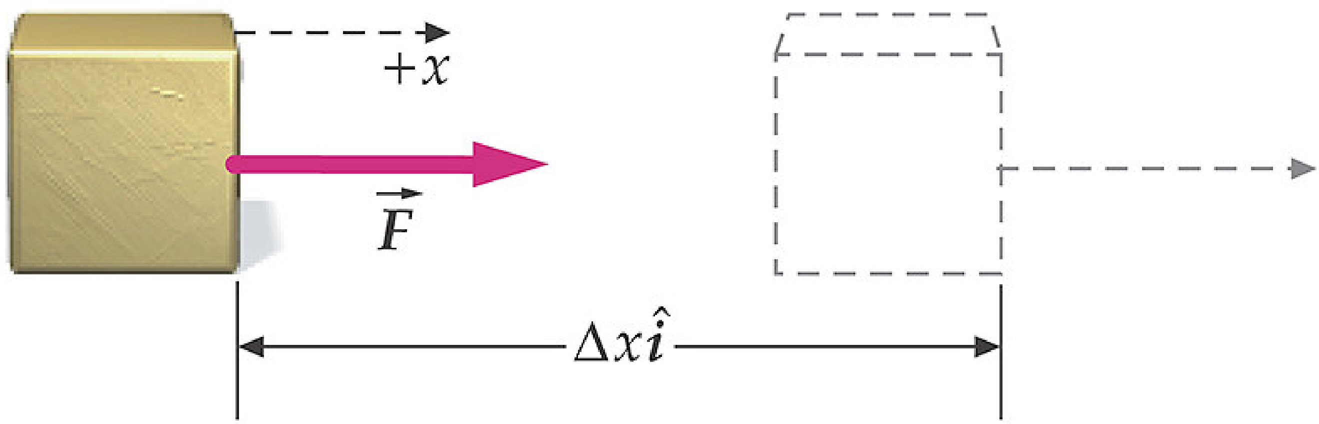
Chapter 7

Work and Kinetic Energy

Work and Kinetic Energy

- Work Done by a Constant Force
- Work Done by a Variable Force - Straight Line Motion
- The Scalar Product
- Work-Kinetic Energy Theorem

