Chapter 6

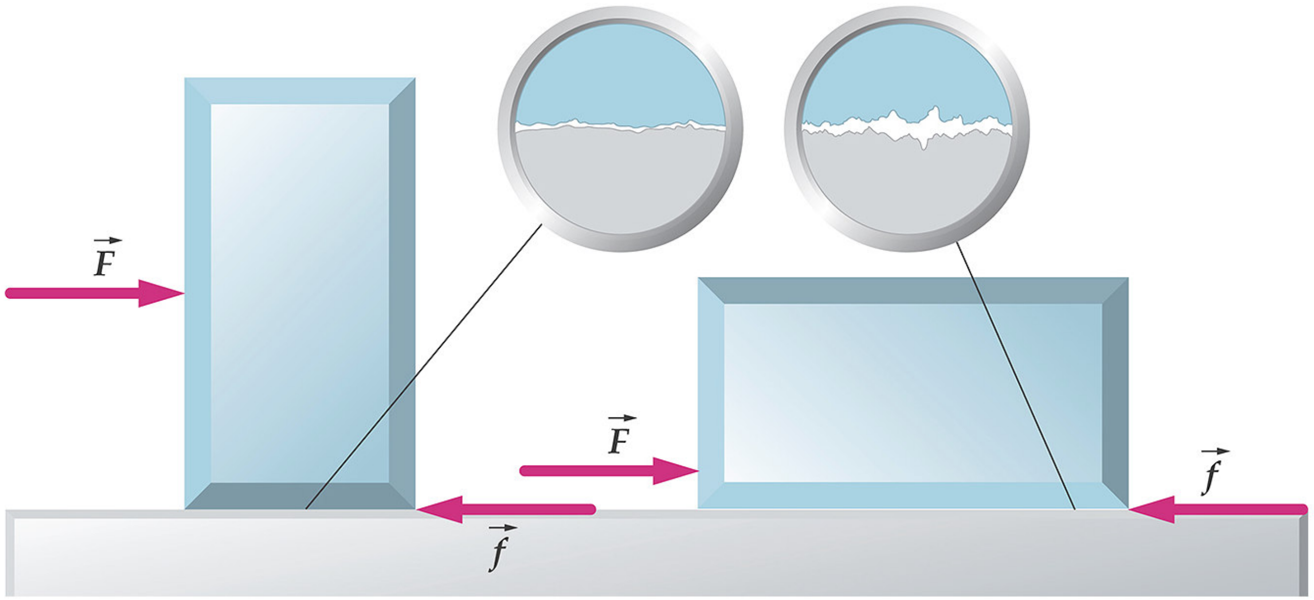
Applications of Newton’s Laws

Applications of Newton's Laws

- Friction
- Drag Forces
- Motion Along a Curved Path
- The Center of Mass

Microscopic Surface Area

The microscopic area of contact is proportional to the normal force.



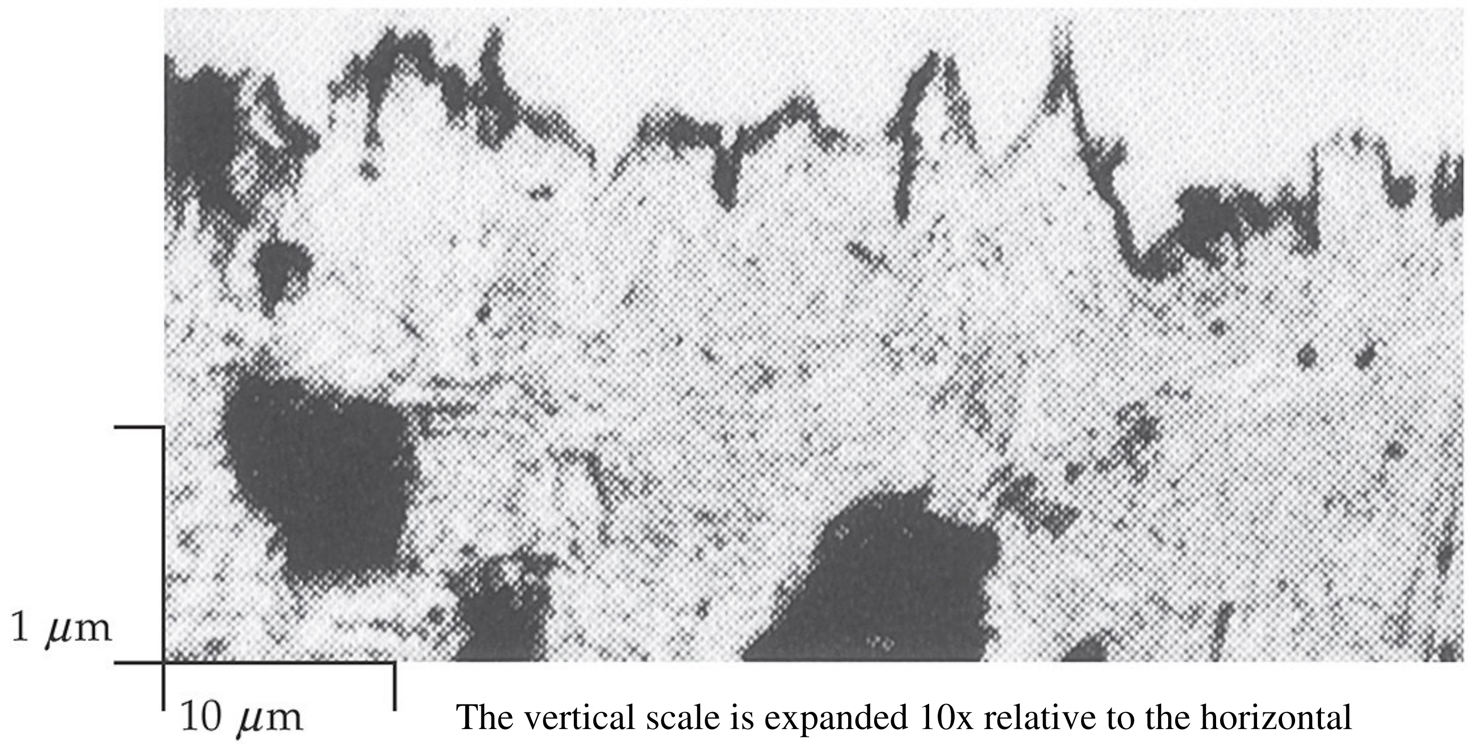
The normal force is the same in both of the above orientations.

Microscopic Surface Area

- When one flat surface rests on another it is only the high points of each surface that are actually in physical contact.
- The actual physical contact area can be less than 1%
- This has important consequences for heat ransfer in a vacuum.

Polished Steel Surface

83μm

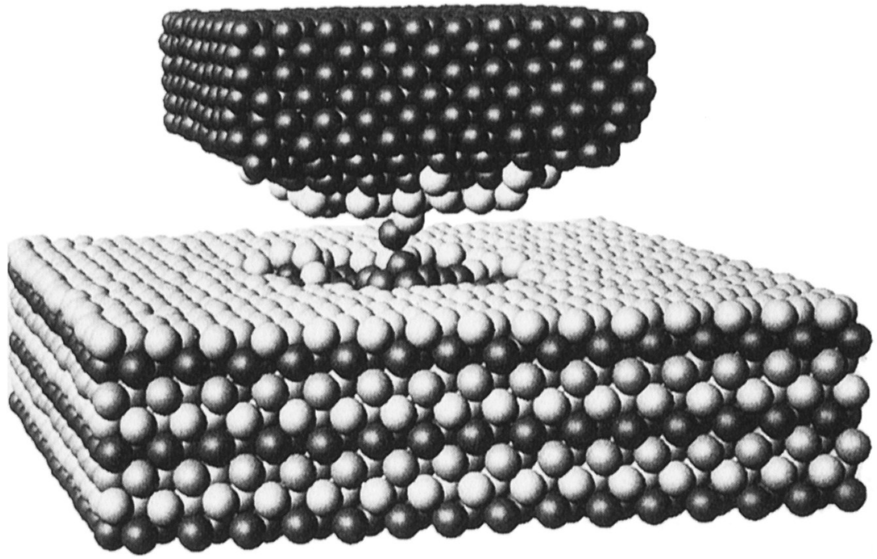


The vertical scale is expanded 10x relative to the horizontal scale.

The diameter of a human hair is on average 100μm.

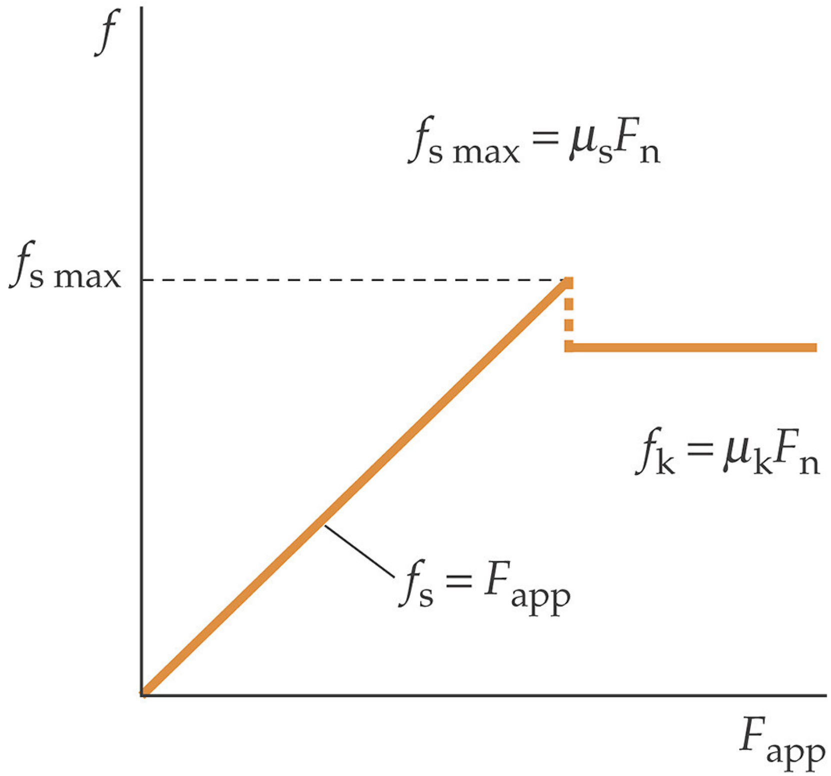
Computer Graphic of Nickel Probe on Gold Substrate

Gold atoms adhere to the nickel probe after contact.



