion

If the object is at rest, it remains at rest (velocity = 0).

If the object is in motion, it continues to move in a straight line with the same velocity.

No force is required to keep a body in straight line motion when effects such as friction are negligible.

An object is in **translational equilibrium** if the net force on it is zero and vice versa.

$$\Sigma F = 0 \longleftrightarrow \textbf{Translational Equilibrium}$$

Inertial Frame of Reference

“If no forces act on an object, any reference frame for which the acceleration of the object remains zero is an inertial reference frame.” -Tipler

If the 1st Law is true in your reference frame then you are in an inertial frame of reference.

Newton's Second Law of Motion
Net Force, Mass, and Acceleration

Newton's 2nd Law:

The acceleration of a body is directly proportional to the net force acting on the body and inversely proportional to the body's mass.

Mathematically: $\mathbf{a} = \dfrac{\mathbf{F}_{\text{net}}}{m}$ or $\mathbf{F}_{\text{net}} = m\mathbf{a}$

$\mathbf{F}_{\text{net}} = m\mathbf{a}$ This is the workhorse of mechanics

Newton's Second Law of Motion

An object's mass is a measure of its inertia. The more mass, the more force is required to obtain a given acceleration.

The net force is just the vector sum of all of the forces acting on the body, often written as $\Sigma\mathbf{F}$.

If $\mathbf{a} = 0$, then $\Sigma\mathbf{F} = 0$. This body can have:

Velocity = 0 which is called *static equilibrium*, **or**

Velocity $\neq$ 0, but constant, which is called *dynamic equilibrium*.

Newton's Third Law of Motion
Interaction Pairs

Newton's 3rd Law:

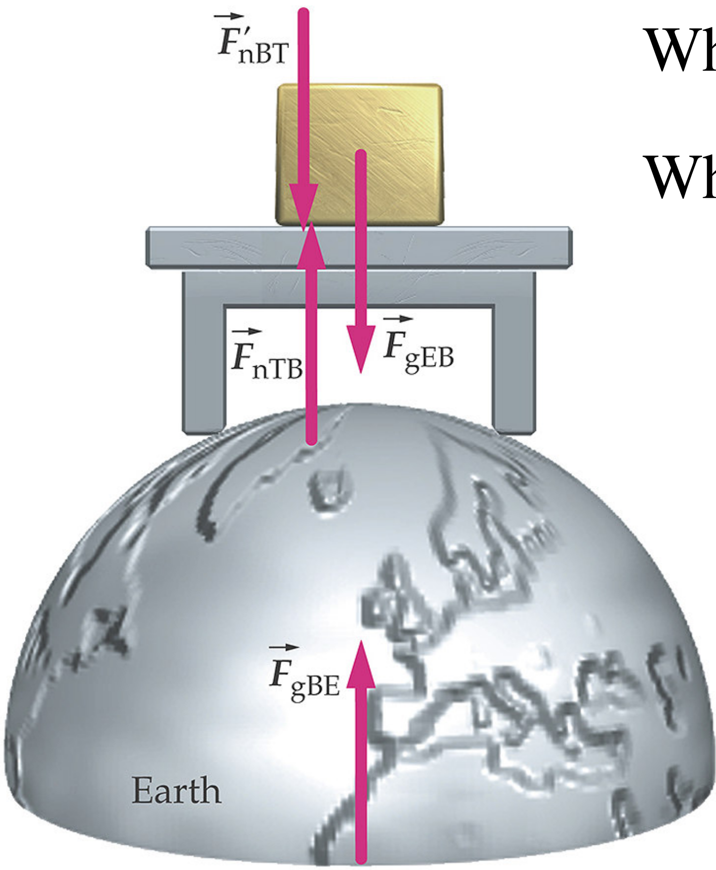
When 2 bodies interact, the forces on the bodies, due to each other, are always equal in magnitude and opposite in direction.

In other words, these interaction forces come in pairs.

Mathematically: $\mathbf{F}_{21} = -\mathbf{F}_{12}.$

$\mathbf{F}_{21}$ designates the force on object 1 due to object 2.

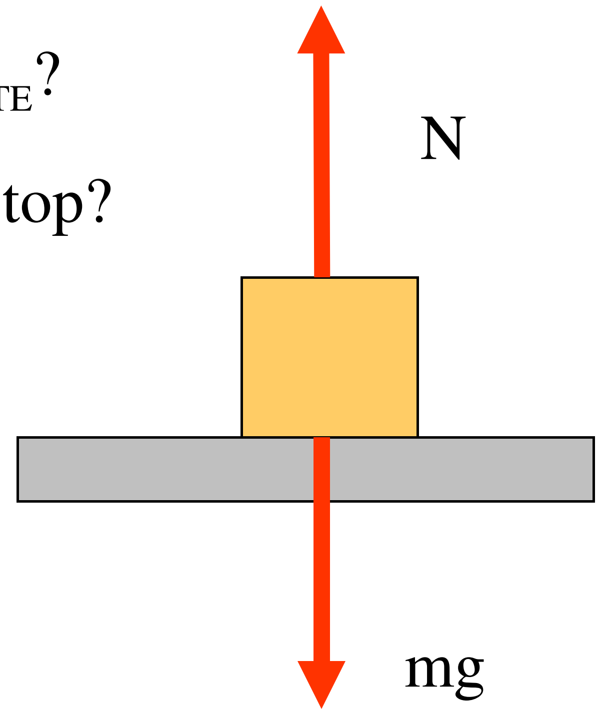
Action-Reaction Pairs (3rd Law)



What about F_{gET}?

What about F_{gTE}?

Where do we stop?



This is all we really need!

Types of Forces

Contact forces:

Normal Force & Friction

Tension

Strings & Springs

Gravitational Force - non contact

Contact Forces

Contact forces: these are forces that arise due to of an interaction between the atoms in the surfaces of the bodies in contact.

Frictional Forces

Friction: a contact force *parallel* to the contact surfaces.

Static friction acts to prevent objects from sliding.

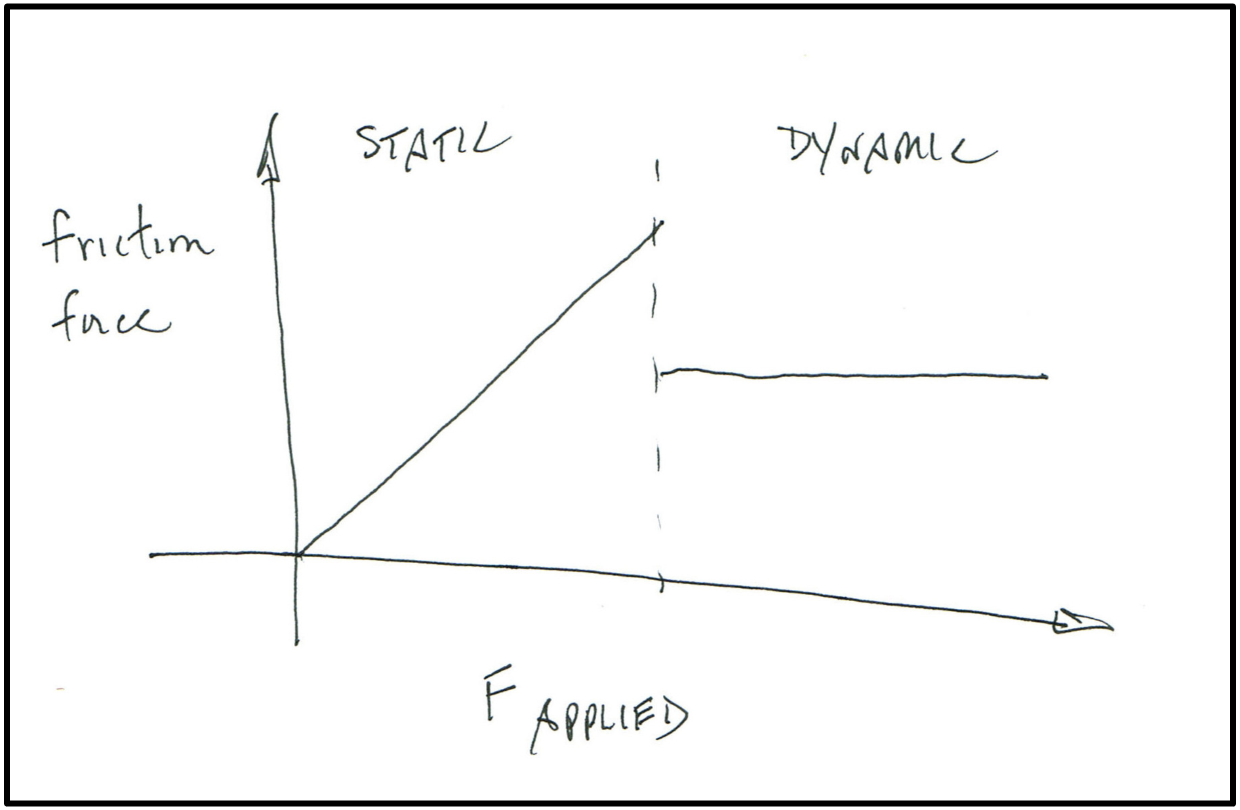
$$f_s^{max} = \mu_s N$$

Kinetic friction acts to make sliding objects slow down. Sometimes called Dynamic friction.

$$f_d = \mu_d N$$

N is the normal force of the surface pushing back on the object.

Frictional Forces



Tension

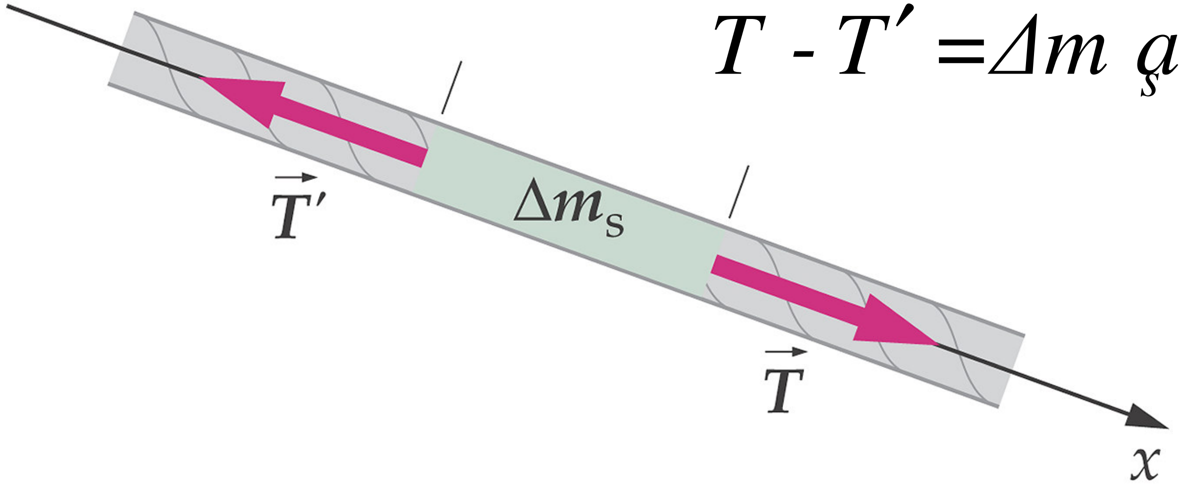
This is the force transmitted through a "rope" or string from one end to the other.

An **ideal** cord has zero mass, does not stretch, and the tension is the same throughout the cord.

The Massless String

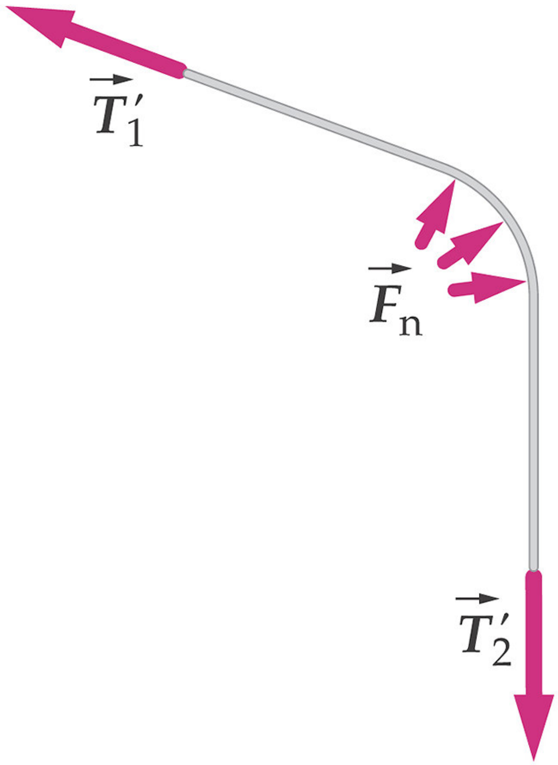
For the string to move, tensions T and T' must be different.

$$T - T' = \Delta m_s a$$



If we assume that the string is massless ($\Delta m_s = 0$) then the two tensions are equal.

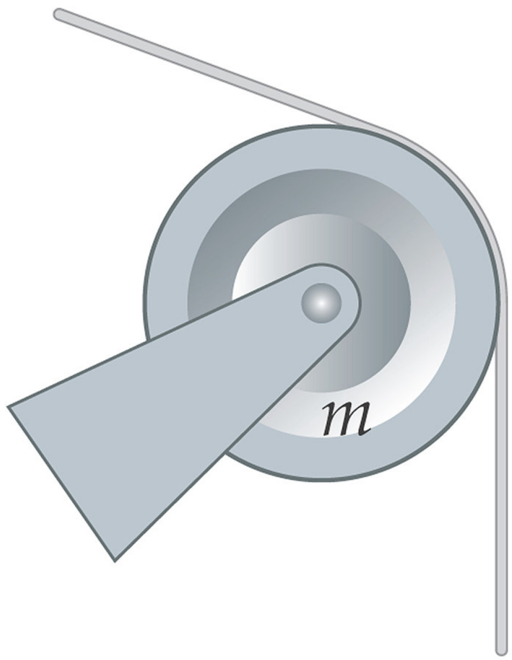
The Massless String



When a massless string, which is under tension, passes over a curved surface the direction of the tension follows the string with no change in magnitude.

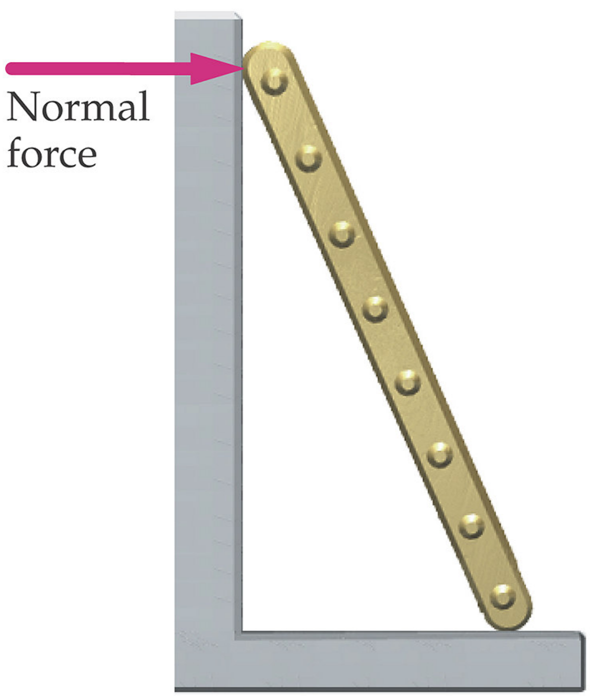
$$\left|T_1'\right| = \left|T_2'\right|$$

The Massless Pulley



- A massive pulley represents inertia in the mass-string system.
- Since we haven't covered rotational motion and moments of inertia we want to avoid dealing with massive pulleys until a later chapter.
- This is accomplished by using massless or relatively small mass pulleys.