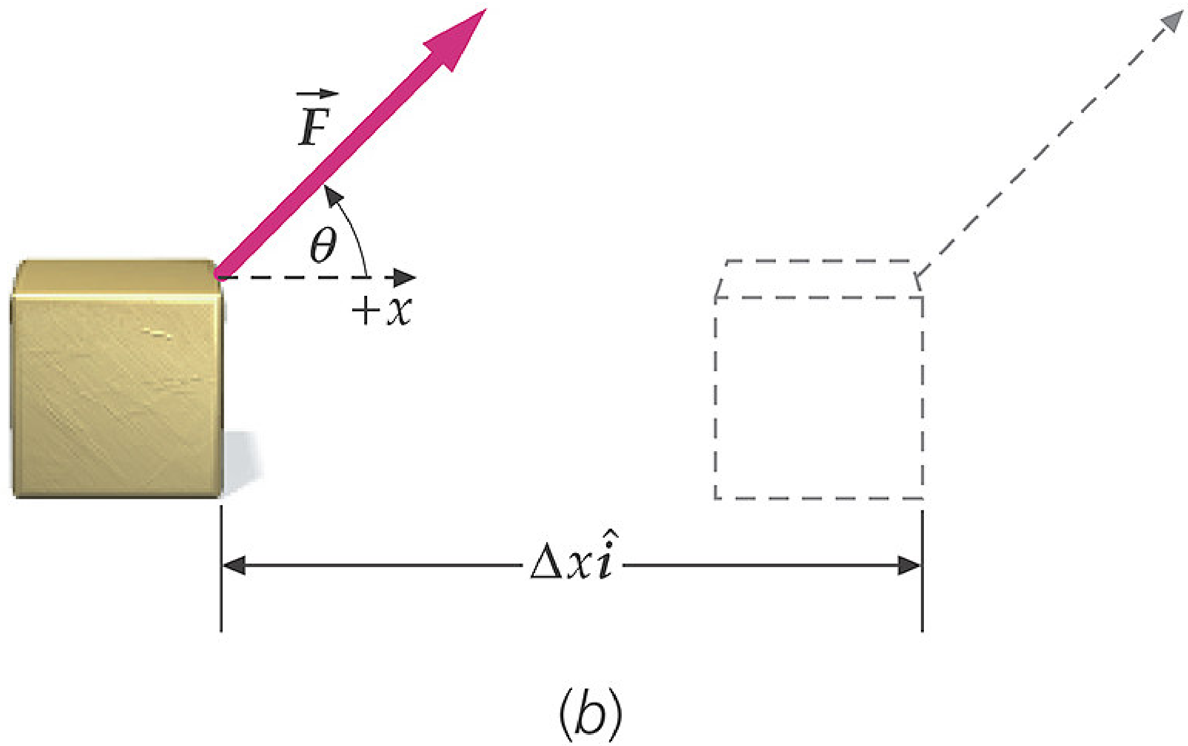
Work - As simple As It Gets



(a)

Work = Force x Distance

Work - With A Different Angle



Work - From a Vector Point of View

$$\vec{F} = F_x \hat{i} + F_y \hat{j}$$

$$\Delta\vec{x} = \Delta x \hat{i}$$

$$Work = \vec{F} \bullet \Delta\vec{x} = \left(F_x \hat{i} + F_y \hat{j} \right) \bullet \left(\Delta x \hat{i} \right)$$

$$Work = F_x \Delta x$$

Only the part of F that is parallel to Δx contributes to the work.
The result of "dotting" two vectors together is a scalar.