This is a microscopic example of the adhesion that contributes to the force of frition

Static and Kinetic Friction



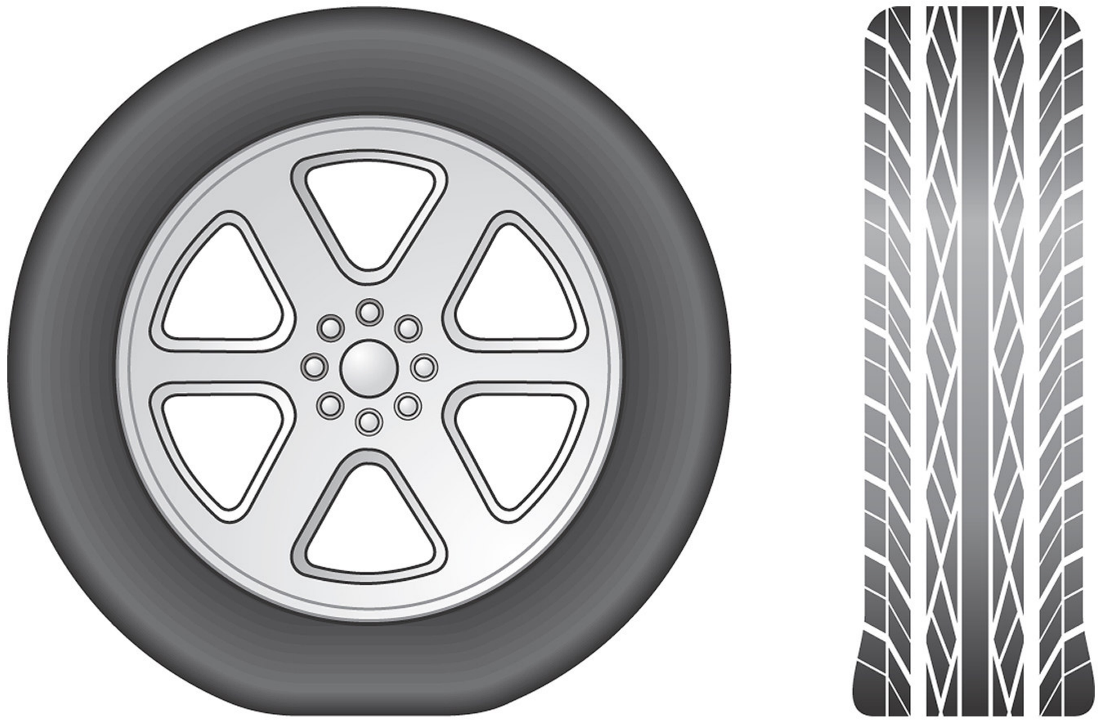
Static friction has a range of values up to a maximum

Frictional Coefficients

Table 5-1 Approximate Values of Frictional Coefficients

Materials	μ_s	μ_k
Steel on steel	0.7	0.6
Brass on steel	0.5	0.4
Copper on cast iron	1.1	0.3
Glass on glass	0.9	0.4
Teflon on Teflon	0.04	0.04
Teflon on steel	0.04	0.04
Rubber on concrete (dry)	1.0	0.80
Rubber on concrete (wet)	0.30	0.25
Waxed ski on snow (0°C)	0.10	0.05

Rolling Friction



$$f_r = \mu_r N$$

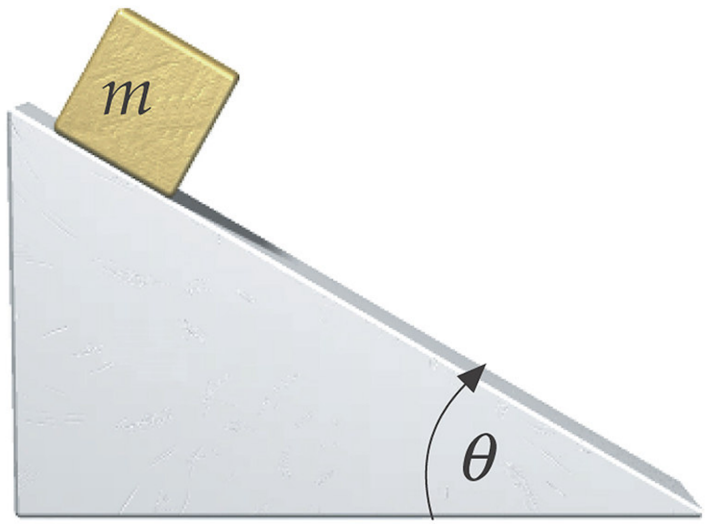
Rolling Friction

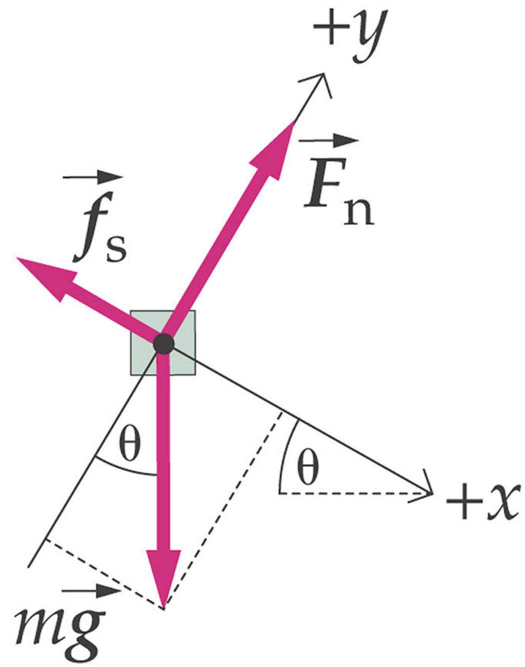
The tire will adhere to the road to some extent. The peeling away of the tire from the road is the source of rolling friction

$$0.01 \le \mu_r \le 0.02 \quad \textit{Tires on concrete}$$

$$0.001 \le \mu_r \le 0.002 \quad \textit{Steel wheels on a steel rail}$$

Finding μ_s with Tanθ

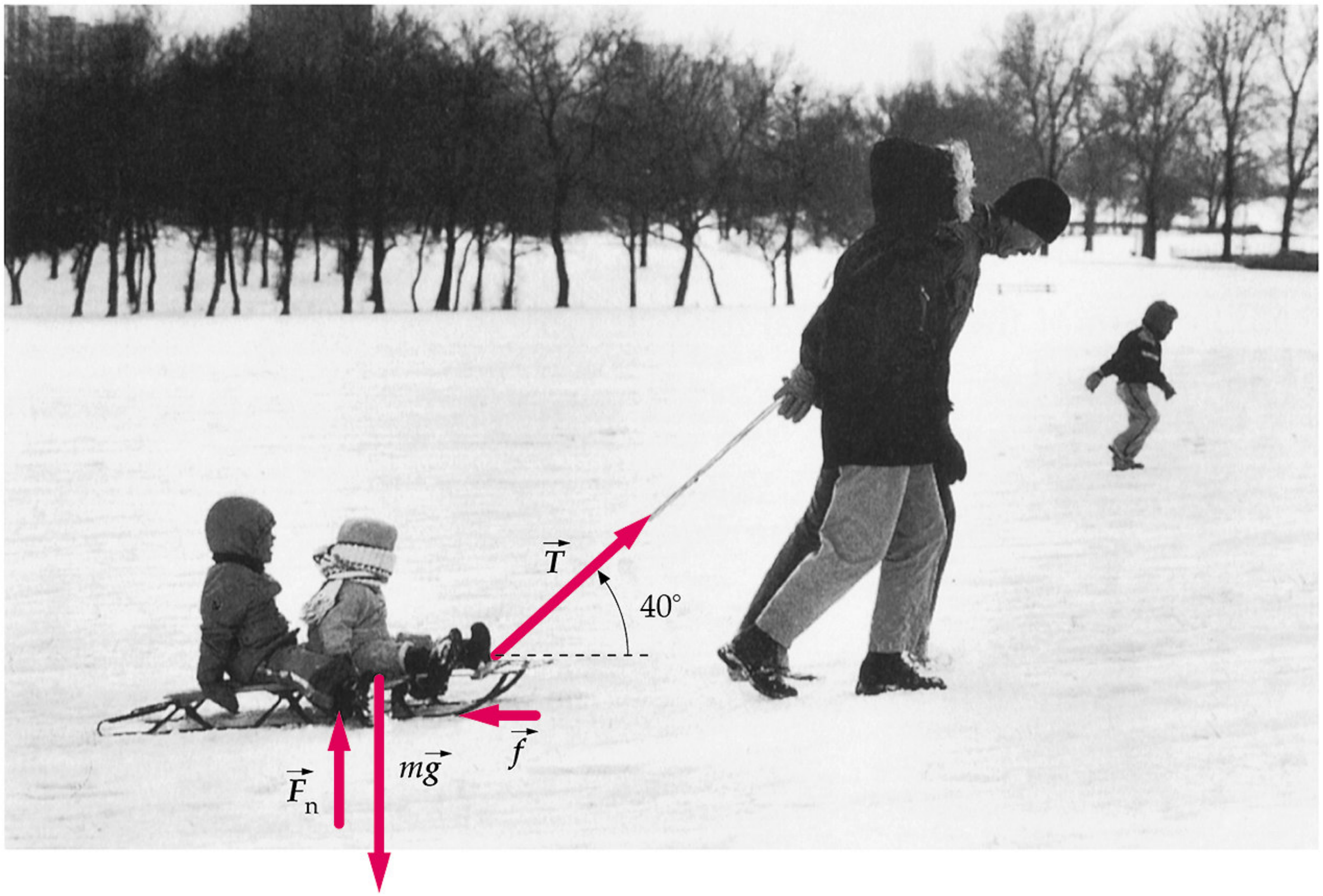




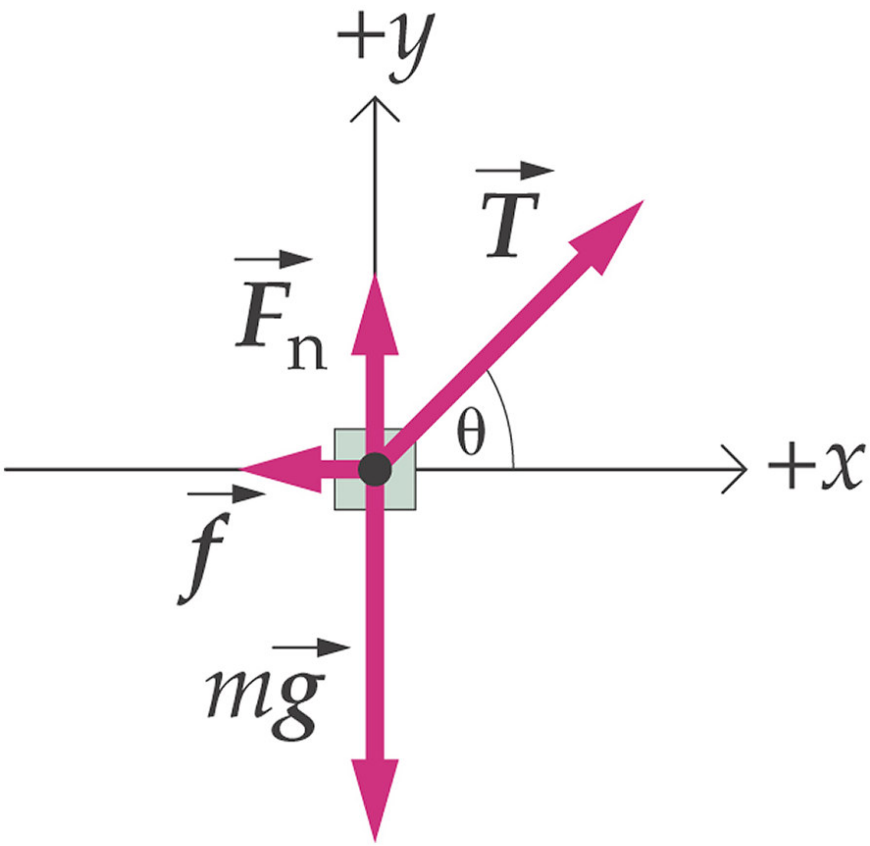
$$f_s = \mu_s mg\cos\theta = mg\sin\theta$$

$$\mu_s = \frac{mg\sin\theta}{mg\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Is This Analysis Realistic?