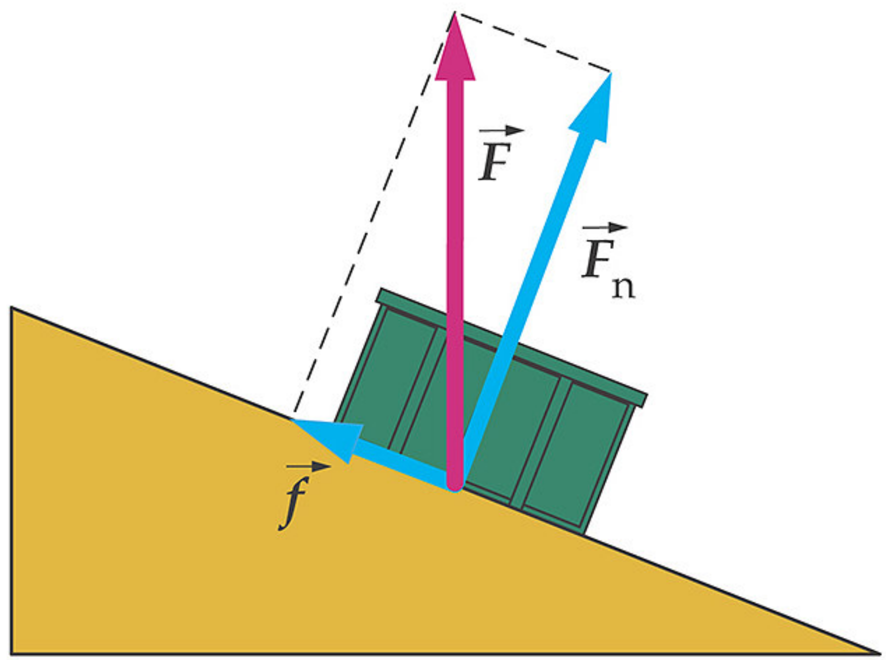
Normal Force



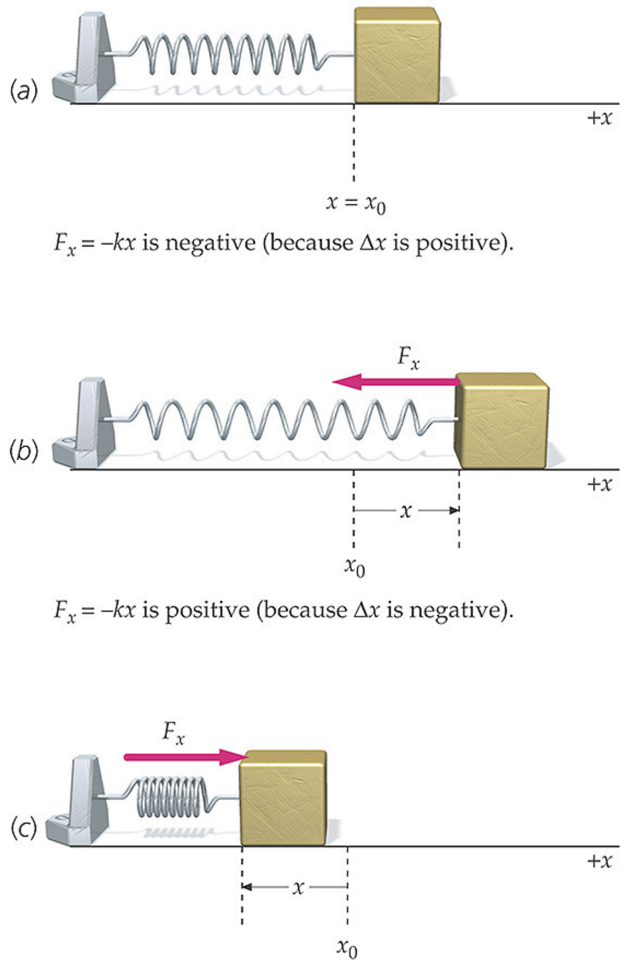
- A normal force is a contact force.
- It acts perpendicular to the surface that is the source of the force.
- Its value is determined by the problem and it assumes the value needed to satisfy the conditions in the problem.

Normal Force

On an incline the surface contact force has two components.
The normal (perpendicular) component is the normal force.
The parallel component is the friction.



Spring - Contact Force



The spring force is an example of a restoring force

$$\vec{F} = -k\vec{x}$$

The magnitude of the force is proportional to the stretch X.

The direction of the force is opposite to the direction of $\vec{x}$

Action at a Distance Forces

Gravitation

Electric Force

Magnetic Force

Weak Interaction

Strong Interaction

Electric Force + Magnetic Force } Electro-magnetic Force

Electromagnetic Force + Weak Interaction } Electroweak

Electroweak + Strong Interaction } Standard Model

Researchers are still trying to include gravity

Action at a distance is what it appears to be at this level. There is actually an exchange of carrier particles (field quanta) that mediate the force.

Gravitation - Weight

What we commonly call the weight of an object is the force due to the gravitational pull of the earth acting on the object.

If we drop an object and only gravity is acting on the object we say that the object is in Free Fall.

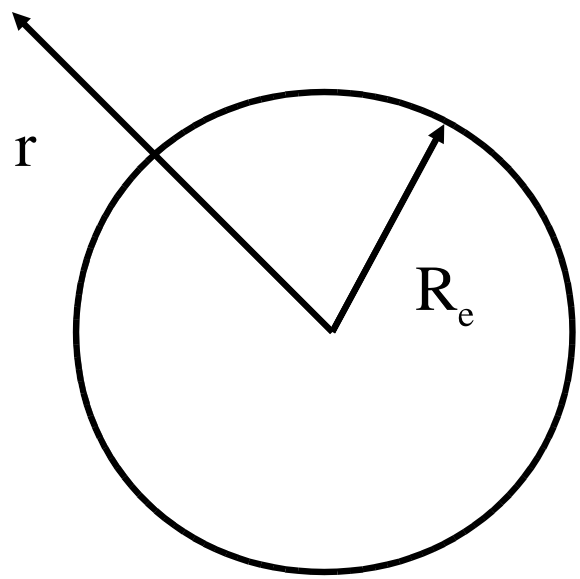
$$\vec{F}_g = m\vec{g}$$

"g" is the acceleration due to gravity an equals 9.8 m/s^2

In the British system "g" is 32 ft/s^2

Gravitation - Weight

The value of “g” varies slightly from one place to another on th surface of the earth. As one leaves the surface of the earth the inverse square dependency on the distance from the center of the earth becomes noticeable.



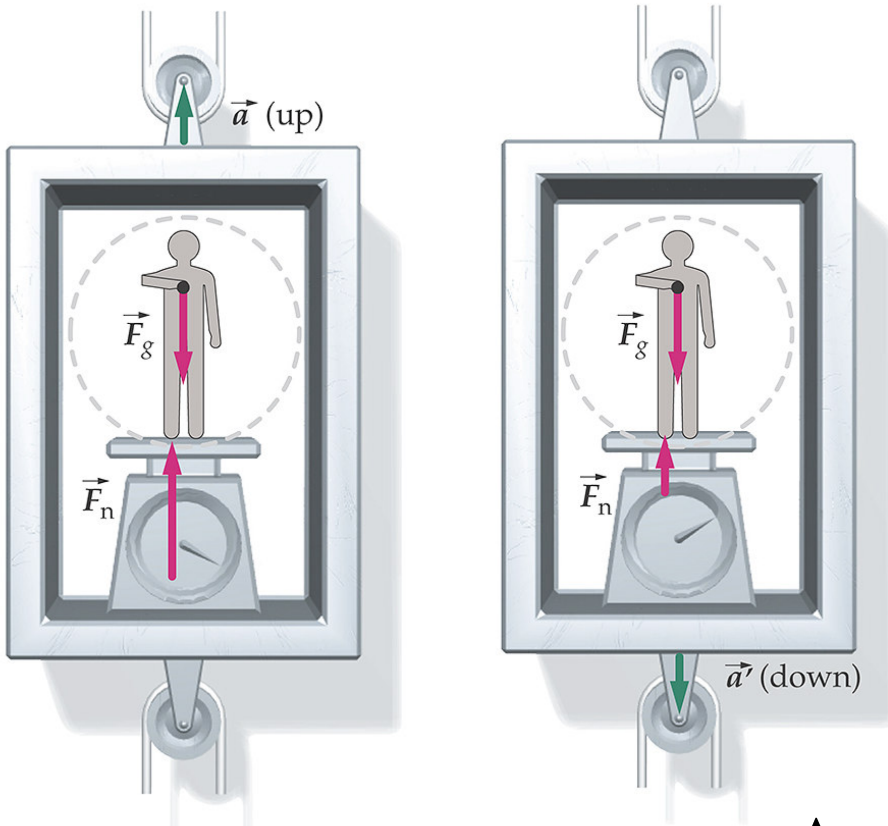
$$F_g = \frac{GM_e m}{r^2} = mg$$

$$g = \frac{GM_e}{r^2}$$

Apparent Weight

Acceleration Up

Scale reading *higher* than when at rest.

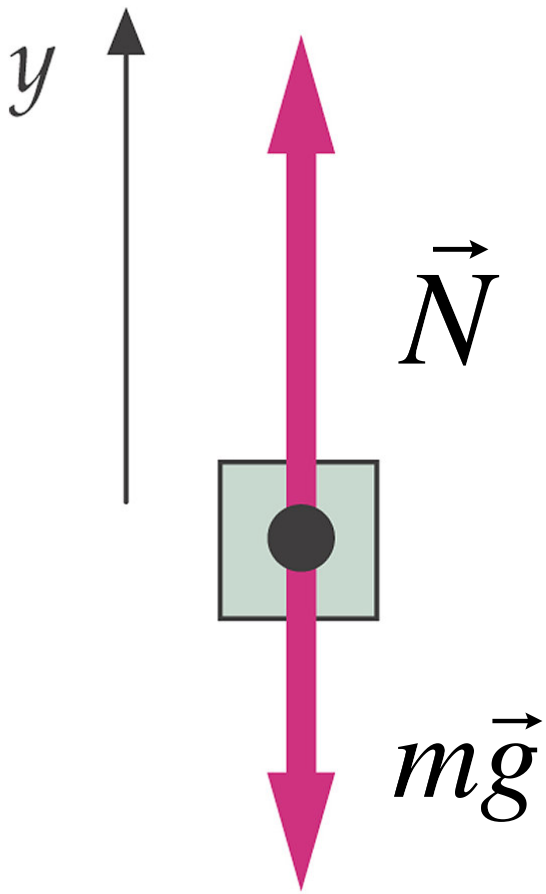


(a) (b)

Scale reading *lower* than when at rest.

Acceleration Down

Apparent Weight



$$\sum F = \vec{N} - m\vec{g} = m\vec{a}$$

Applying Newton's Second Law

The one equation everyone remembers!

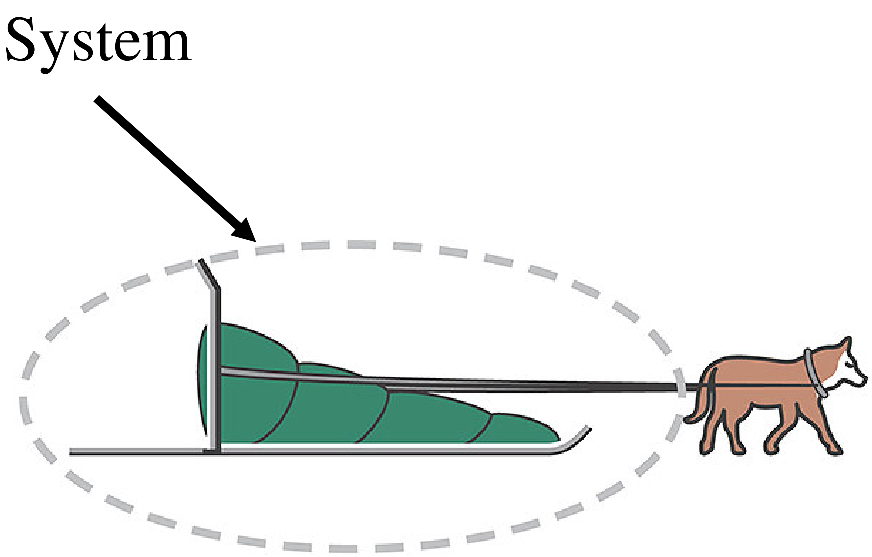
$$\Sigma \mathbf{F} = m\mathbf{a}$$

Sum of the forces acting on the objects in the system	"m" is the System Mass	"a" is the *System Response*

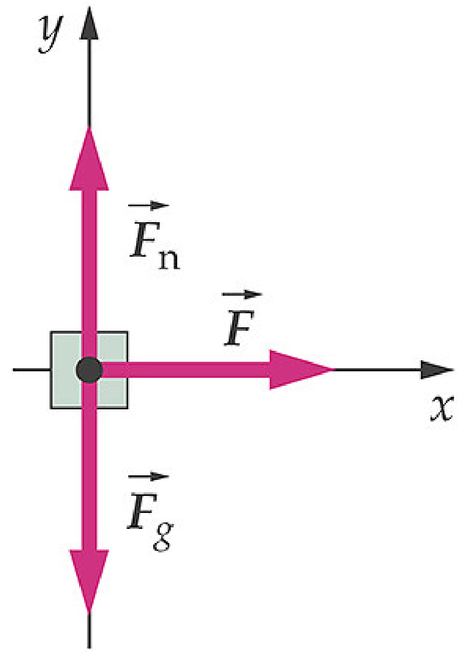
This equation is just the tip of the "iceberg" of the mechanics problem. The student will need to anlyze the forces in the problem and sum the force vector components to build the left hand side of the equation.