Definitions

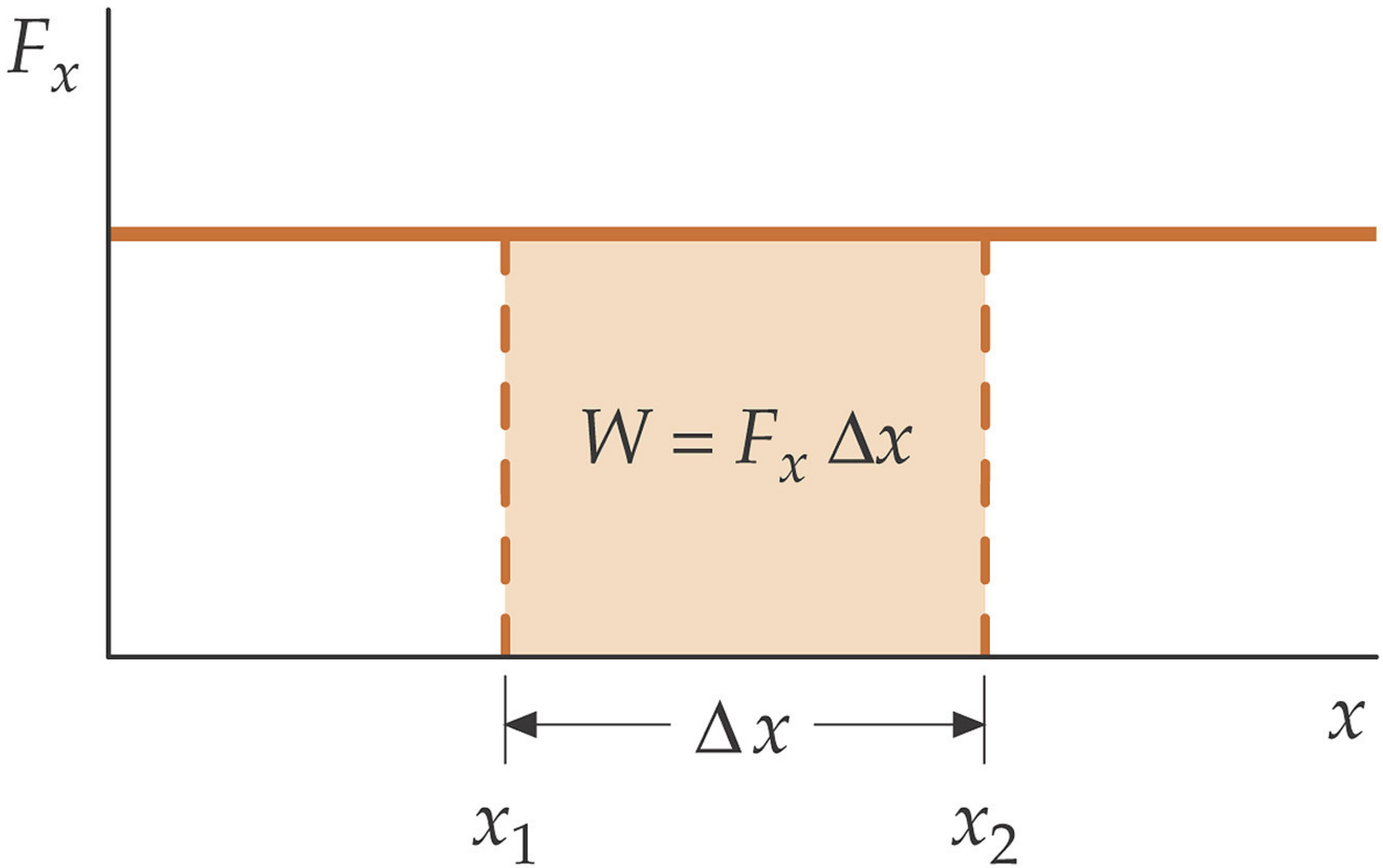
1 Joule = 1 N-m

1 Ft-lbs = 1.356 J

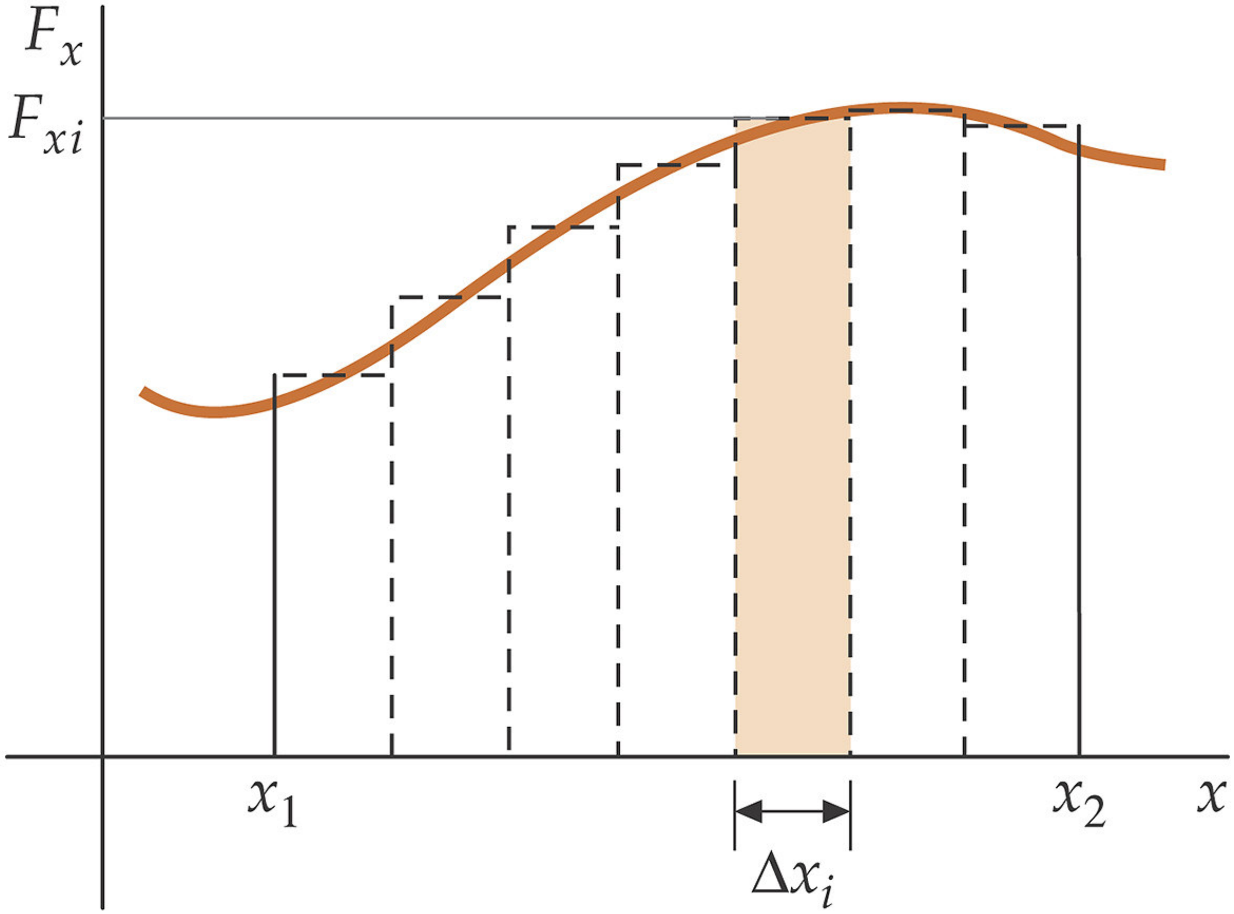
1ev = 1.602×10^{-19} J

The ev (electron volt) is the amount of energy that an electron gains in falling through an electrical potential difference of 1 volt.