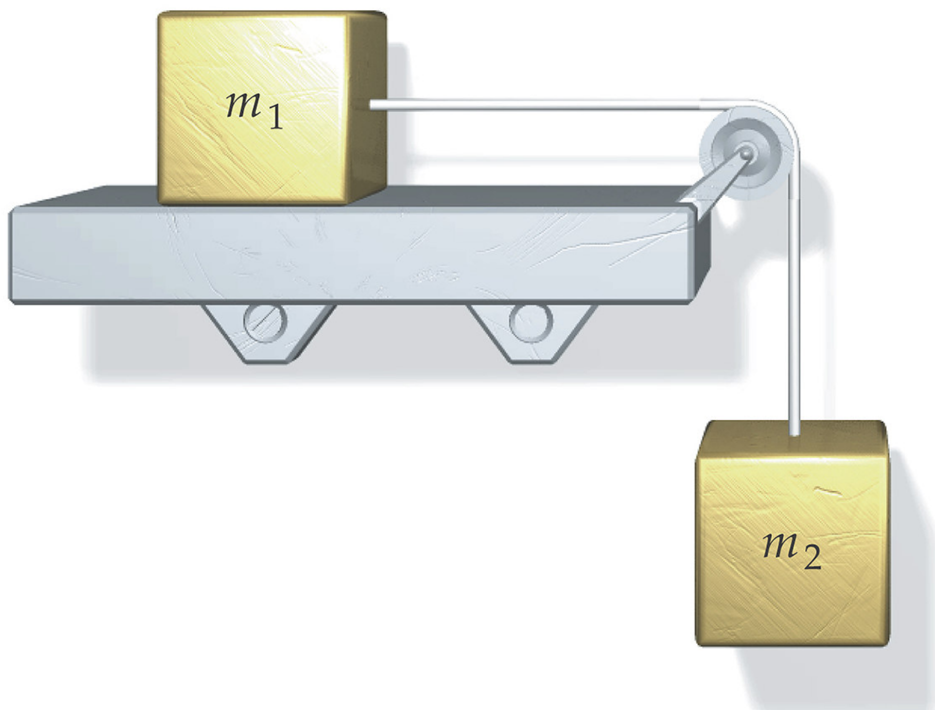
Force Diagram for - Is It Realistic?



- Part of T, the vertical component, is offsetting the weight of the sled and reducing the size of the normal force.
- The horizontal component of T appears larger than the frictional force f.
- The unbalanced force in the x-direction causes an acceleration of the sled. Can old Dad keep the tension constant?

With Friction - All Set to Slide

Find μ_s and the acceleration

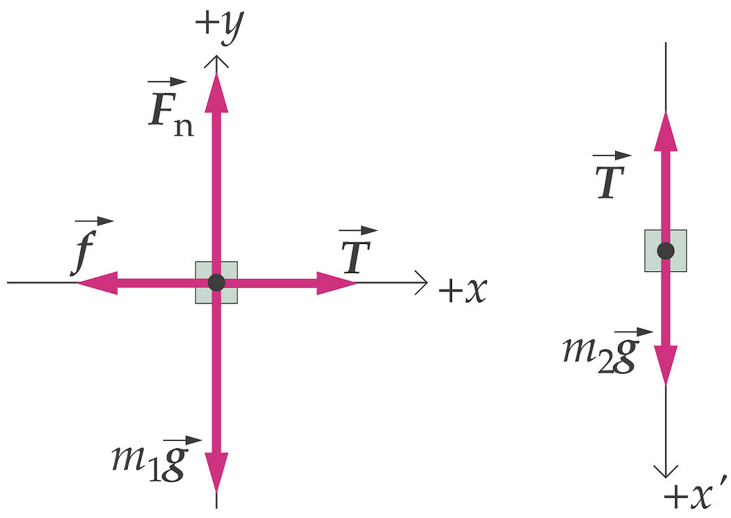


$\mu_k = 0.54$

$m_1 = 7$ kg

$m_2 = 5$ kg

All Set to Slide means $f_s = f_s^{max}$



Block 1 Block 2

Static friction

$$\sum F_x = -\mu_s m_1 g + m_2 g = (m_1 + m_2)a$$

$$-\mu_s m_1 g + m_2 g = 0$$

$$\mu_s = \frac{m_2}{m_1} = \frac{5}{7} = 0.71$$

Dynamic friction

$$-\mu_k m_1 g + m_2 g = (m_1 + m_2)a$$

$$a = \frac{m_2 - \mu_k m_1}{m_1 + m_2} g = 1.0 \tfrac{m}{s^2}$$

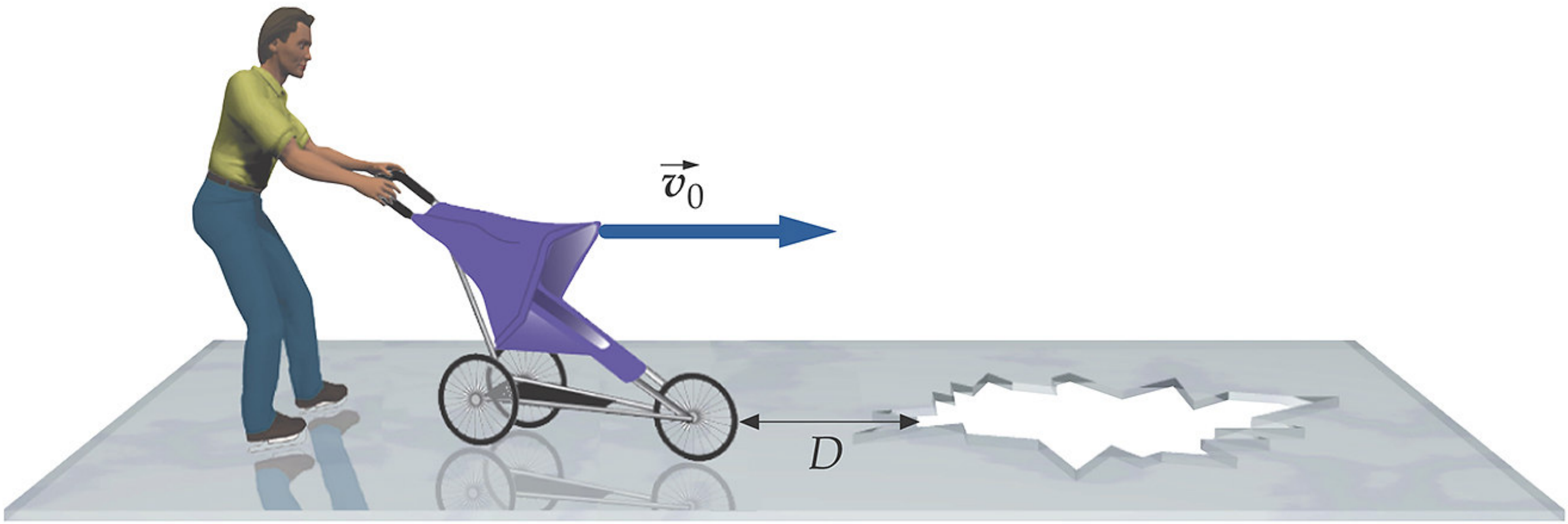
The Toboggan Problem is the Milk Carton Problem

Except we didn't want the milk carton to travel with the table cloth but we do want the children to travel with the toboggan.

The Runaway Buggy

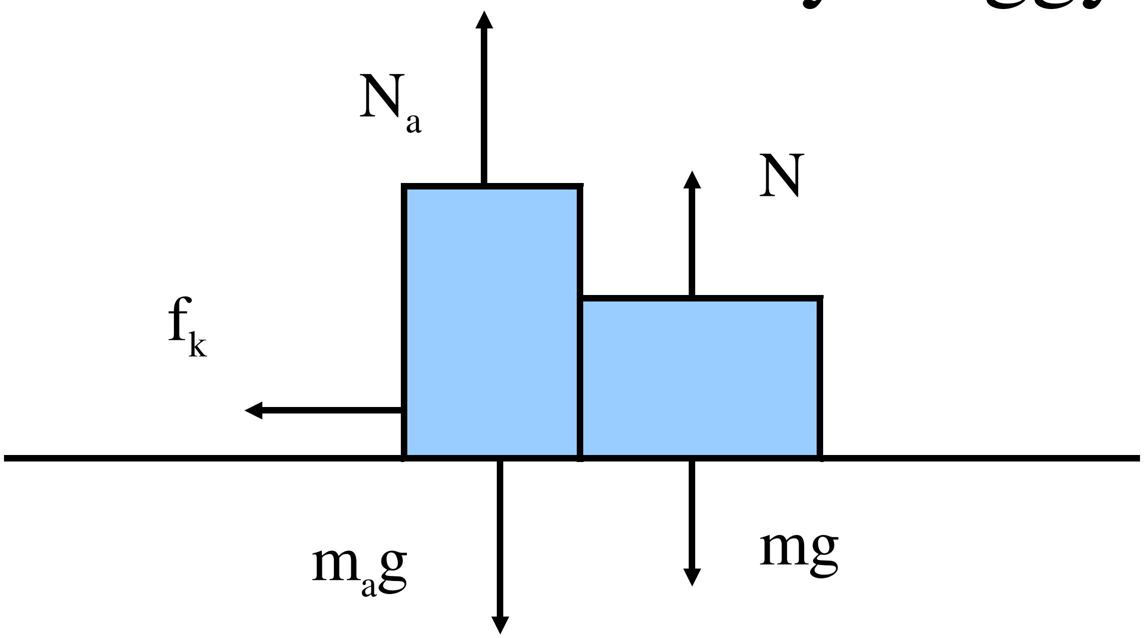
Questions: What is the minimum stopping distance, D?

What is the force exerted on the buggy?



There is only friction between the skates and the ice while the buggy slides with no friction

The Runaway Buggy

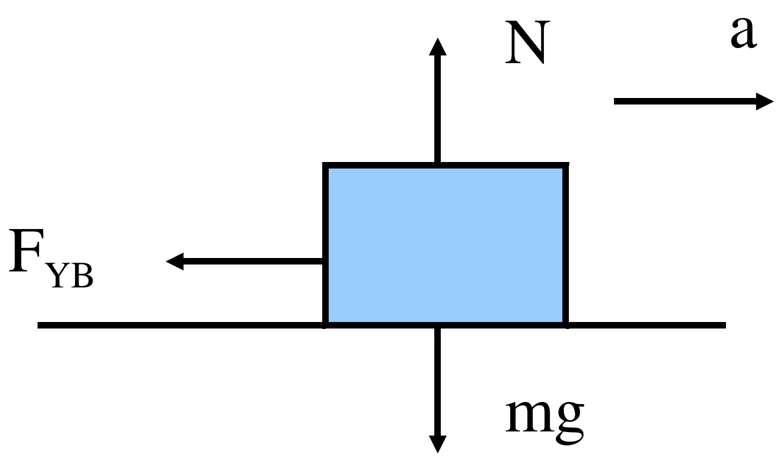


Two masses loosely coupled together. Only the adult's skates experience friction.

Treated as one mass for inertial purposes

Treated as separate masses for normal force and friction considerations.

The Runaway Buggy - The Buggy Alone

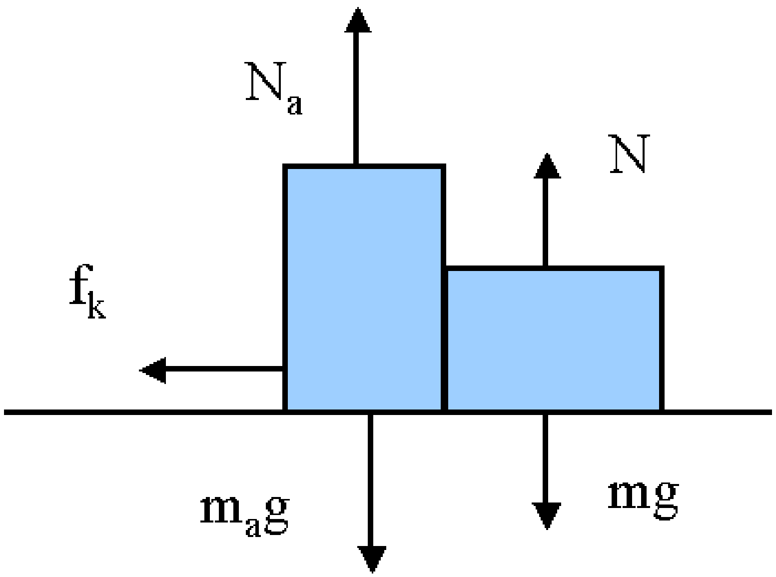


Two masses loosely coupled together. Only the adult's skates experience friction.