Free Body Diagram

Tipler's Notation



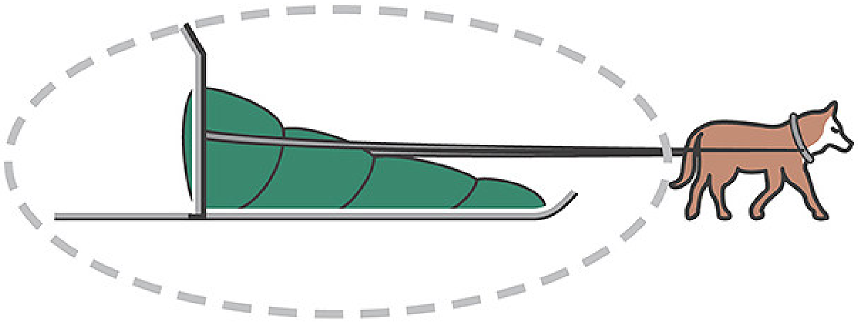
(a)



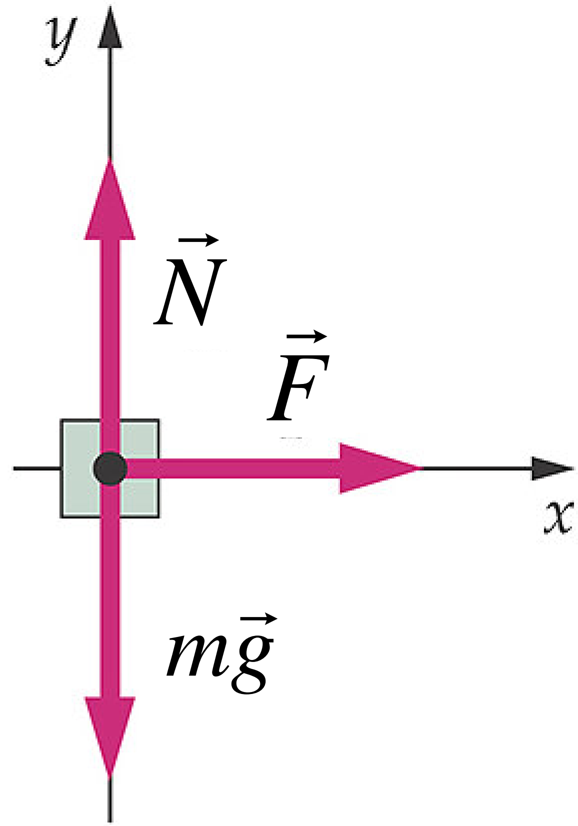
All these vectors represent forces. Using the letter F for vector names forces attention to the subscripts for differentiation.

Free Body Diagram

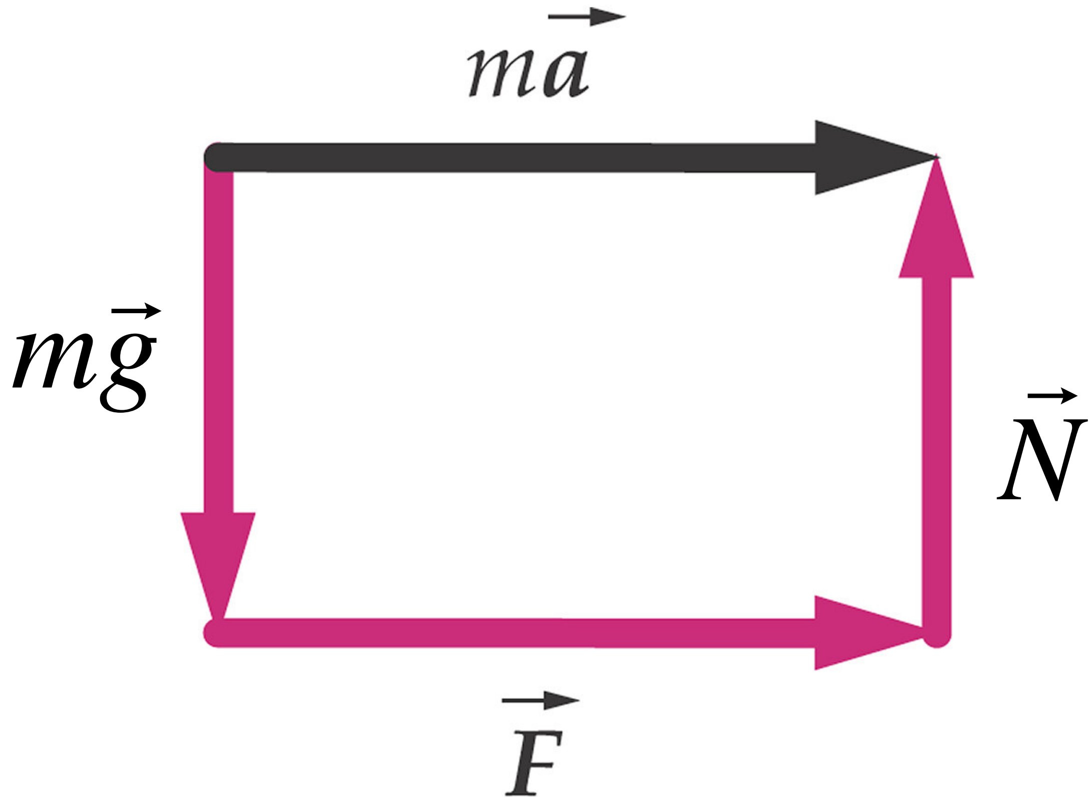
A Better Notation



(a)

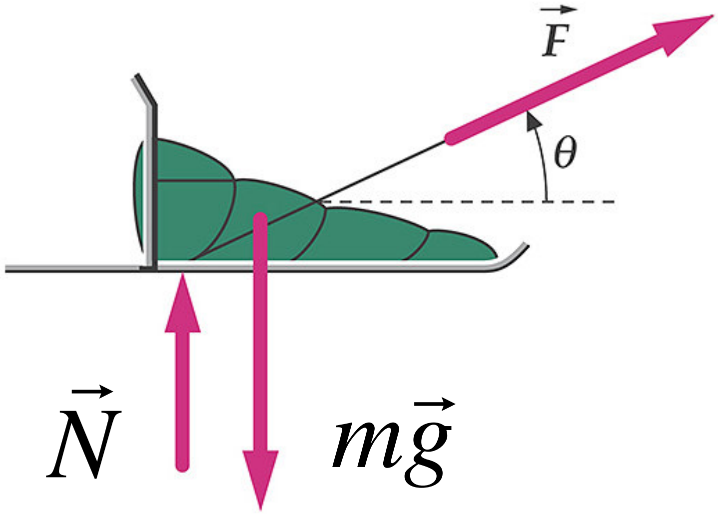


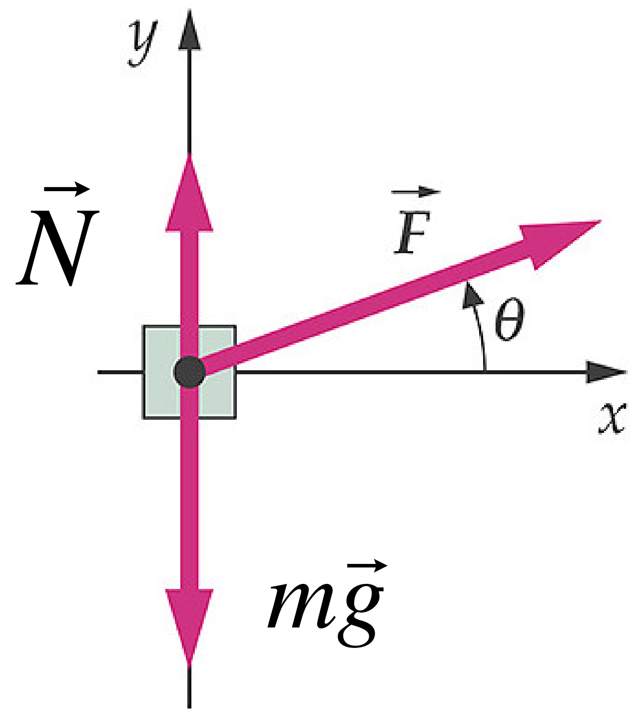
$\Sigma \mathbf{F} = m\mathbf{a}$

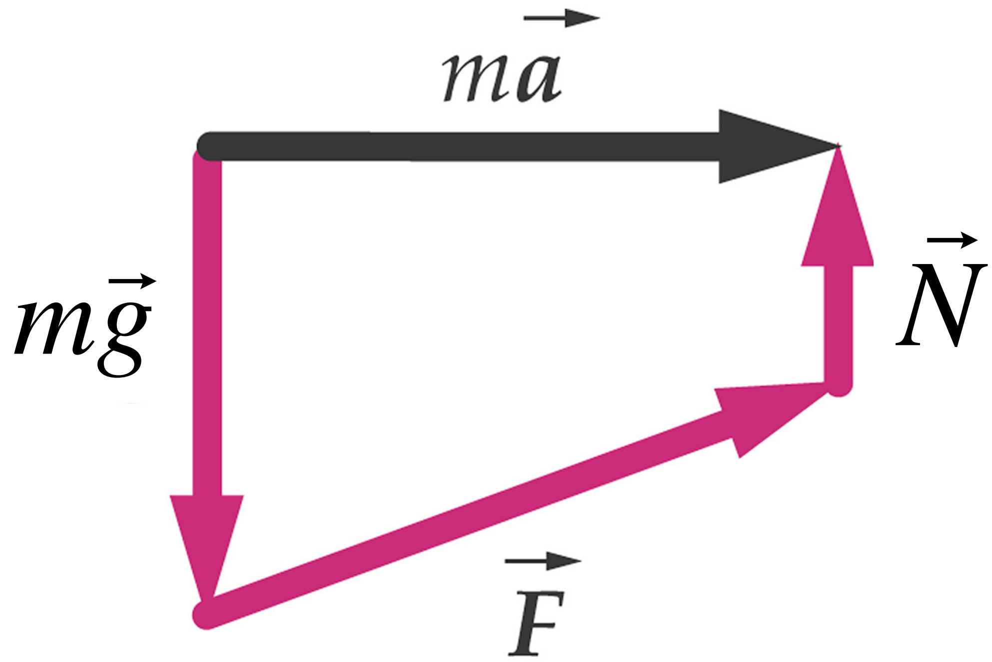


This is not a methodology to solve for the acceleration. It is just graphically demonstrating that the net force is ma

Same problem but the applied force is angled up



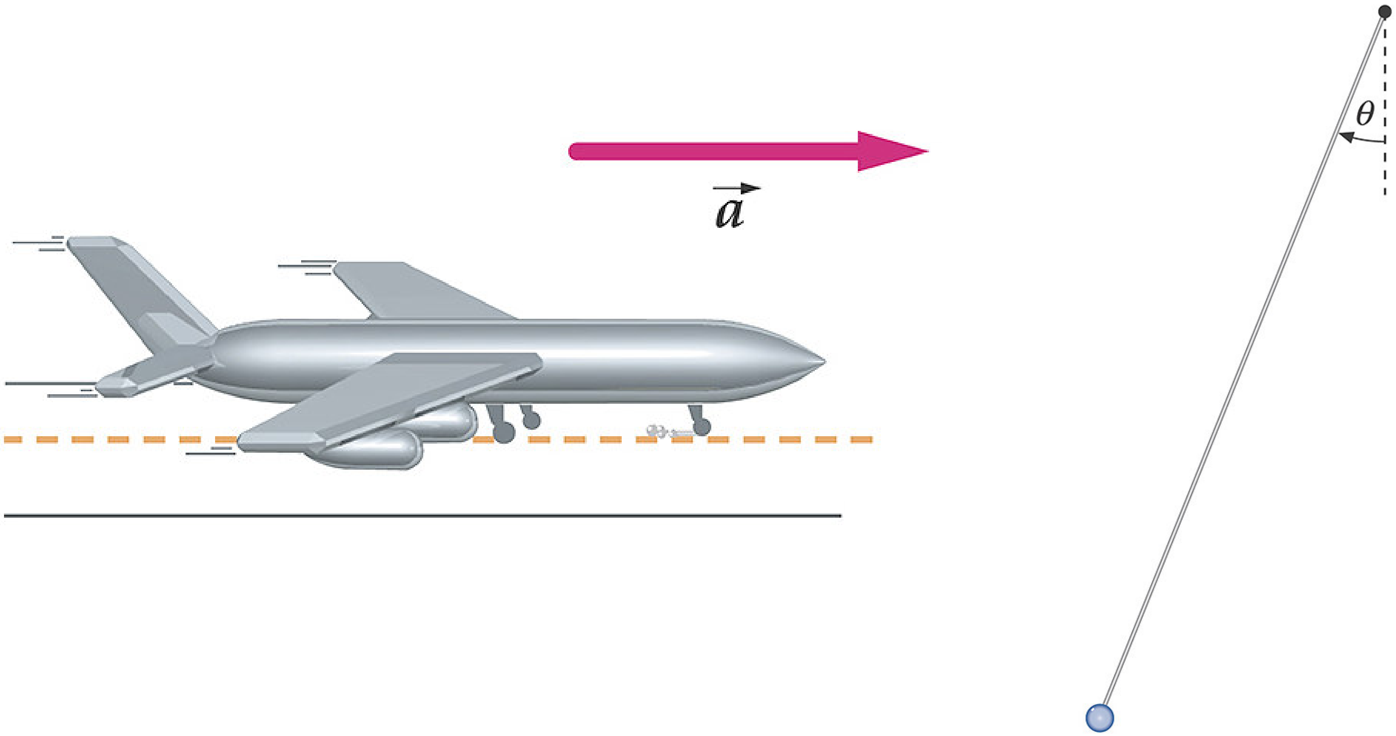




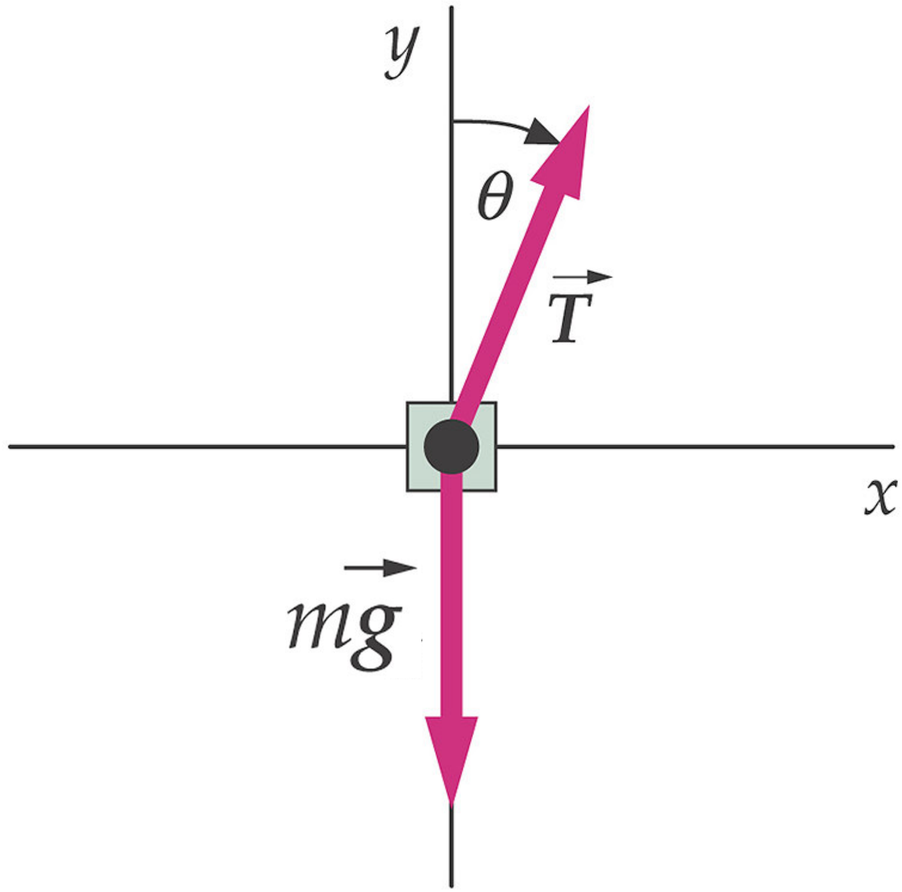
The normal force, N, is smaller in this case because the upward angled applied force reduces the effective weight of the sled.

Equilibrium Problem

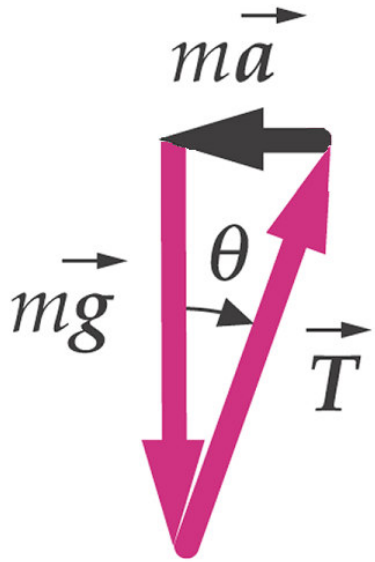
Hanging Mass in an Accelerating Plane



Equilibrium Problem



Equilibrium Problem

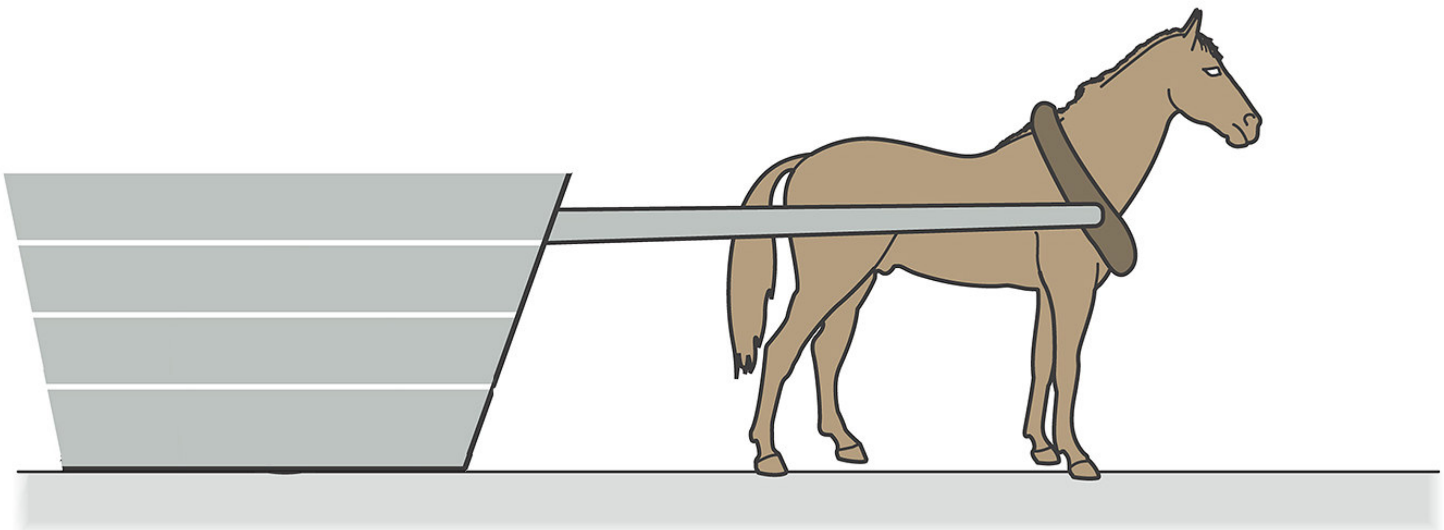


This is an example of three-vector equilibrium problem. It lends itself to a simple solution because the vector sum of the three vectors closes on itself (equilibrium) and forms a triangle

Net Force Example

Eliminated the wheel - we haven't dealt with rotation yet.

The text had the friction force in the wrong place.



(a)

Net Force Example

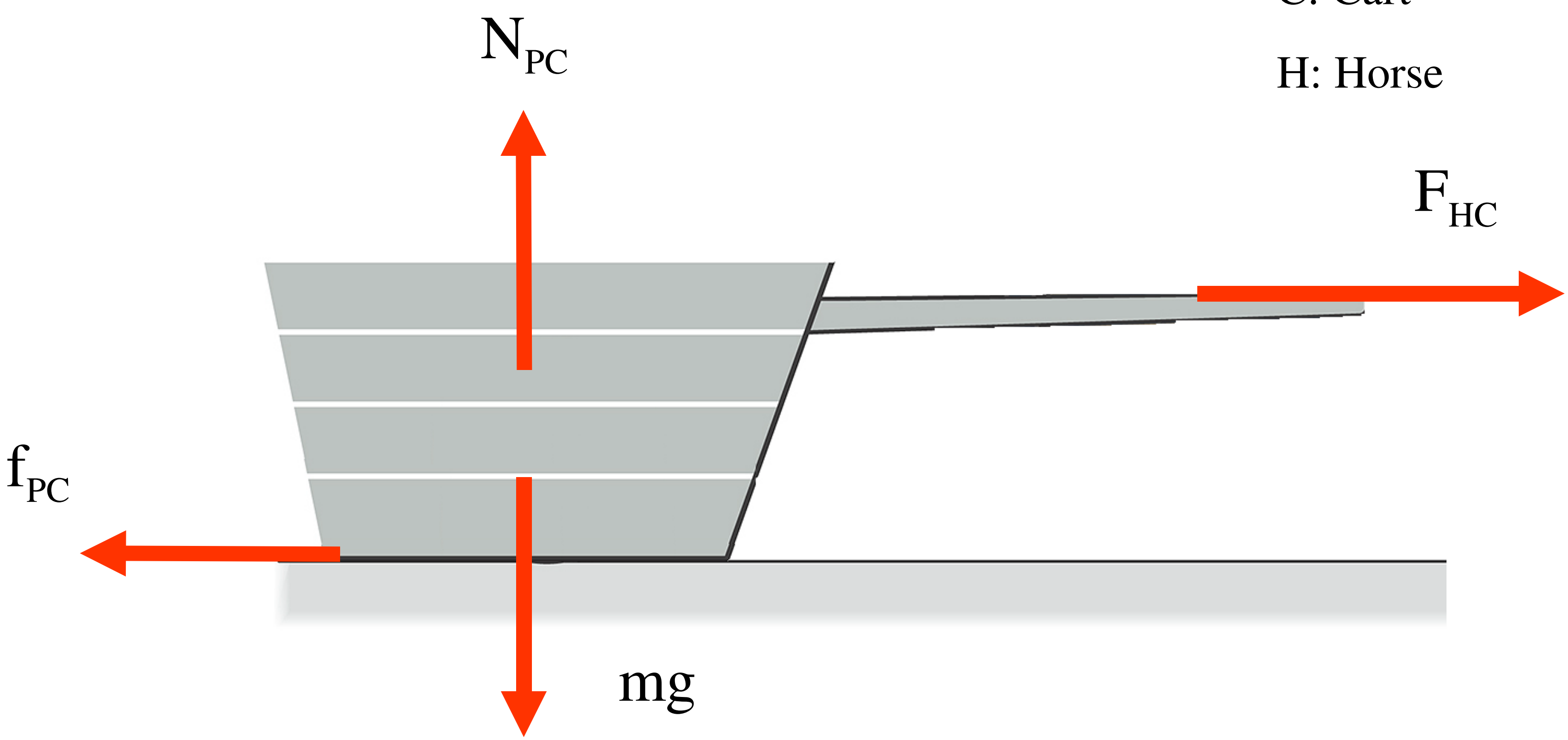
Force subscript interpretation

Force $_{\text{source, receiver}}$ -- N_{PC} is the normal force due to the Pavement acting on the Cart

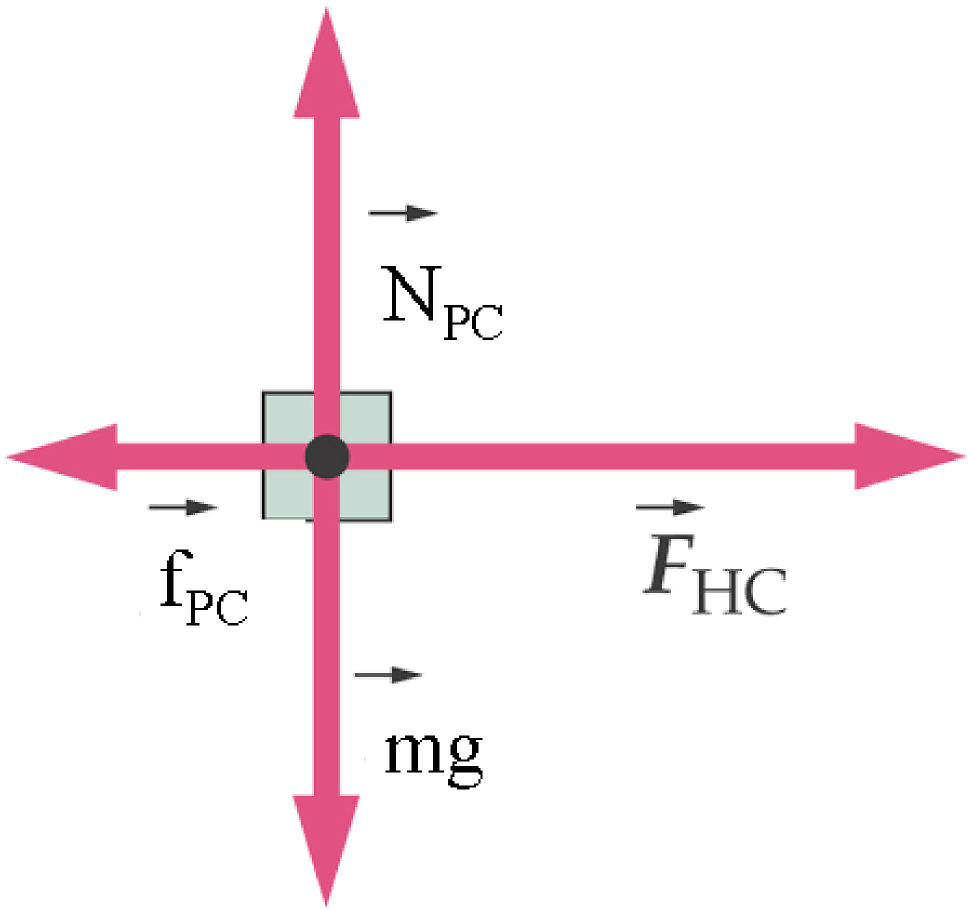
P: Pavement

C: Cart

H: Horse



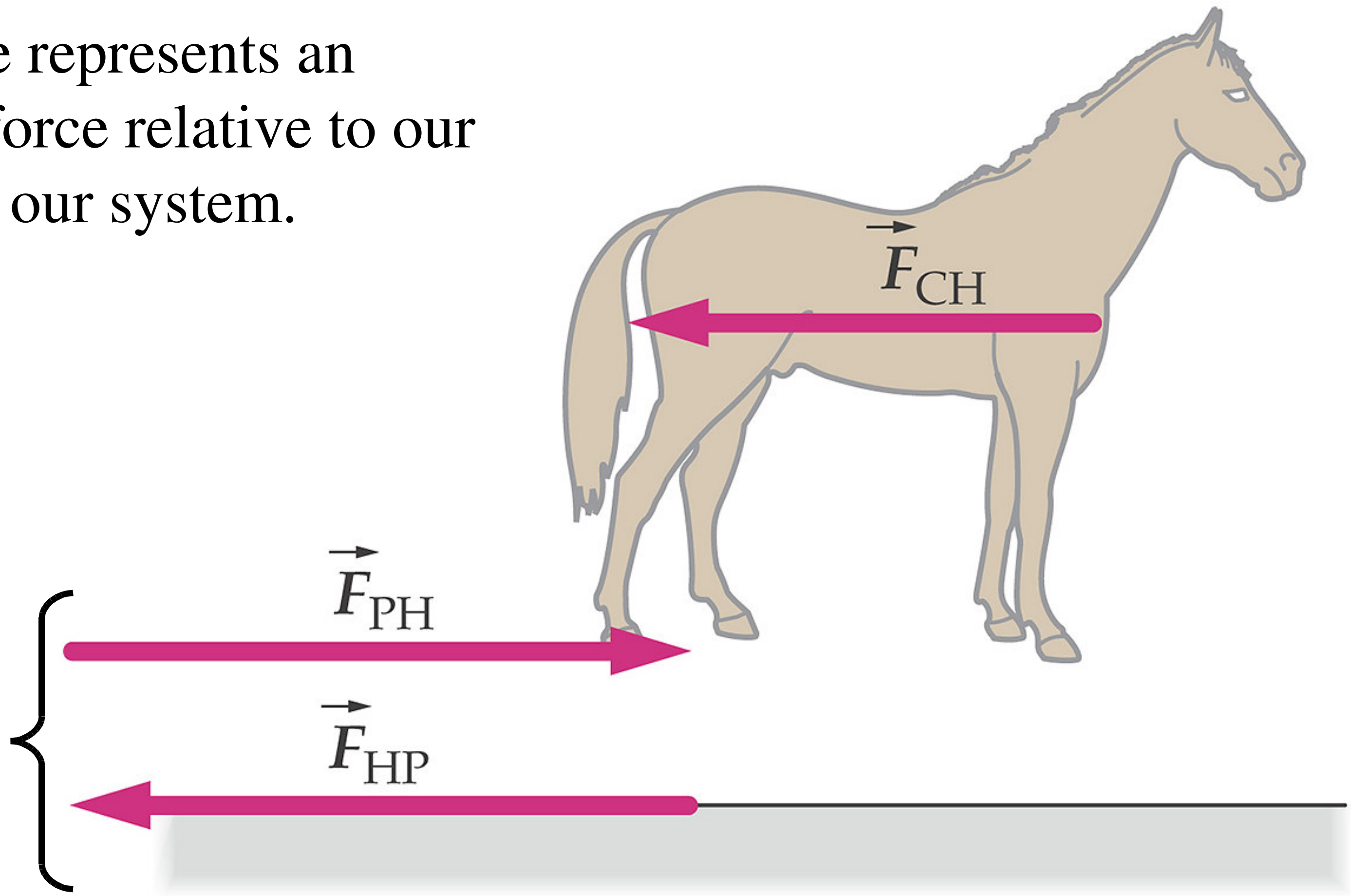
Net Force Example



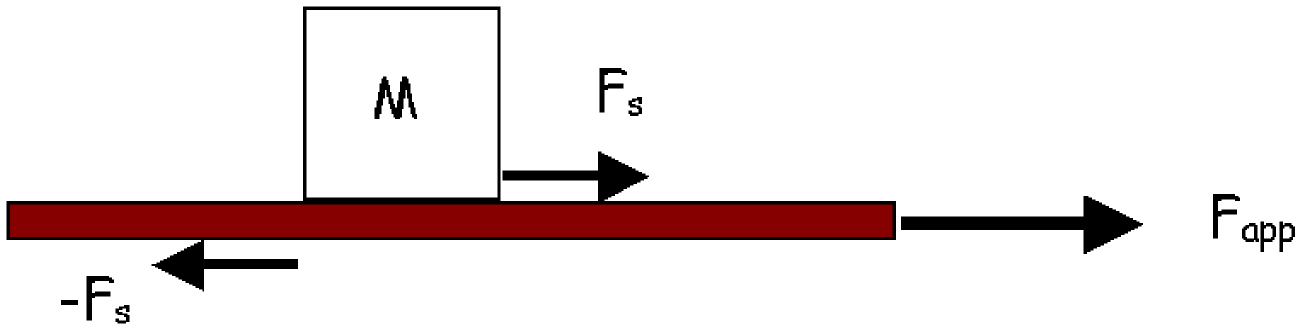
Net Force Example

The horse represents an external force relative to our choice of our system.