Treated as one mass for inertial purposes

Treated as separate masses for normal force and friction considerations.

The Runaway Buggy - A Neater Solution

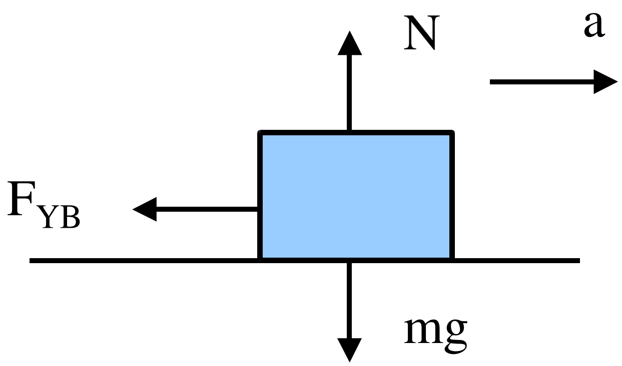


$$f_k = \mu_k N_a = \mu_k m_a g$$

$$\sum F_x = -\ f_k = (m_a + m)a$$

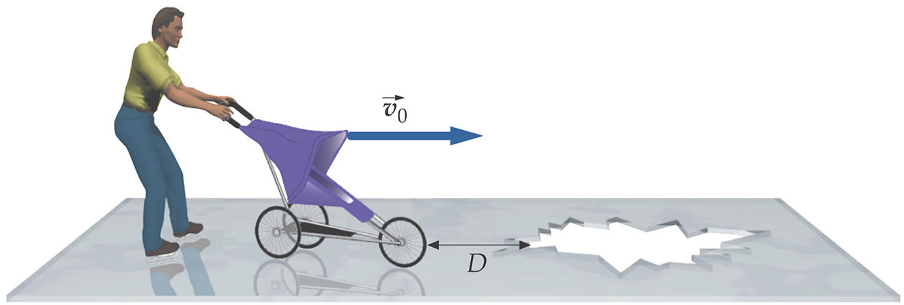
$$-\mu_k m_a g = (m_a + m)a$$

$$a = \frac{-\mu_k m_a}{m_a + m} g$$



$$F_{YB} = m|a| = \frac{\mu_k m}{1 + \dfrac{m}{m_a}} g$$

The Runaway Buggy - Stopping Distance



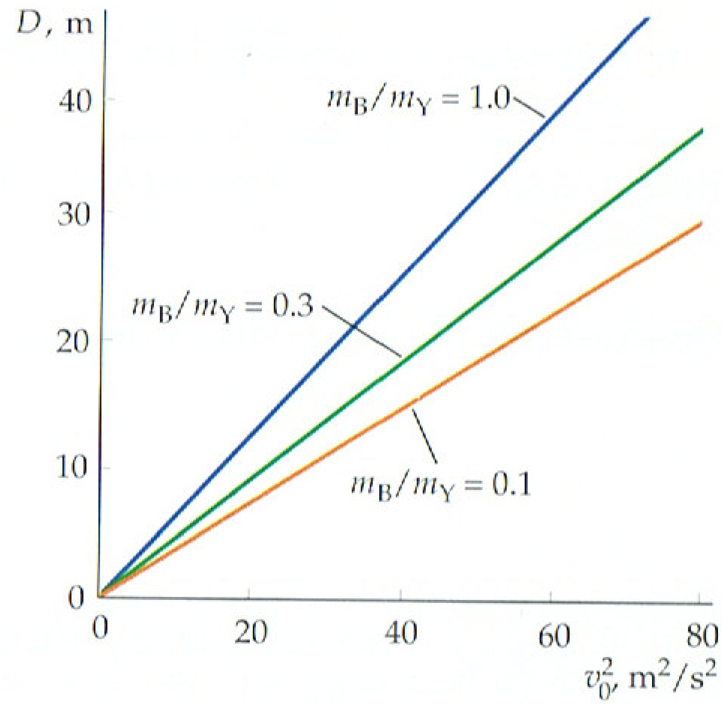
Let D be the stopping distance. Since we have velocities, acceleration and a distance we choose the following:

$$v_f^2 = v_o^2 + 2a\Delta x$$

$$0 = v_o^2 + 2aD$$

$$D = \frac{-v_o^2}{2a} = \left(1 + \frac{m}{m_a}\right)\frac{v_o^2}{2\mu_k g}$$

The Runaway Buggy Example



$$\frac{m_B}{m_Y} \Rightarrow \frac{m}{m_a}$$

TAKING IT FURTHER The minimum value of D is proportional to v_0^2 and inversely proportional to μ_k. Figure 5-14 shows the stopping distance D versus initial velocity squared for values of m_B/m_Y equal to 0.1, 0.3, and 1.0, with $\mu_k = 0.5$. Note that the larger the mass ratio m_B/m_Y, the greater the distance D needed to stop for a given initial velocity. This is akin to braking to a stop in a car that is pulling a trailer that does not have its own brakes. The mass of the trailer increases the stopping distance for a given speed.

Air Resistance

Air Resistance

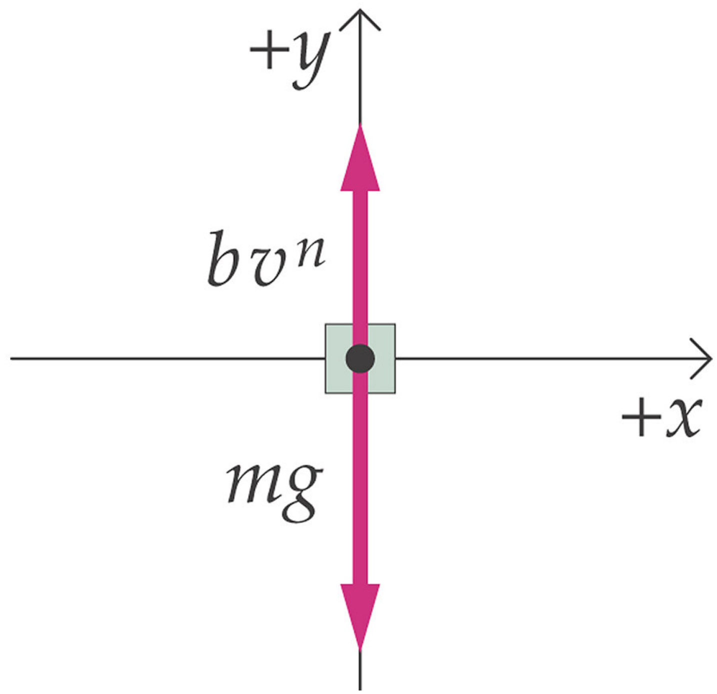
When the drag force due to air resistance equals the force due to gravity the net force on the falling object is zero

There is no more acceleration.

The velocity stays constant from that point on. This is referred to as the terminal velocity.

Air Resistance

The force of air resistance is proportional to a power of the velocity of the falling object.



Air Resistance Models

Linear Model

$$F(v) = bv$$

$$\sum F_y = mg - bv = 0$$

$$v = \frac{mg}{b}$$

Quadratic Model

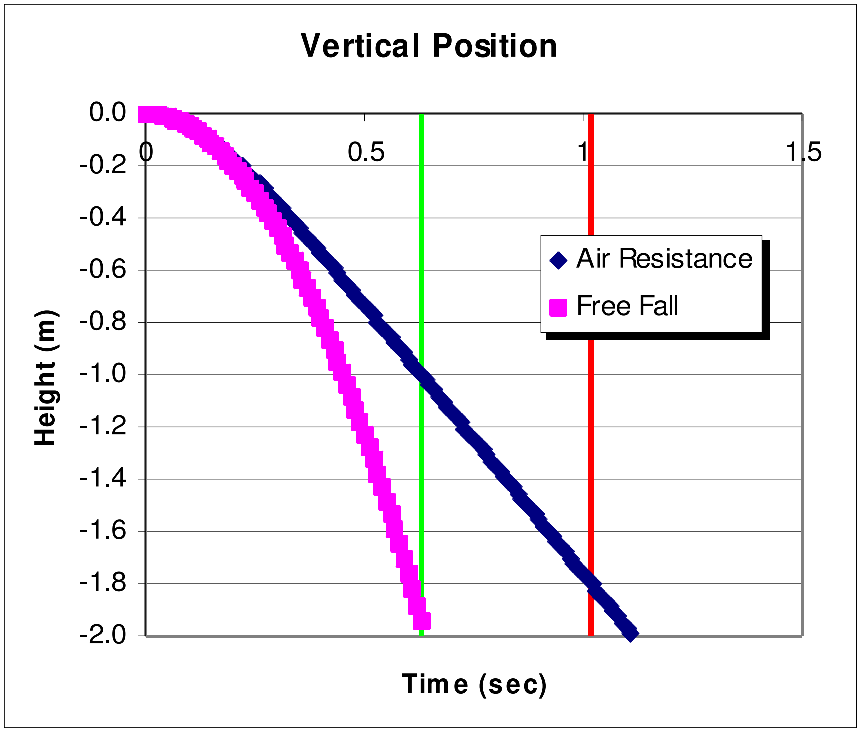
$$F(v) = \tfrac{1}{2} C \rho A v^2$$

$$\sum F_y = mg - \tfrac{1}{2} C \rho A v^2 = 0$$

$$v = \left(\frac{2mg}{C \rho A} \right)^{\frac{1}{2}}$$

where ρ is the density of the medium through which the object falls, A is the cross sectional area of the object, and C is a constant known as the drag coefficient and is related to the shape and texture of an object.

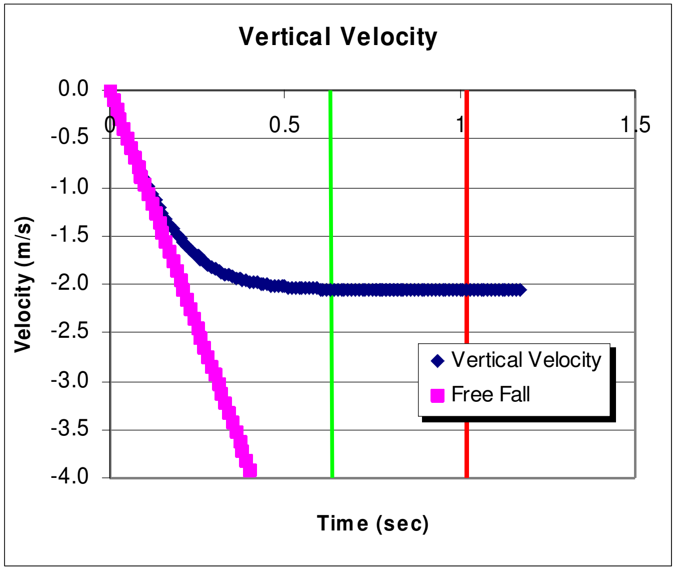
Air Resistance



The free fall curve has a term proportional to t^2. With air resistance the acceleration goes to zero. Its distance curve is proportional to t.