Action-Reaction



Milk Carton Problem



$F_s^{(max)} = \mu_s*N = \mu_s*M*g$ Max static friction force

$F_c = M*a \le \mu_s*M*g,$ Non-slip limit on applied force

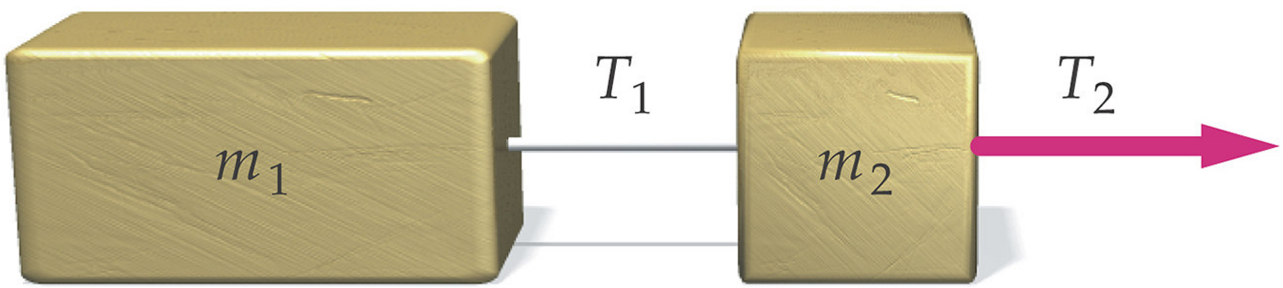
which, for $\mu_s = 0.7$, implies $a \le \mu_s*g = 0.7g$.

Assumptions to Simplify Problems

These are our special ***massless*** and ***unstretchable*** ropes.

Tension T_1 and T_2 are ***not equal*** because they are not part of the same rope.

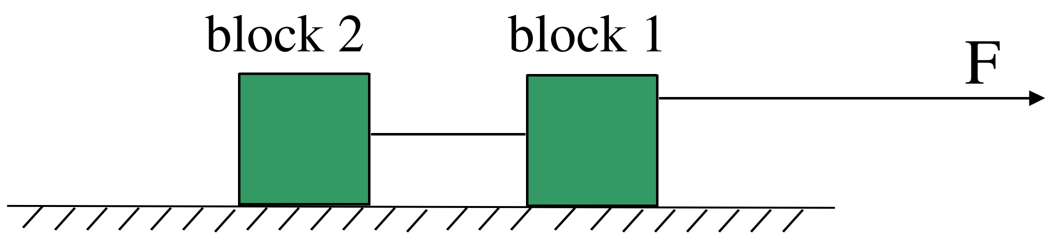
When m_1 moves, m_2 moves in the exact same manner: same ***velocity***, same ***acceleration***.



Applying Newton's Second Law

Example: A force of 10.0 N is applied to the right on block 1. Assume a frictionless surface. The masses are $m_1 = 3.00$ kg and $m_2 = 1.00$ kg.

Find the tension in the cord connecting the two blocks as shown.



Assume that the rope stays taut so that both blocks have the same acceleration.

Free Body Diagram

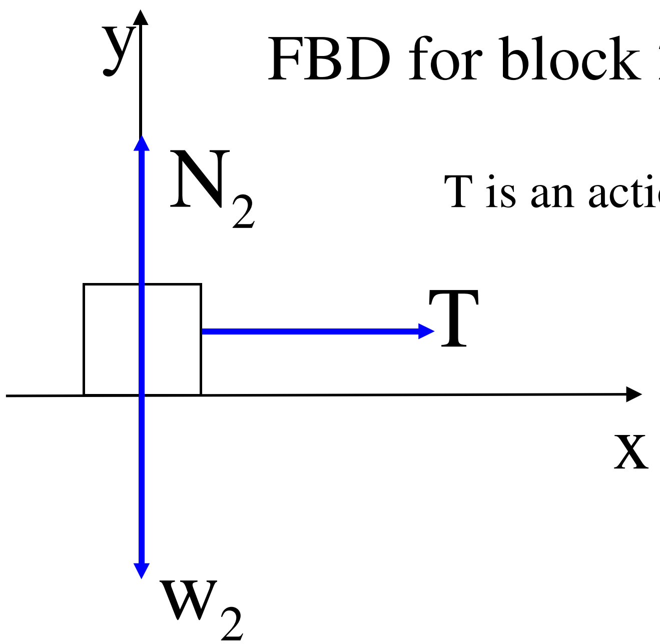
A free body diagram is a method of isolating a body and examining only the forces that are acting on it.

However, it can isolate you from the overall problem.

We will label all forces on all the bodies in the problem and then select the portion we want to deal with at that time and this will be our free body diagram.

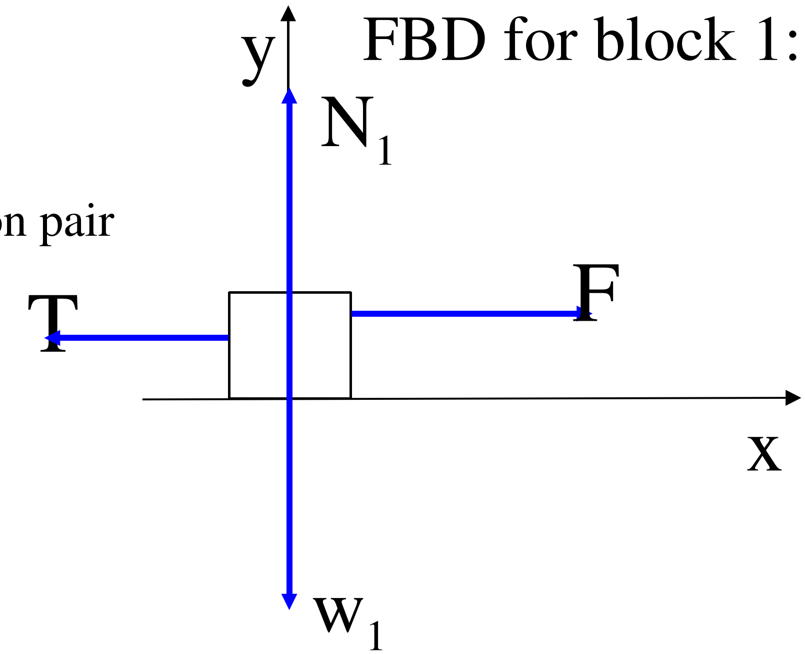
Free Body Diagrams

FBD for block 2:



T is an action-reaction pair

FBD for block 1:



Apply Newton's 2nd Law to each block:

$$\sum F_x = T = m_2 a$$

$$\sum F_y = N_2 - w_2 = 0$$

$$\sum F_x = F - T = m_1 a$$

$$\sum F_y = N_1 - w_1 = 0$$

Example continued:

$$F - T = m_1 a \quad (1)$$
$$T = m_2 a \quad (2)$$

These two equations contain the unknowns: *a* and *T*.

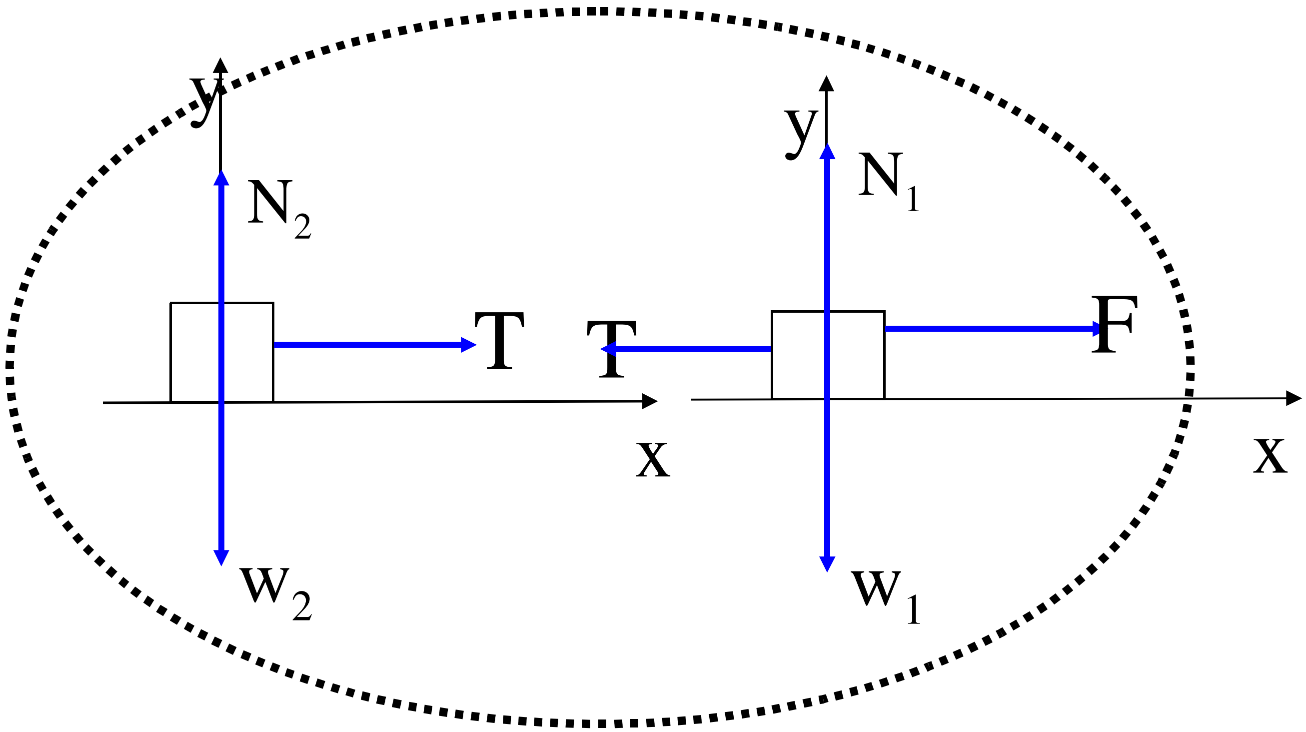
To solve for T, a must be eliminated. Solve for a in (2) and substitute in (1).

$$F - T = m_1 a = m_1 \left(\frac{T}{m_2} \right)$$

$$F = m_1 \left(\frac{T}{m_2} \right) + T = \left(1 + \frac{m_1}{m_2} \right) T$$

$$T = \frac{F}{\left(1 + \frac{m_1}{m_2} \right)} = \frac{10\ \text{N}}{\left(1 + \frac{3\ \text{kg}}{1\ \text{kg}} \right)} = 2.5\ \text{N}$$

Pick Your System Carefully

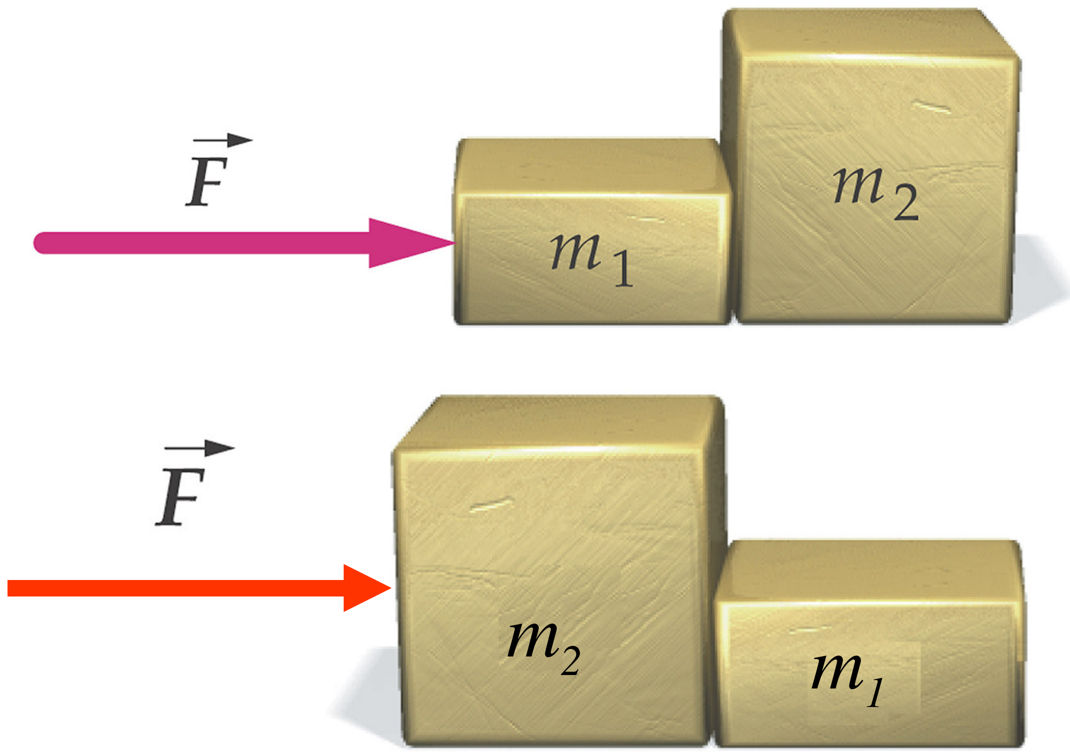


Include **both** objects in the system. Now when you sum the x-components of the forces the tensions cancel. In addition, since there is no friction, y-components do not contribute to the motion.

A Simple Thought Problem

$F = 30N$; $m_1 = 5$ kg; $m_2 = 10$ kg

Which case has the larger action-reaction forces?