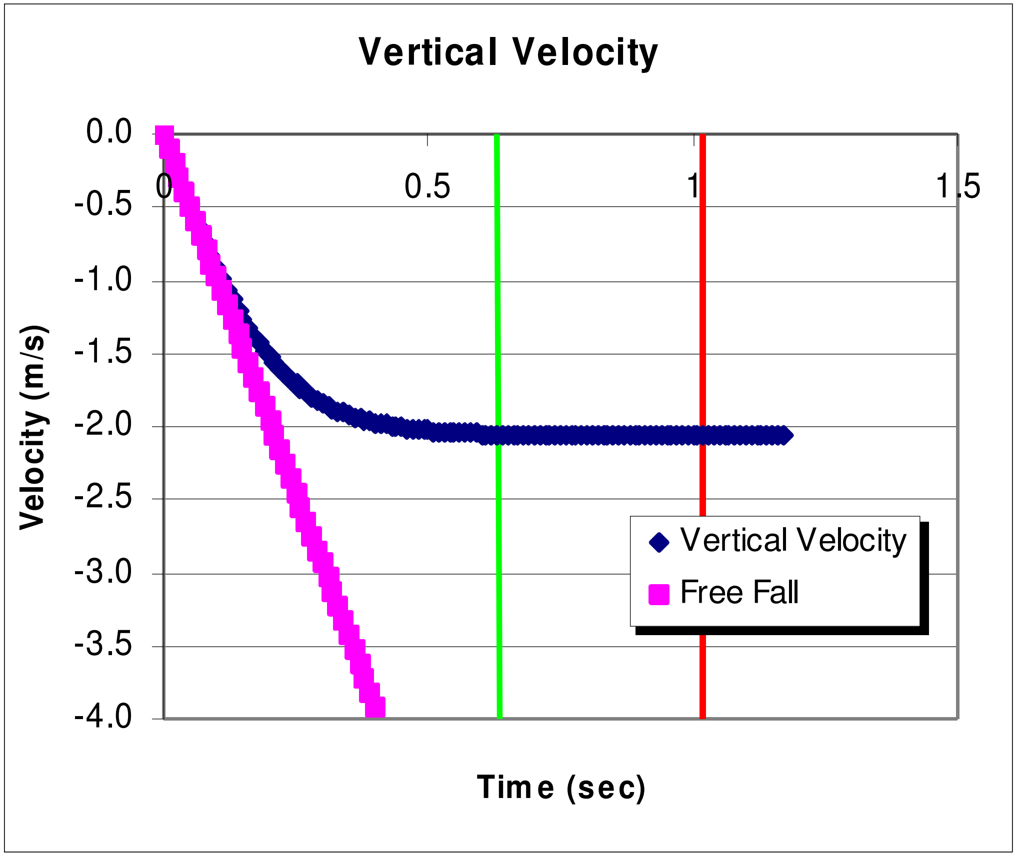
Air Resistance Time Constant

- The time constant represents the characteristic time interval in the problem. This is the size of the time interval over which important event in the problem take place.
- It takes a time interval about three (3) time constants in length for the velocity of the filter to reach 95% of terminal velocity. This time is indicated by the first vertical line in the Distance and Velocity graphs.
- The second vertical line represents the total fall time of the filter.
- The interval between these two lines is the time period available for making measurements of the terminal velocity.



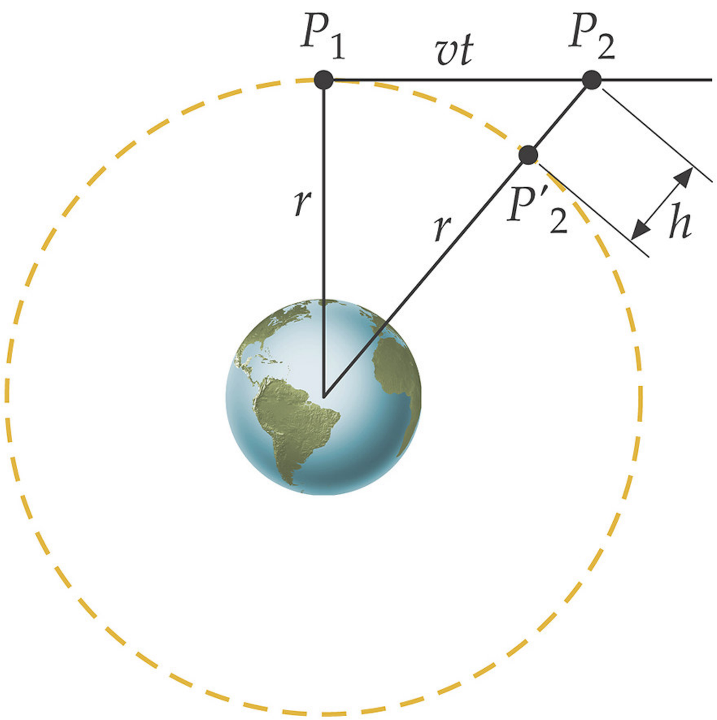
$$\tau = \left(\frac{2m}{gC\rho A} \right)^{1/2}$$

Air Resistance - Velocity vs Time Graph



Movement Along a Curved Path

Movement Along a Curved Path



The derivation shows that the centripetal acceleration is

$$a_c = \frac{v^2}{r}$$

If there is circular motion then the acceleration has this form.

Relationships for Circular Motion

$$a_c = \frac{v^2}{r}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

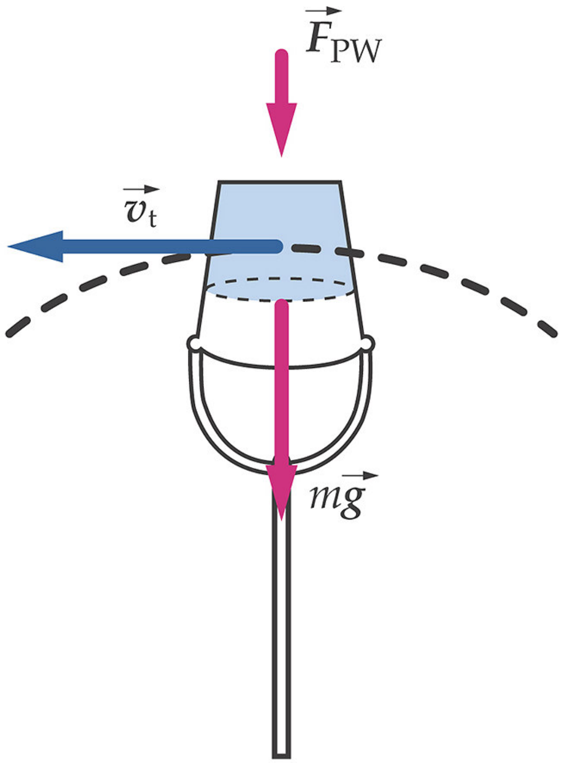
v is the linear (tangential) velocity (m/s).

r is the radius of the motion

f is the frequency (rev/s)

T is the period of the motion (s)

Centripetal Force



$$\sum F_r = F_{PW} + mg = ma_r$$

$$F_{PW} = ma_r - mg$$

$$F_{PW} = m\left(\frac{v^2}{r} - g\right)$$

$$\sum F_r = F_{PW} + mg = ma_r$$

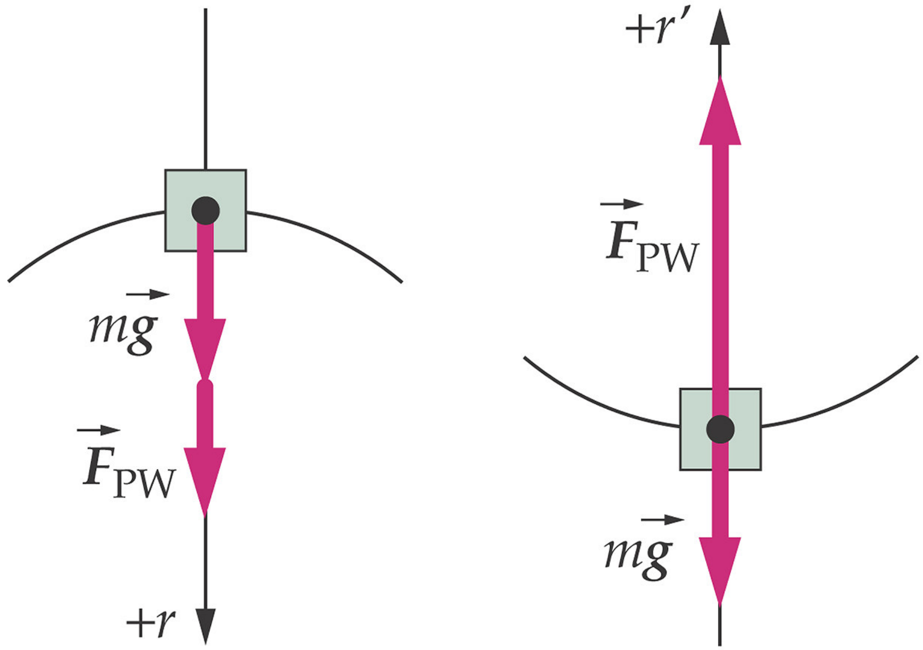
$$F_{PW} = ma_r - mg$$

$$F_{PW} = m\left(\frac{v^2}{r} - g\right)$$

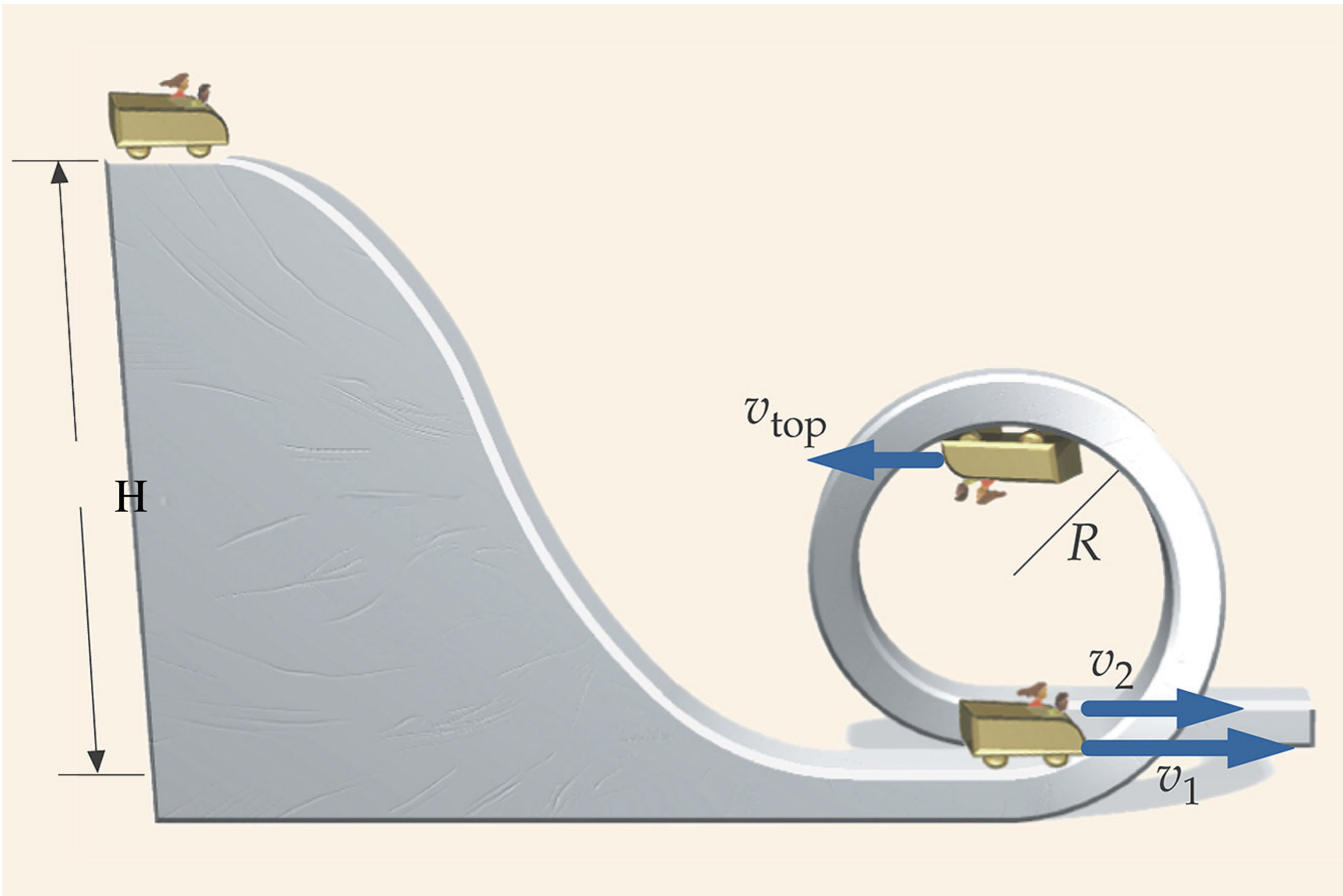
$$\sum F_r = F_{PW} - mg = ma_r$$

$$F_{PW} = ma_r + mg$$

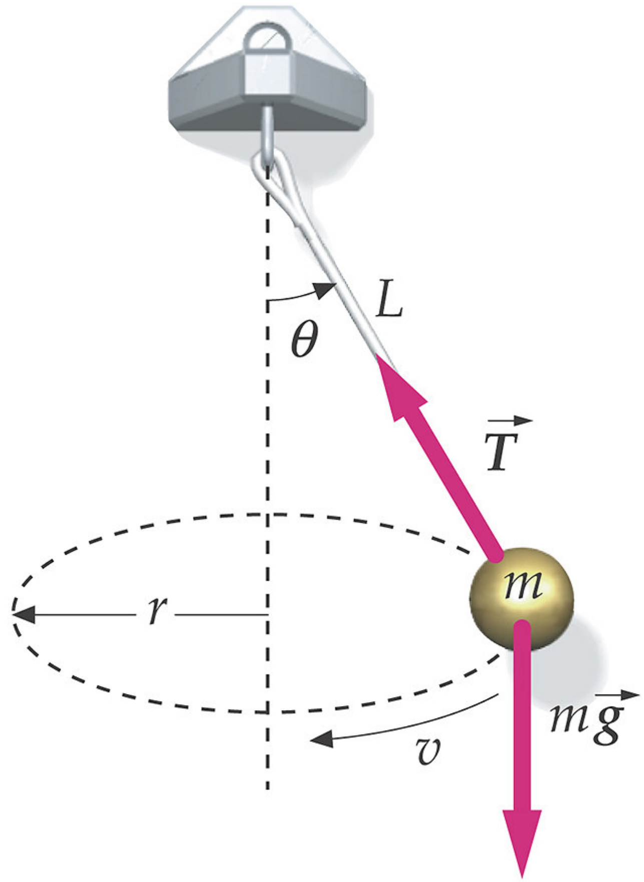
$$F_{PW} = m\left(\frac{v^2}{r} + g\right)$$



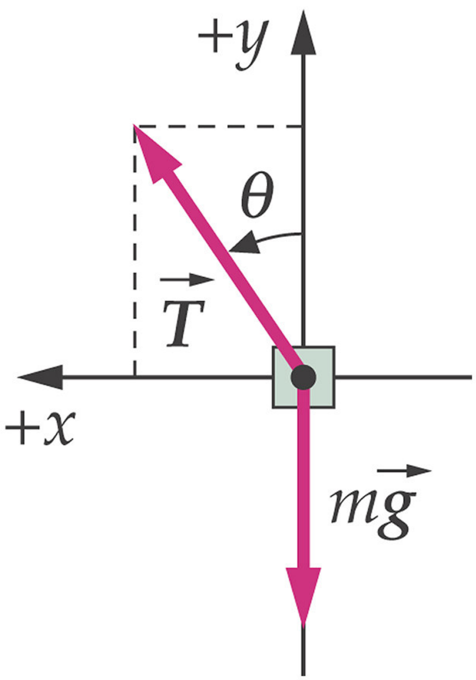
Roller Coaster



Conical Pendulum



Conical Pendulum



$$\sum F_y = T\cos\theta - mg = 0$$

$$T = \frac{mg}{\cos\theta}$$

$$\sum F_x = T\sin\theta = ma_x = m\frac{v^2}{r}$$

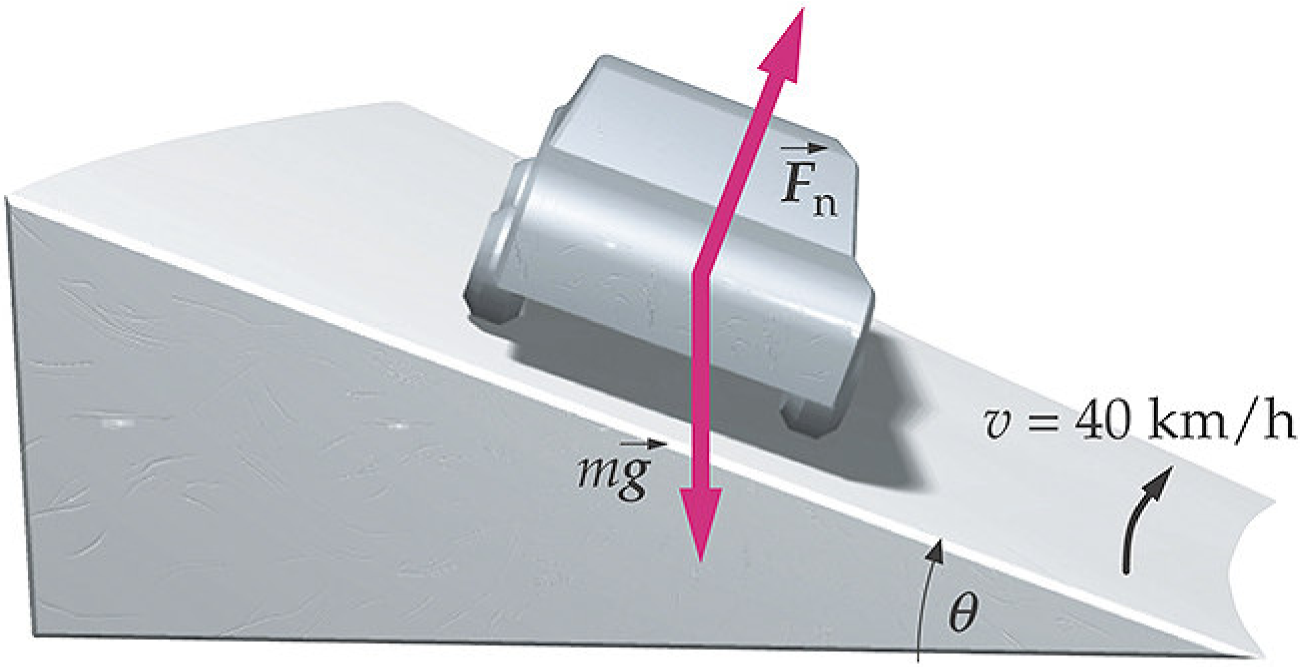
$$\frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{r}$$

$$g\tan\theta = \frac{v^2}{r}$$

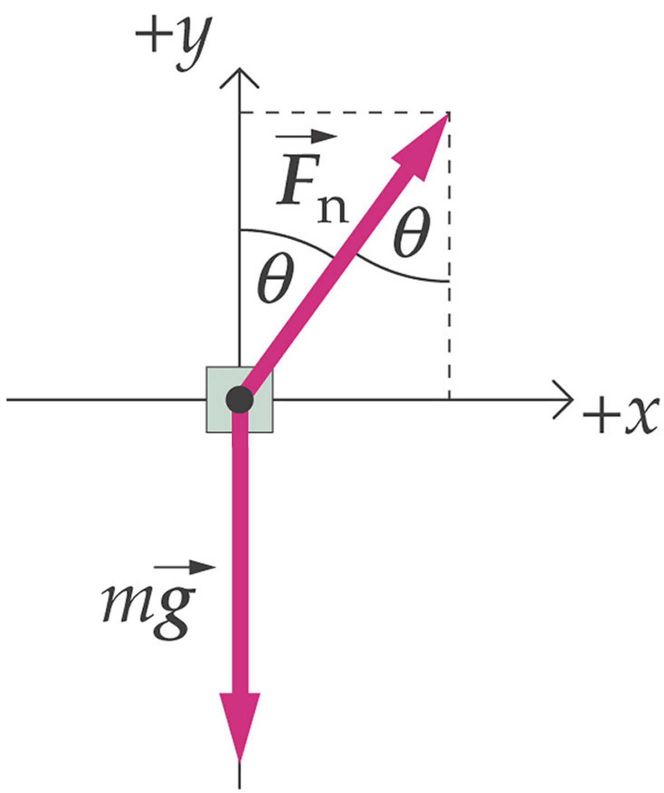
$$v = \sqrt{rg\tan\theta}$$

Banked Tracks

Banked Track - No Friction



Banked Track - No Friction



The component of the normal force along the x-axis is the centripetal force. This is $F_n\sin\theta$.

F_n is equal to $mg/\cos\theta$.

$F_n\sin\theta = mg\sin\theta/\cos\theta$

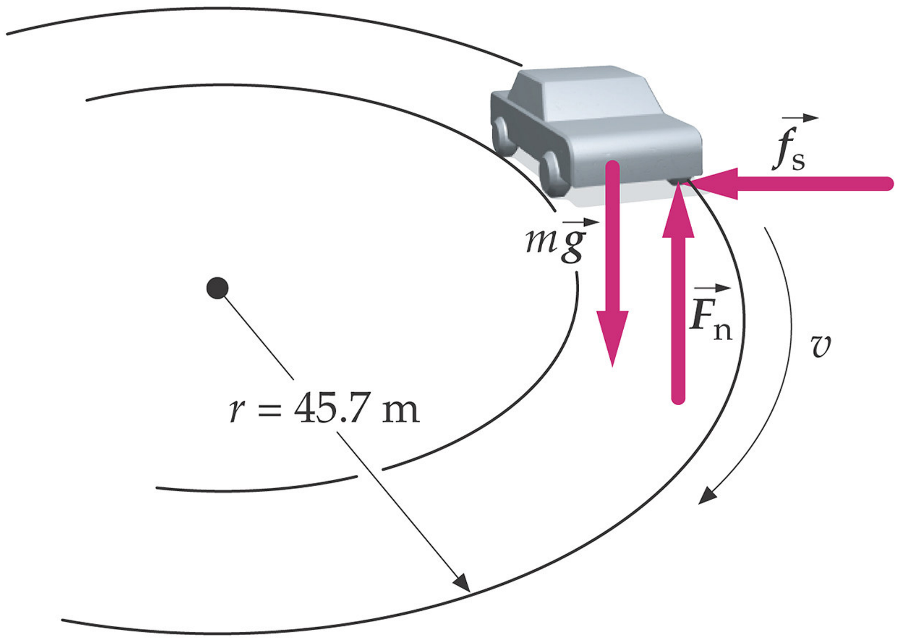
$$\frac{mv^2}{r} = mg\tan\theta$$

$$\tan\theta = \frac{v^2}{rg}$$

The same results as the conical pendulum

Flat Tracks

Flat Track - With Friction

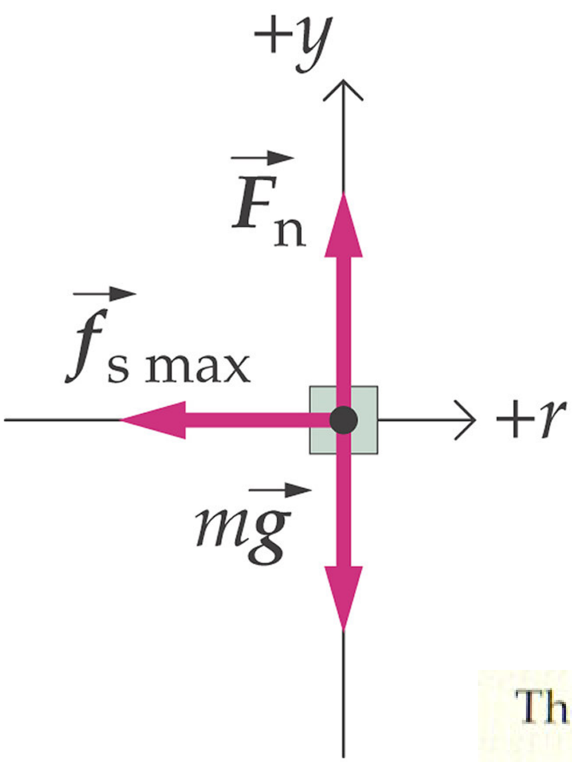


In a flat track situation the driver relys on friction between his tires and the track to stay on the curve.