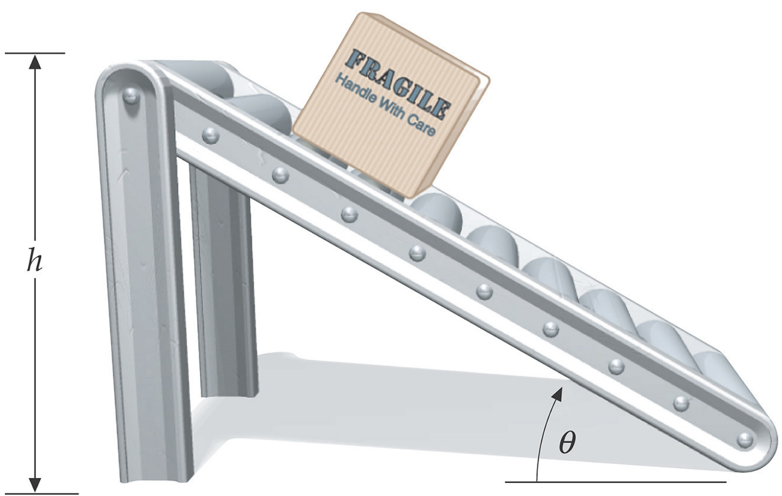


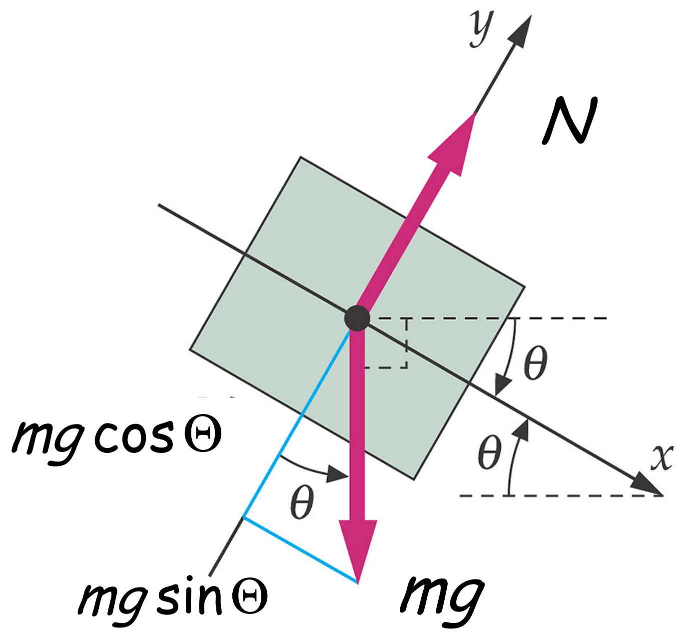
Case 1

Case 2

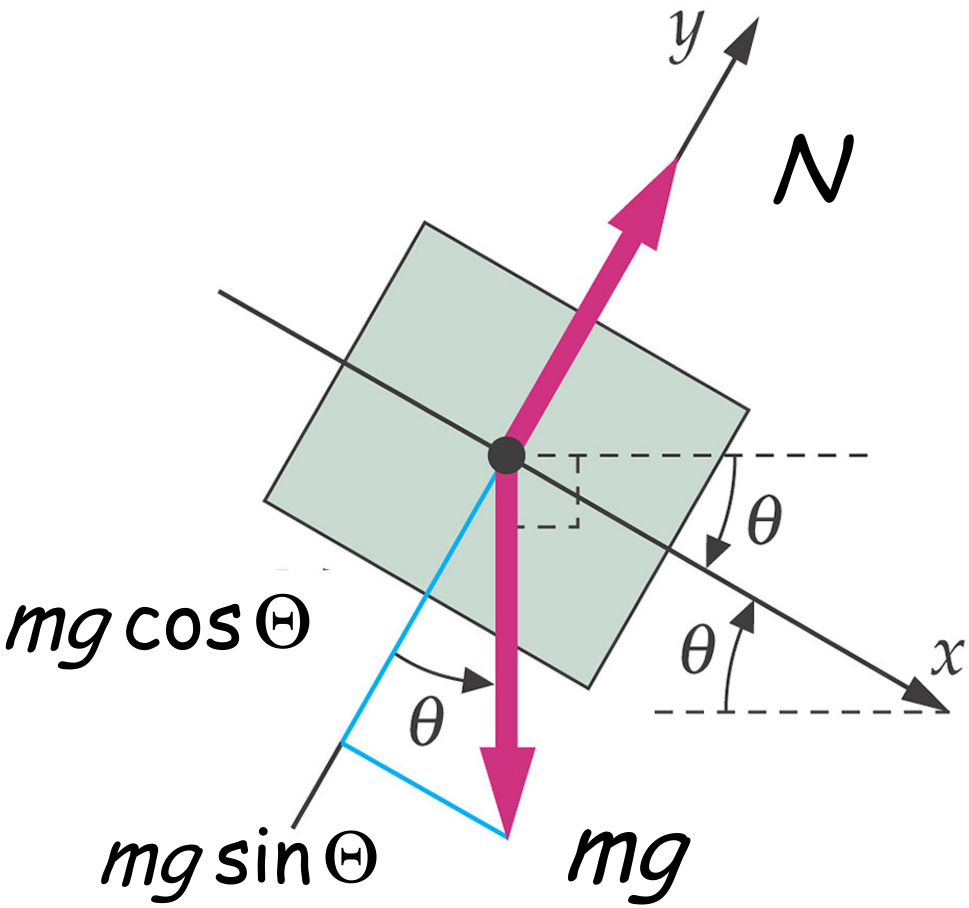
Example

The box can survive a drop from a height of 1 foot. Its velocity just before hitting the floor, after a drop from that height would be 2.50 m/s. The angle of the ramp is to be selected so that the vertical velocity is < 2.50 m/s.

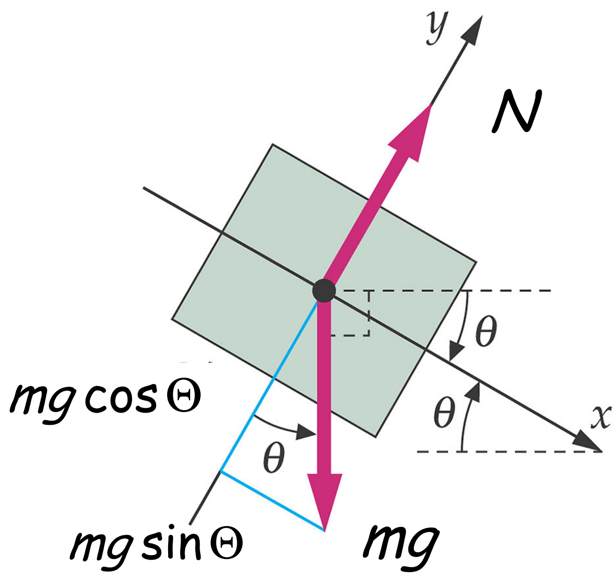




Example - continued



Example - continued



$$\sum F_x = mg\sin\Theta = ma$$

$$\sum F_y = N - mg\cos\Theta = 0$$

$$a = g\sin\Theta$$

$$N = mg\cos\Theta$$

All motion is along the x-axis.

$$v_f^2 = v_0^2 + 2a\Delta x$$

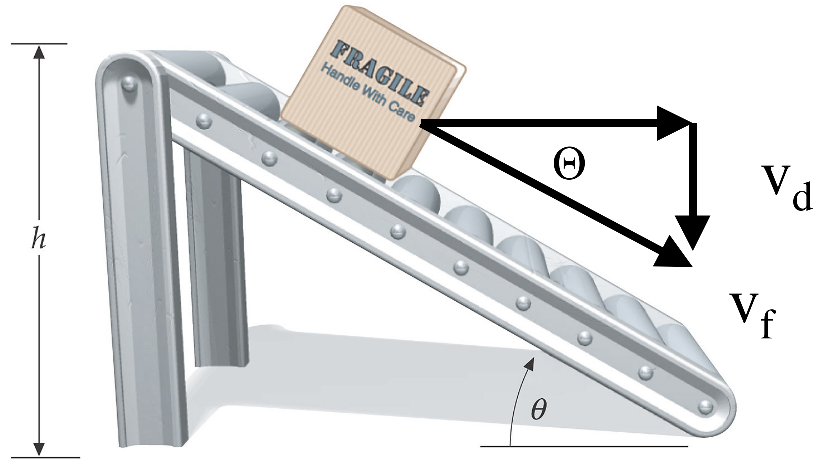
$$v_f^2 = 0 + 2(g\sin\Theta)\Delta x$$

$$v_f^2 = 0 + 2g\Delta x\sin\Theta$$

$$v_f^2 = 0 + 2gh$$

$$v_f = \sqrt{2gh}$$

Example - continued

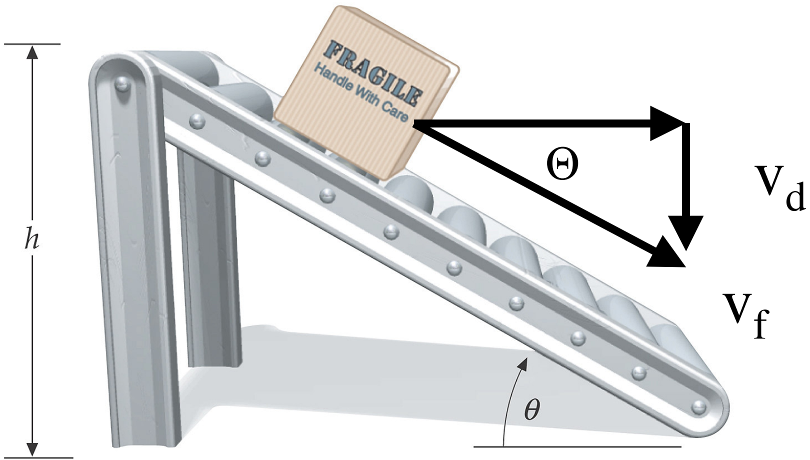


$$v_f^2 = v_0^2 + 2a\Delta x$$
$$v_f^2 = 0 + 2(g\sin\Theta)\Delta x$$
$$v_f^2 = 0 + 2g\Delta x\sin\Theta$$
$$v_f^2 = 0 + 2gh$$
$$v_f = \sqrt{2gh}$$

$$v_d = v_f \sin\Theta \leq 2.50\, m/s$$
$$\sin\Theta \leq \frac{2.50}{v_f} = \frac{2.50}{\sqrt{2gh}}$$

The solution provides a condition on the angle since the height of the ramp is dictated by the truck height.

Example - continued



The problem would seem to have a fatal flaw in the logic of the constraint. As in the famous sky diving joke it's not the fall that will kill you it's the sudden stop at the end.

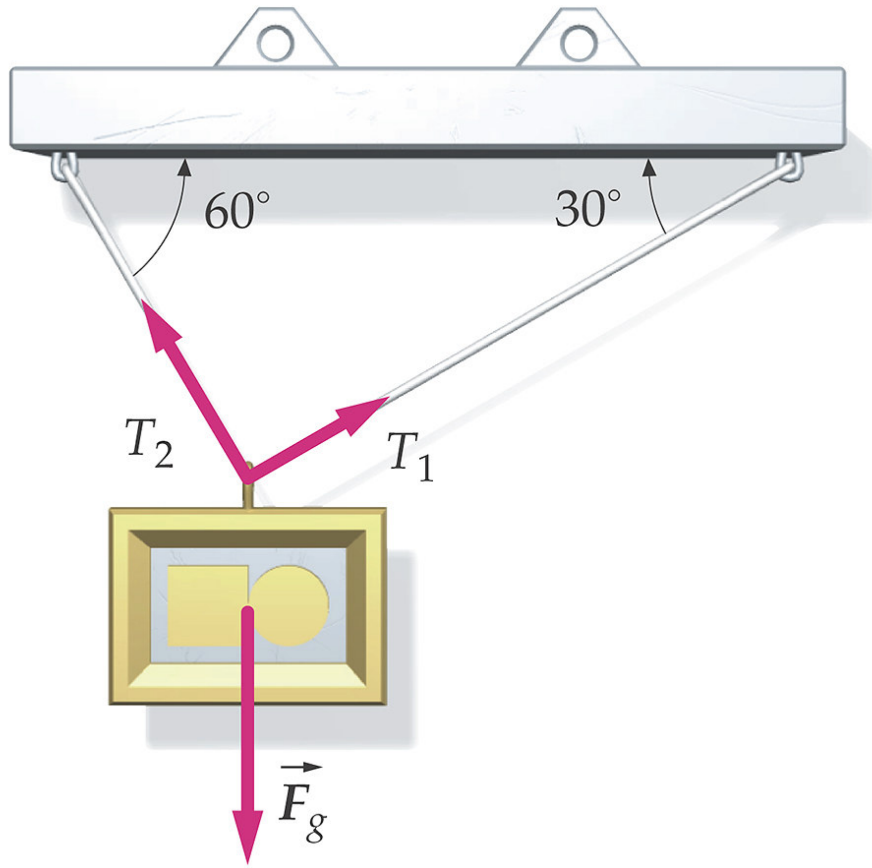
As long as the package doesn't stop suddenly then there shouldn't be any problem. In many cases a horizontal section at the bottom of this angled ramp would allow the package to decelerate gradually.