For some reason the author ignores his center of mass obsession on a problem where it might be useful

Where should r be measured? Inner tires, outer tires, center of mass?

Flat Track - With Friction



$$v = \frac{2\pi r}{T} = \frac{2\pi(45.7\text{ m})}{15.2\text{ s}} = \boxed{18.9\text{ m/s}}$$

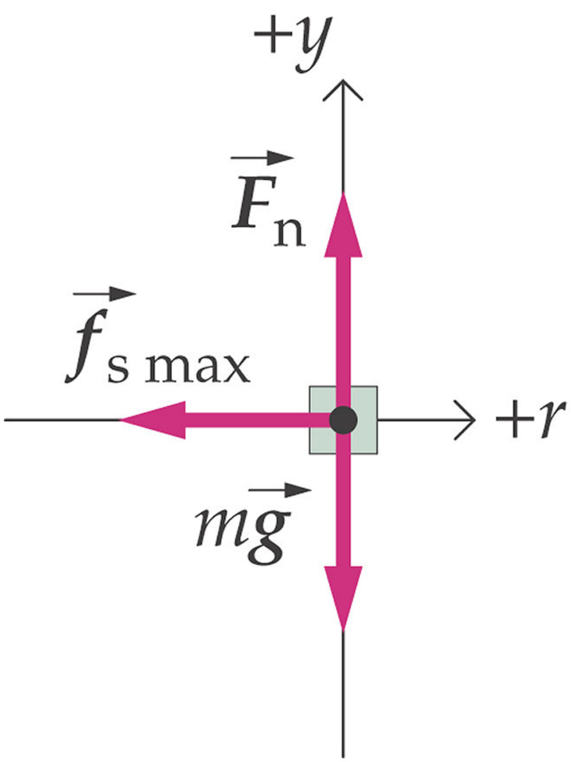
$$a_c = \frac{v^2}{r} = \frac{(18.9\text{ m/s})^2}{(45.7\text{ m})} = \boxed{7.81\text{ m/s}^2}$$

$$a_t = \frac{dv}{dt} = \boxed{0}$$

The acceleration is 7.81 m/s^2 in the centripetal direction.

Require: 1 complete loop
in 15.2s without skidding

Flat Track - With Friction



$$\Sigma F_y = ma_y$$

$$F_n - mg = 0 \quad \text{so} \quad F_n = mg$$

$$\text{and} \quad f_{s\,max} = \mu_s F_n = \mu_s mg$$

$$\Sigma F_r = ma_r$$

$$-f_{s\,max} = m\left(-\frac{v^2}{r}\right) \Rightarrow f_{s\,max} = m\frac{v^2}{r}$$

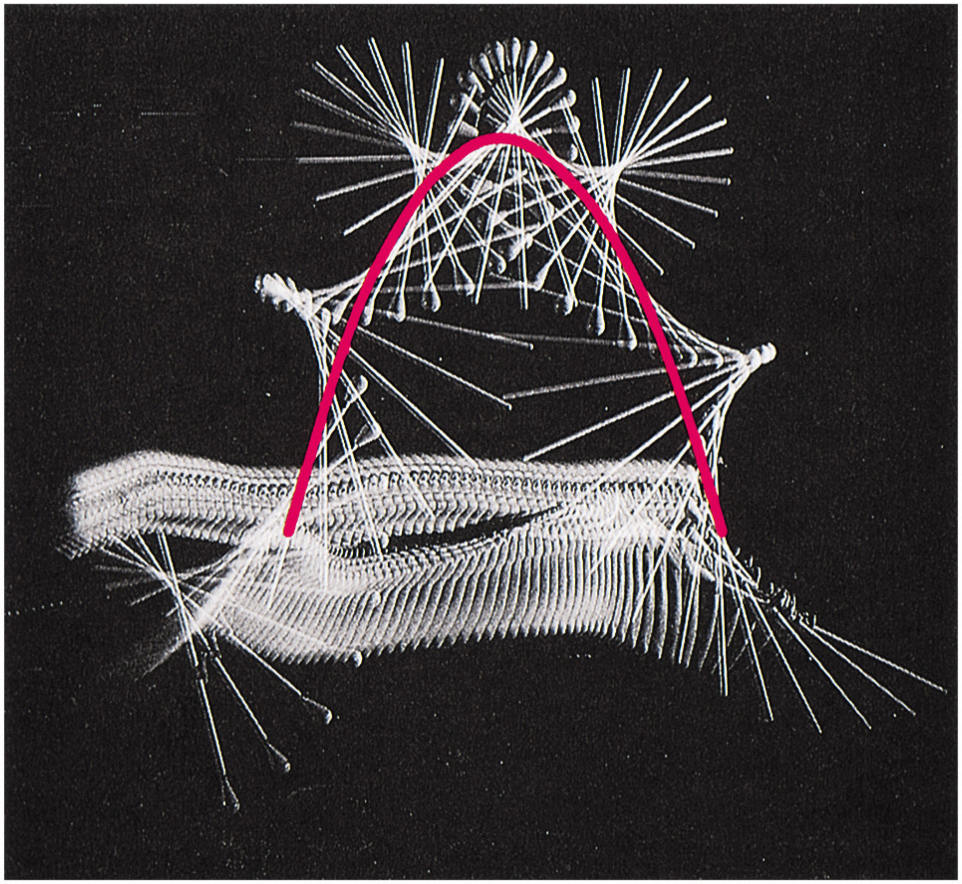
$$\mu_s mg = m\frac{v^2}{r} \Rightarrow \mu_s g = \frac{v^2}{r}$$

$$\mu_s = \frac{v^2}{rg} = \frac{(18.9\ \text{m/s})^2}{(45.7\ \text{m})(9.81\ \text{m/s}^2)} = \boxed{0.796}$$

Require: 1 complete loop in 15.2s without skidding

Center of Mass Motion

The Center of Mass

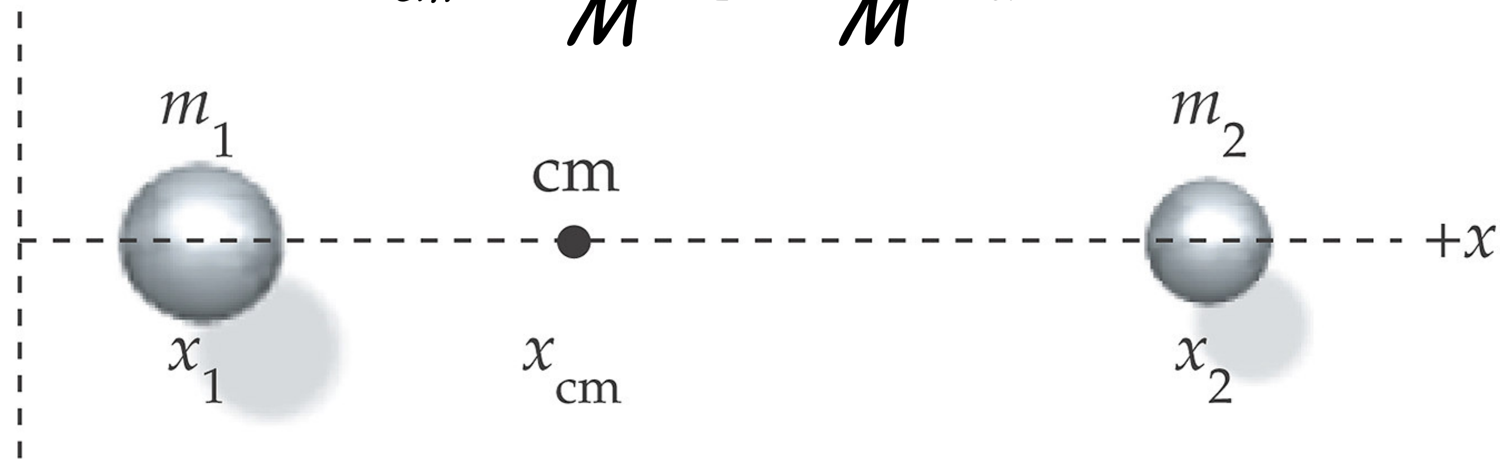


The center of mass follows a parabolic path.

The Center of Mass

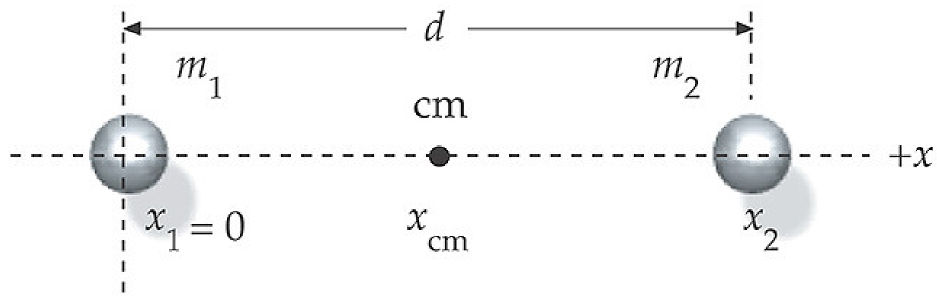
$$Mx_{cm} = m_1 x_1 + m_2 x_2$$

$$x_{cm} = \frac{m_1}{M} x_1 + \frac{m_2}{M} x_2$$

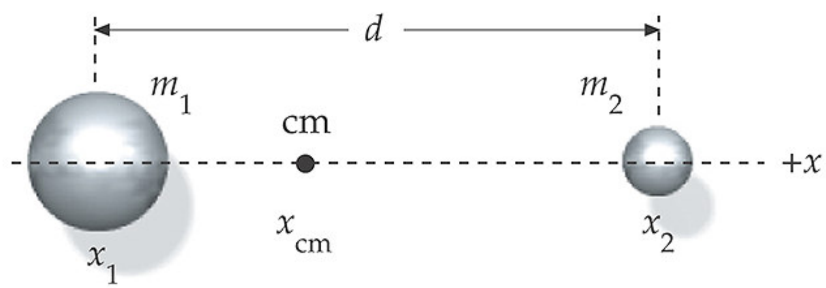


The CM is a mass weighted displacement

The Center of Mass



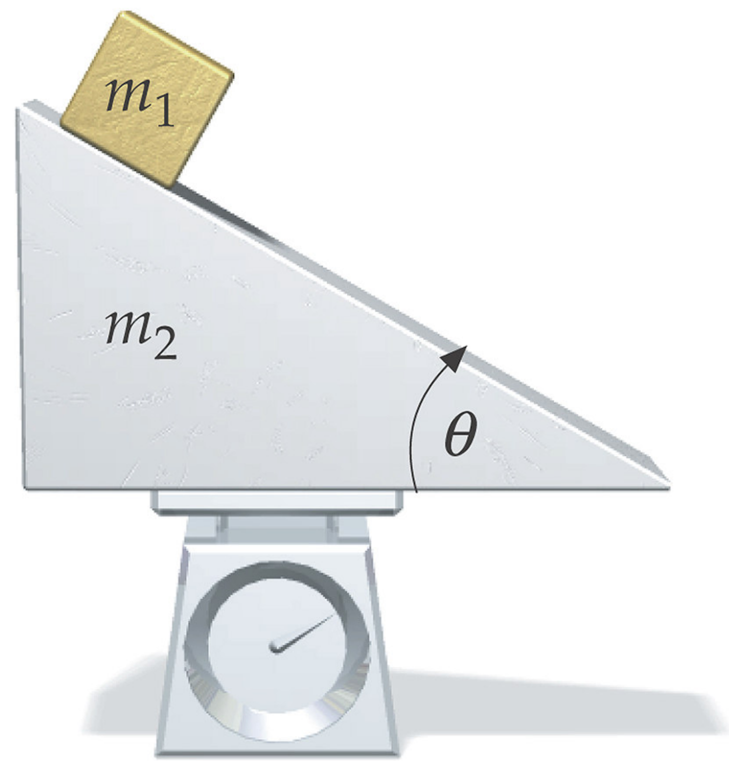
For equal masses the CM is mid way between them.