Hanging Problems

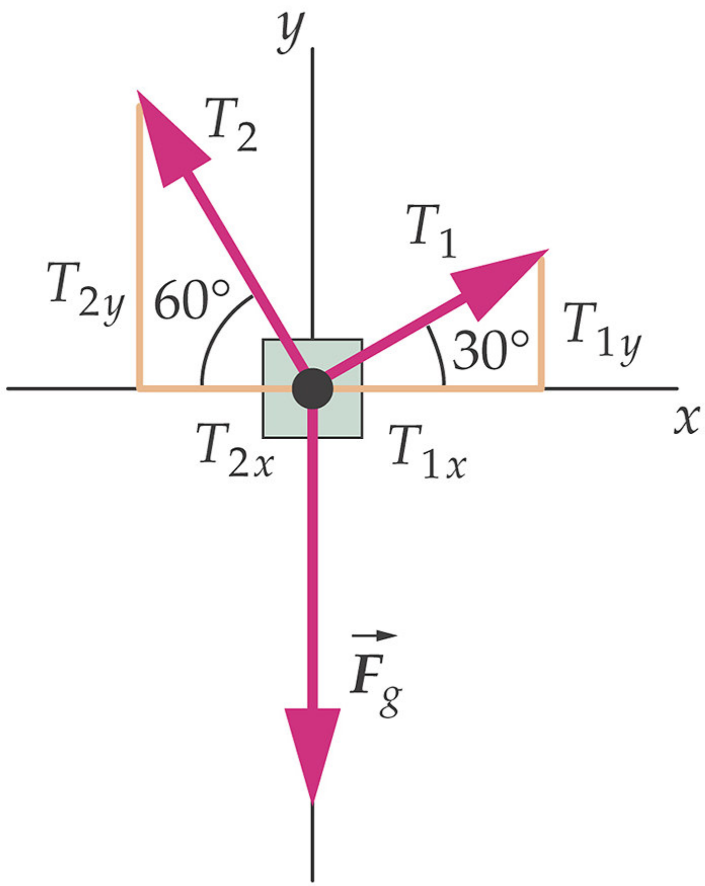
Mass over pulley on edge of table

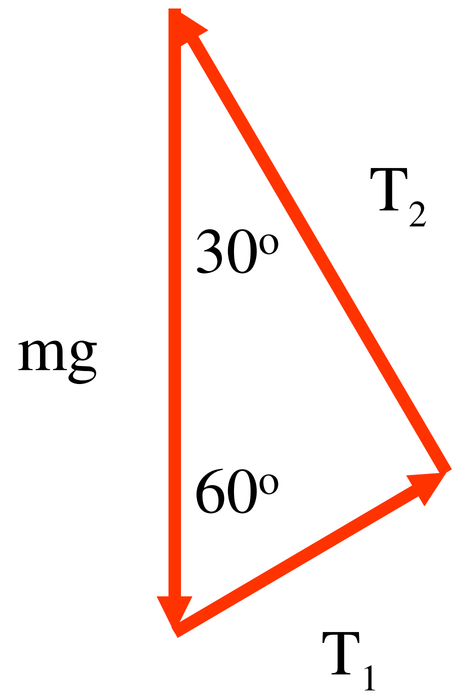
Hanging traffic light

Hanging Picture

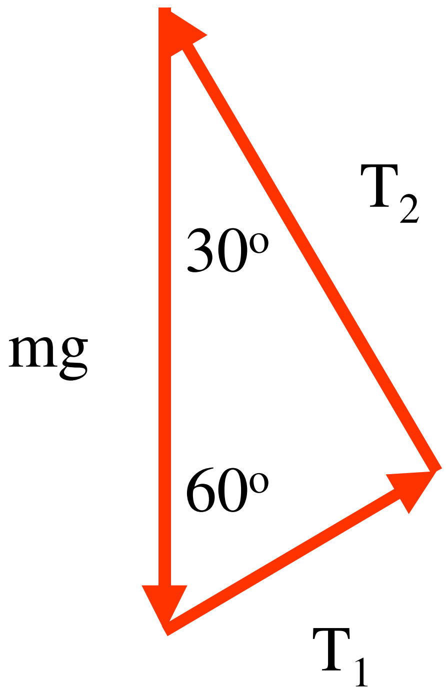


Hanging Picture - Free Body Diagram





Hanging Picture



$$T_1 = mg\cos(60^0)$$

$$T_2 = mg\sin(60^0)$$

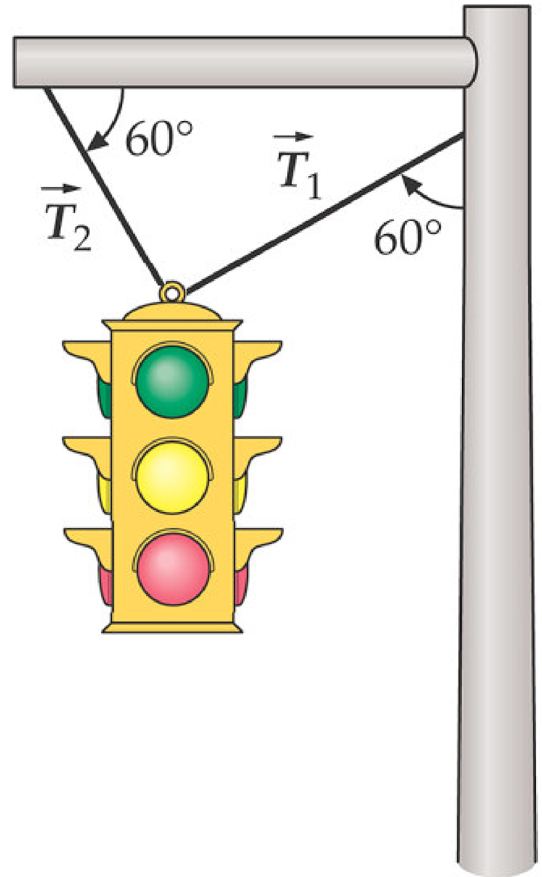
- Since this turned out to be a right triangle the simple trig functions are that is needed to find a solution.
- If the triangle was not a right triangle then the Law of Sines would have been needed.

Hanging Traffic Light

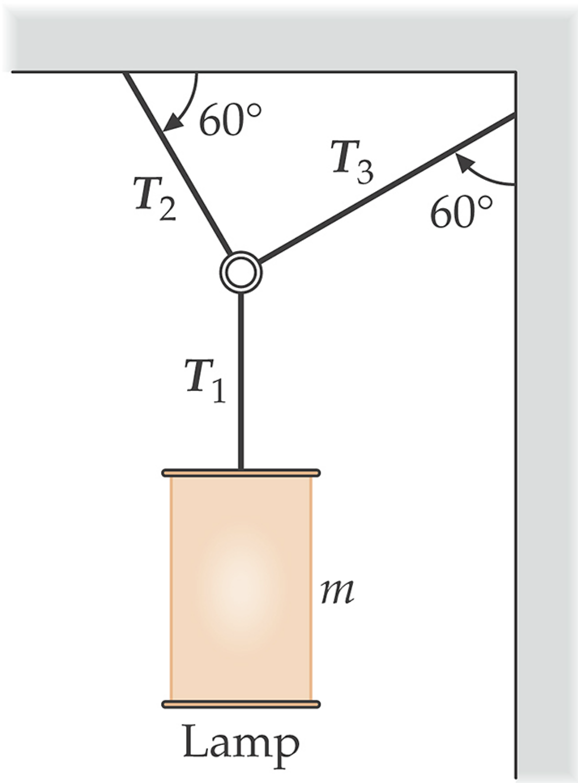
Tension T_1 and T_2 are in general different.

It doesn't matter if the rope is one continuous piece or two separate pieces of rope.

The difference with the pulley situation is that the light can only pull down while the pulley can push back in almost any direction.



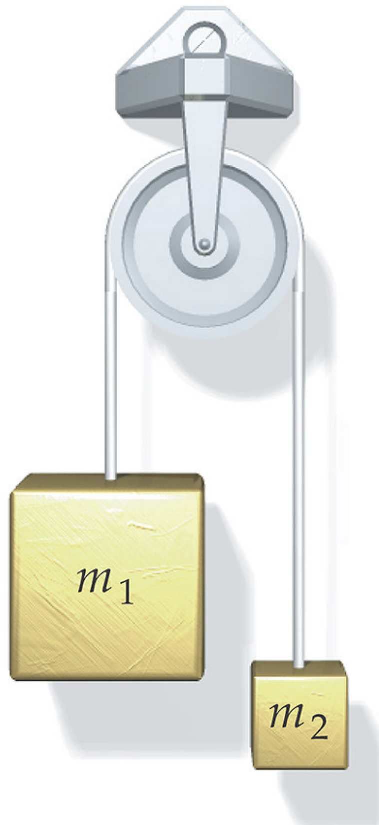
Hanging Traffic Light



Tension vectors are not proportional to the string lengths.

Atwood Machine and Variations

Basic Atwood's Machine



A 2-Pulley Atwood Machine

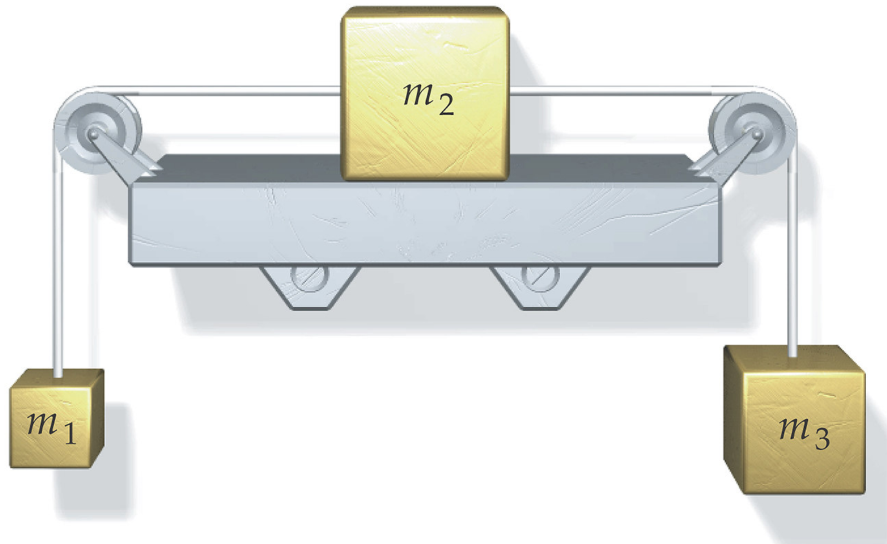
As long as we have massless rope and pulleys these two Atwood machines will yield the same results.

1-Dimensional Equivalent Problem

Atwood machines are equivalent to this one-dimensional problem

Thanks to massless ropes and pulleys

Compare to the Atwood Machine

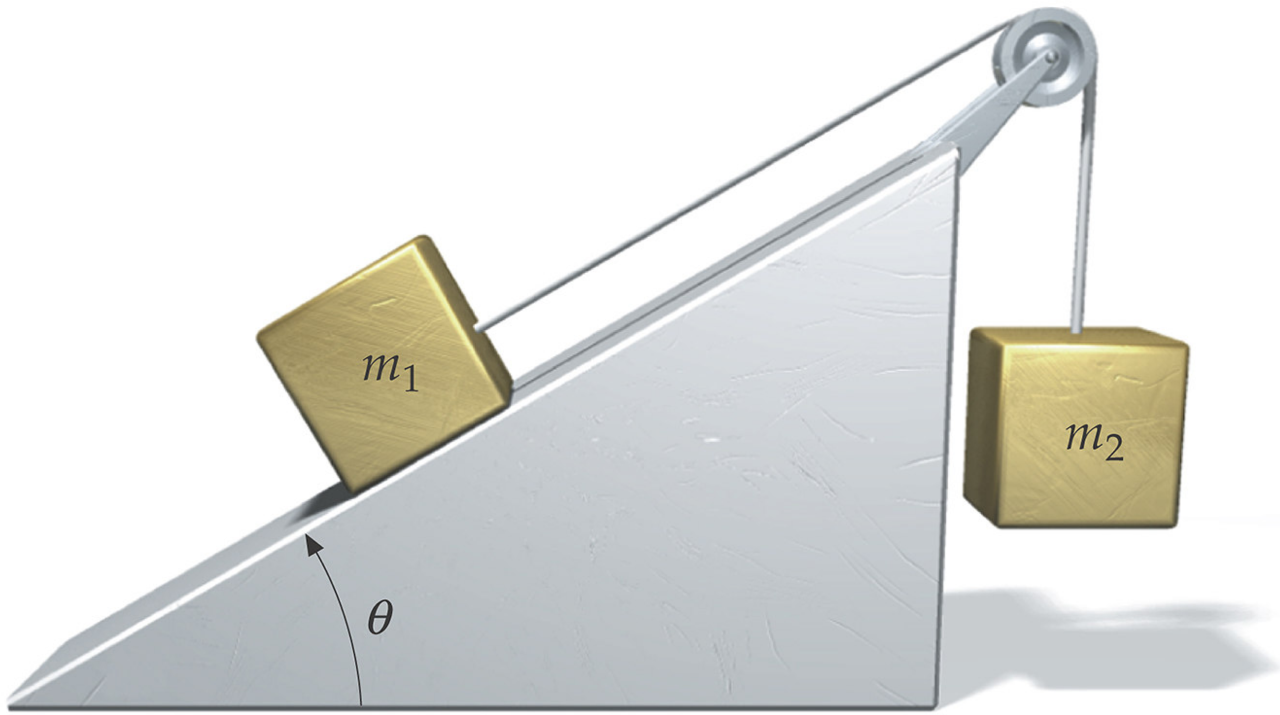


Gravity still affects only masses 1 and 3. With no friction mass 2 doesn't add any forces, only inertial mass.

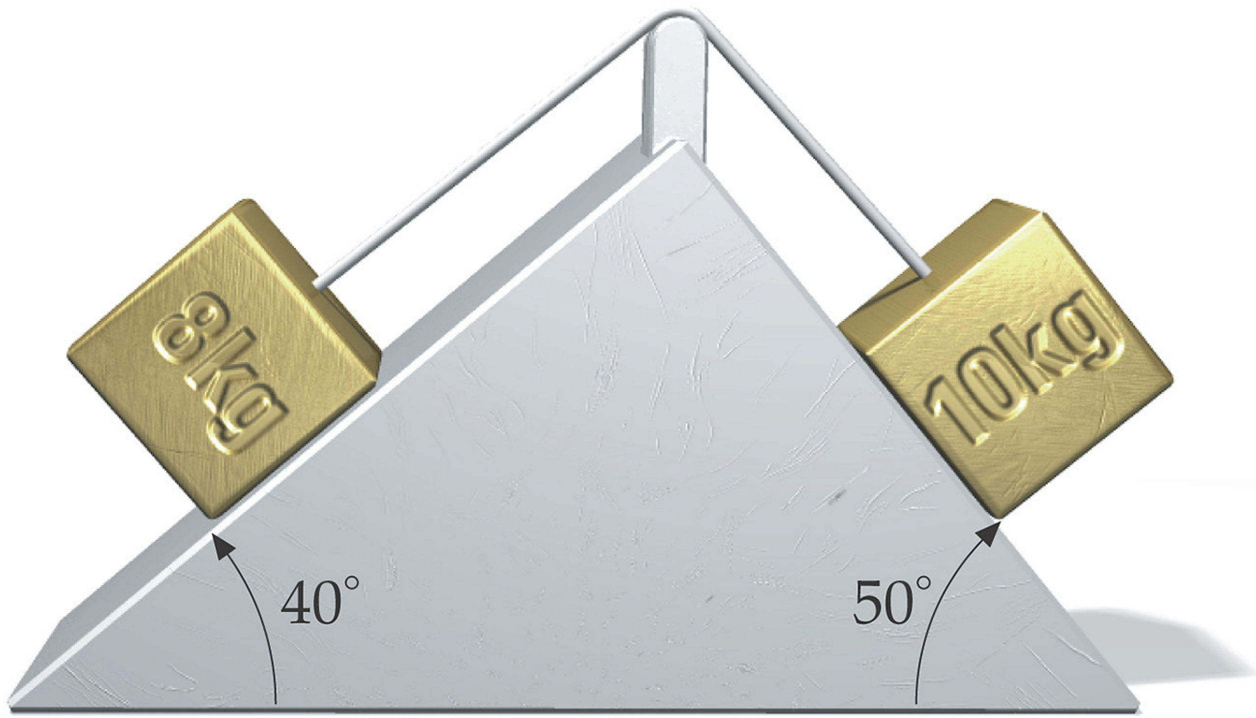
The net force can be the same as a 2-body Atwood Machine but the total mass is higher.

Incline Plane Problems

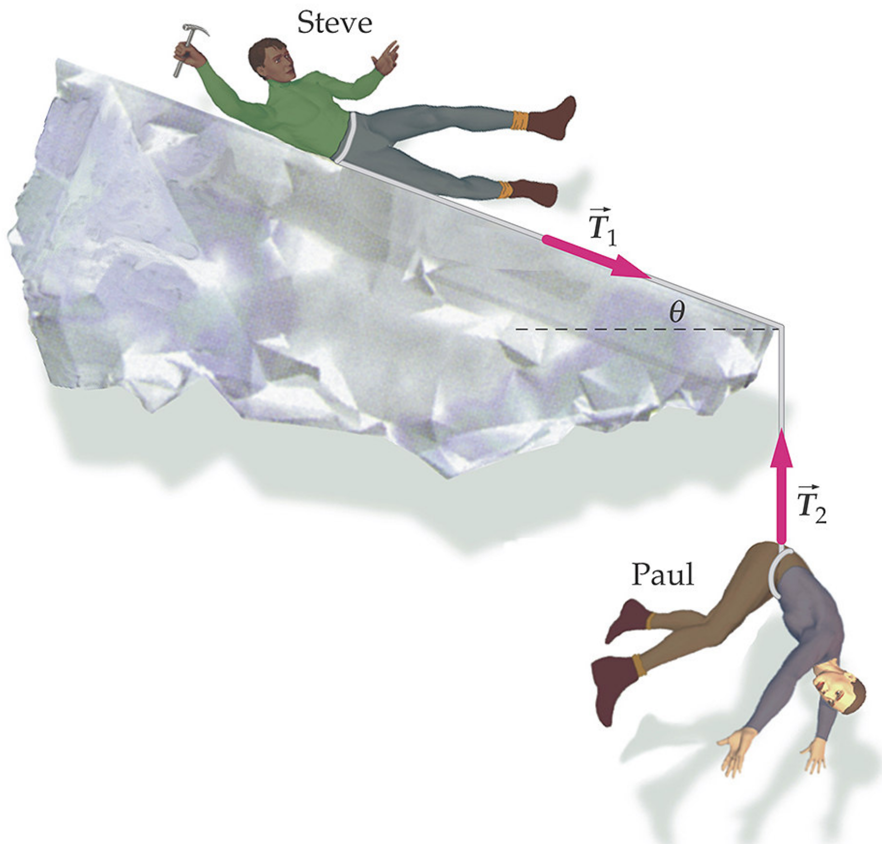
Single Incline Plane Problem



Double Incline Plane Problem



Steve and Paul's Excellent Adventure



Steve and Paul's Excellent Adventure

Apply $\Sigma F_x = ma_x$ in the x direction to Steve:

Apply $\Sigma F_{x'} = ma_{x'}$ in the x' direction to Paul:

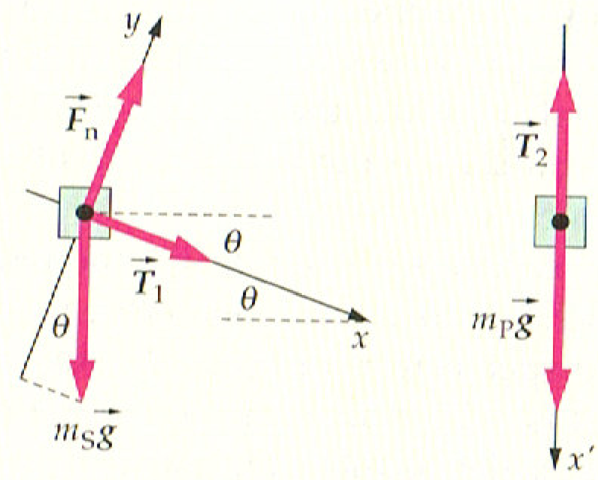


FIGURE 4-30

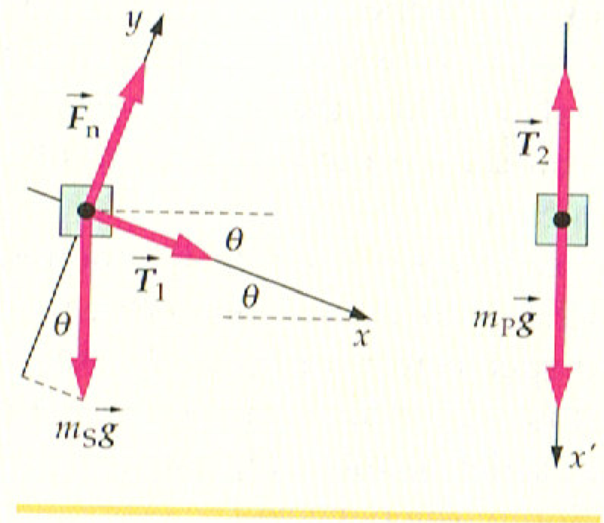
$$F_{nx} + T_{1x} + m_S g_x = m_S a_{Sx}$$

$$T_{2x'} + m_P g_{x'} = m_P a_{Px'}$$

$$a_{Px'} = a_{Sx} = a_t$$

a_t stands for the acceleration component in the *tangential* direction. (The direction of the motion.)

This is my candidate for the physics problem with the most useless notation and variable definitions



5. Because the rope is of negligible mass and slides over the ice with negligible friction, the forces $\vec{T}_1$ and $\vec{T}_2$ are simply related. Express this relation:

$$T_2 = T_1 = T$$

6. Substitute the steps-4 and -5 results into the step-2 and step-3 equations:

$$T + m_S g \sin\theta = m_S a_t$$
$$-T + m_P g = m_P a_t$$

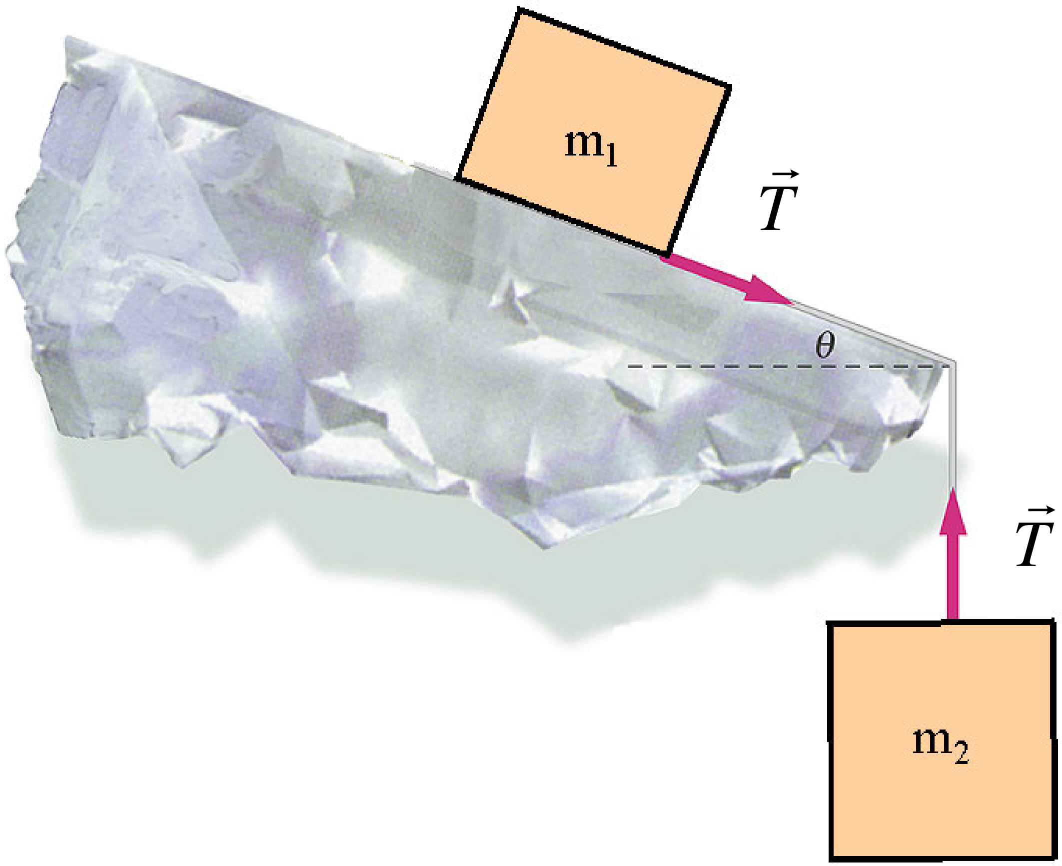
7. Solve the step-6 equations for the acceleration by eliminating T and solving for a_t:

$$a_t = \boxed{\frac{m_S \sin\theta + m_P}{m_S + m_P} g}$$

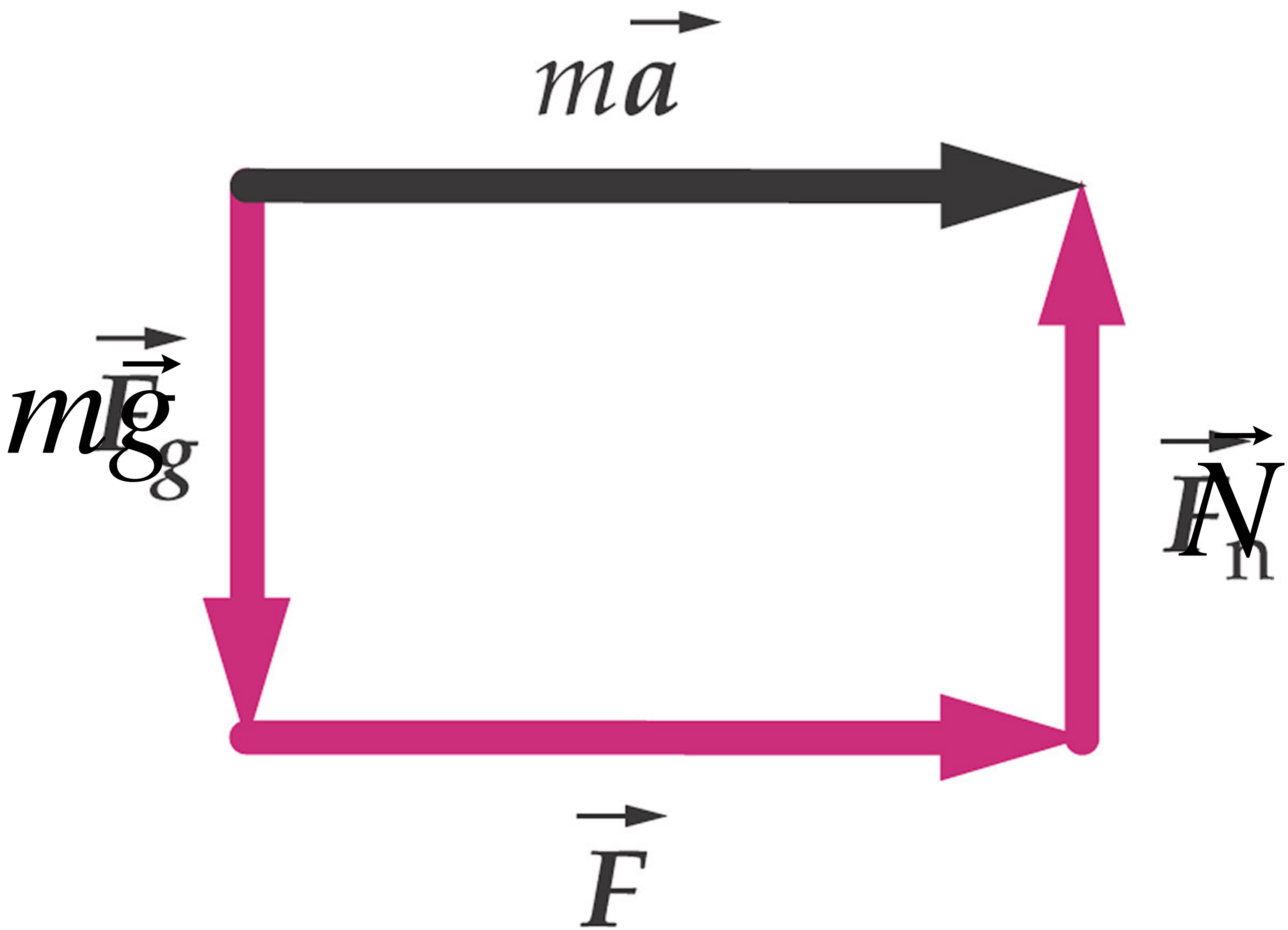
8. Substitute the step-7 result into either step-6 equation and solve for T:

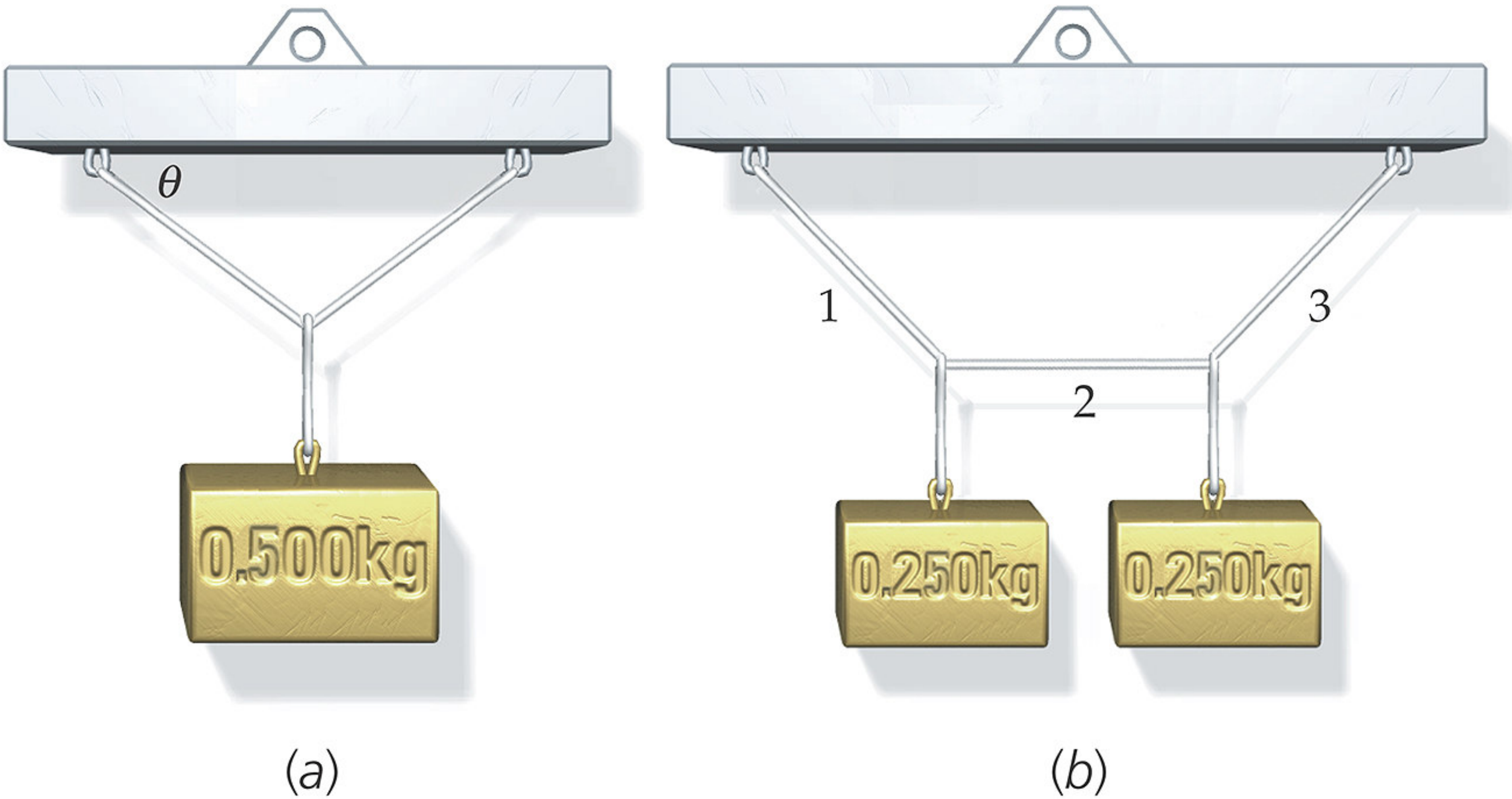
$$T = \boxed{\frac{m_S m_P}{m_S + m_P}(1 - \sin\theta) g}$$

Steve and Paul in Boxes for Easy Shipping



Extra Slides


$m\vec{a}$
$m\vec{g}$
$\vec{F}_g$
$\vec{N}$
$\vec{F}_n$
$\vec{F}$


θ
0.500kg
1
2
3
0.250kg
0.250kg
(a)
(b)