For unequal masses the CM is closer to the larger mass.

Problems

The Sliding Block Problem



What is the scale reading while the block is sliding?

The forces don't depend on the velocity.

The Sliding Block Problem

The center of mass approach is unnecessary since the incline isn't moving.

The inclined problem is analyzed with a non-rotated coordinate system.

Then an acceleration result from a rotated system is pulled in. A component of the result is then taken to get the desired projection.

$$F_n - m_1 g - m_2 g = M a_{cm\,y} = (m_1 + m_2) a_{cm\,y}$$

$$F_n = (m_1 + m_2) g + (m_1 + m_2) a_{cm\,y}$$

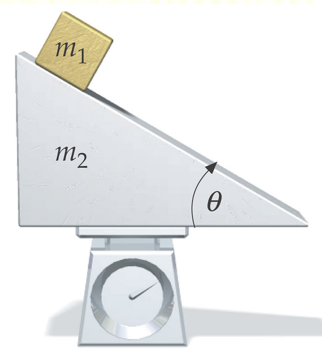
$$M a_{cm\,y} = m_1 a_{1y} + m_2 a_{2y}$$

$$(m_1 + m_2) a_{cm\,y} = m_1 a_{1y} + 0$$

$$a_{cm\,y} = \frac{m_1}{m_1 + m_2} a_{1y}$$

so $a_{1y} = -a_1 \sin\theta$, where $a_1 = g \sin\theta$

$$a_{1y} = -(g \sin\theta) \sin\theta = -g \sin^2\theta$$

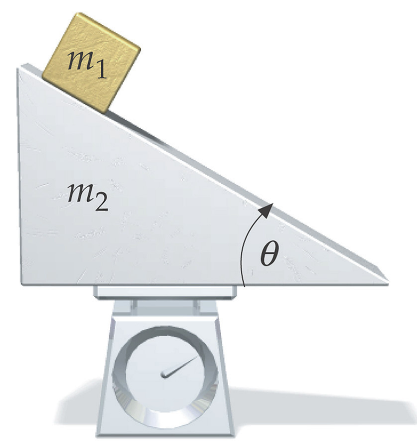


The Sliding Block Problem

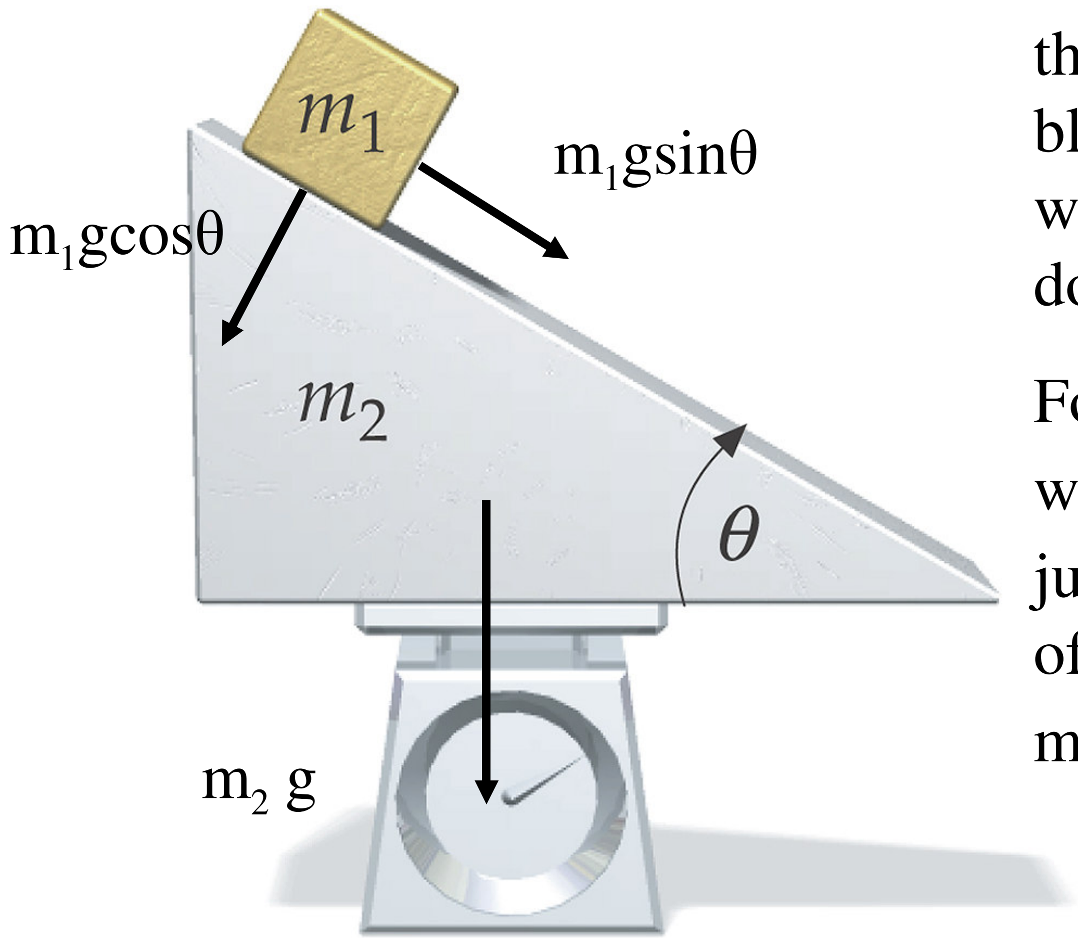
In the end the simplicity of the situation is obscured.

$$a_{\text{cm}\,y} = \frac{m_1}{m_1 + m_2} a_{1y} = -\frac{m_1}{m_1 + m_2} g \sin^2\theta$$

$$\begin{aligned} F_n &= (m_1 + m_2)g + (m_1 + m_2)a_{\text{cm}\,y} \\ &= (m_1 + m_2)g - m_1 g \sin^2\theta = [m_1(1 - \sin^2\theta) + m_2]g \\ &= \boxed{(m_1 \cos^2\theta + m_2)g} \end{aligned}$$



The Sliding Block Problem



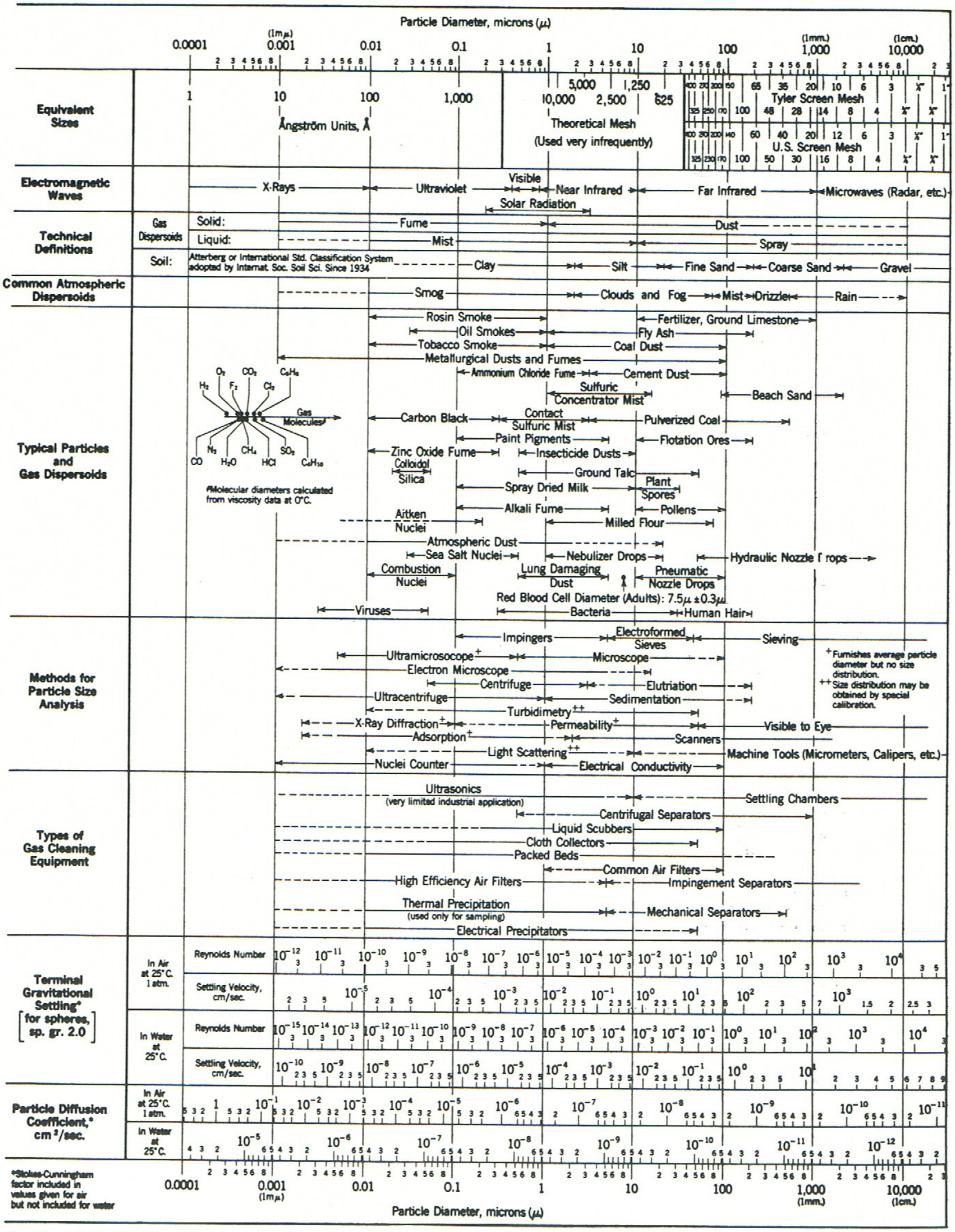
The scale reading is just the normal force of both blocks. How much of their weight is directed straight down?

For m_2 it is the entire weight m_2 g. For m_1 it is just the vertical projection of $m_1 g\cos\theta$ which is $m_1 g\cos^2\theta$

$$F_n = \left(m_1 cos^2\ \theta + m_2\right) g$$

Extra Slides

CHARACTERISTICS OF PARTICLES AND PARTICLE DISPERSOIDS



C. E. Lapple, Stanford Research Institute Journal, Vol. 5, p.95 (Third Quarter, 1961)