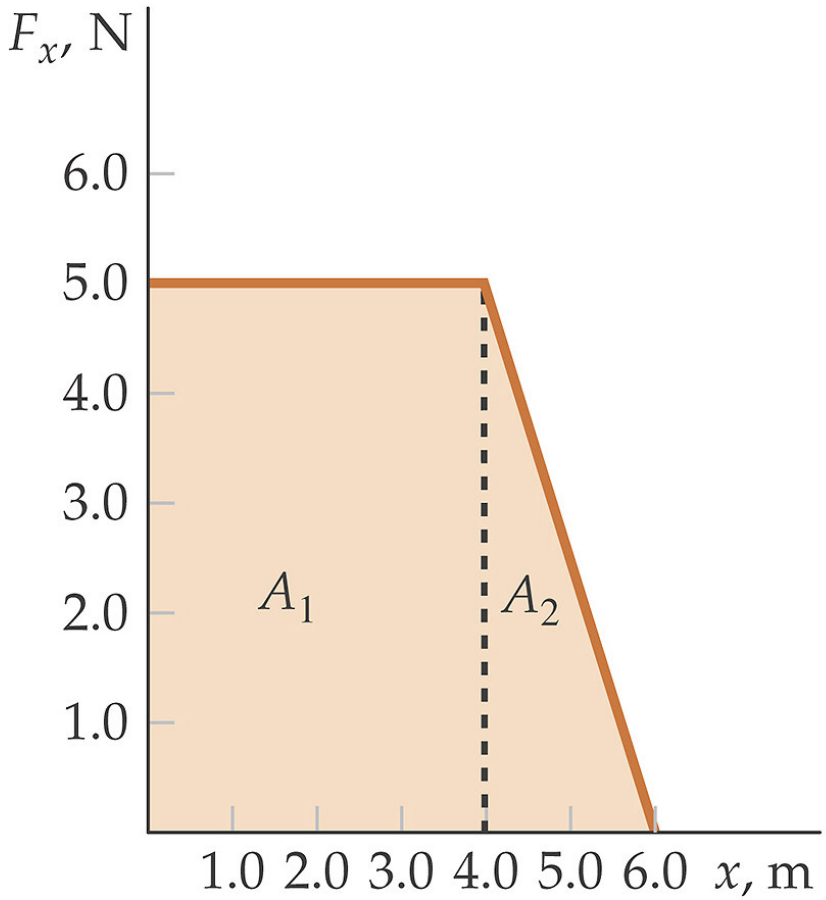
Work Done As Area



A Spatially Varying Force

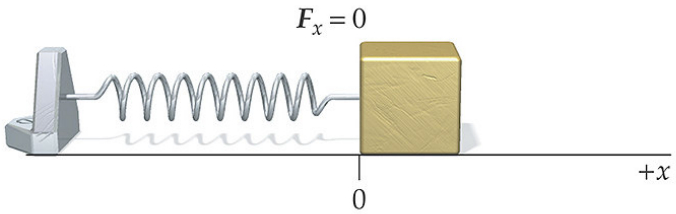


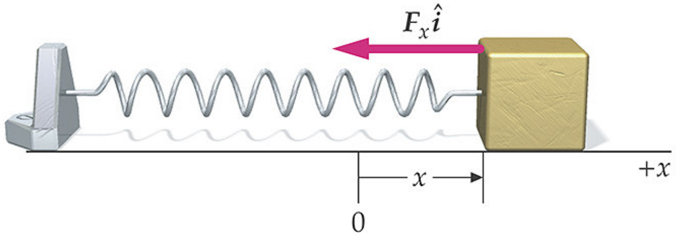
Work Done by A Varying Force



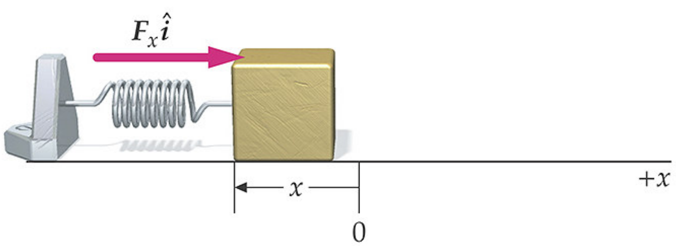
Break the area up into familiarly shaped objects and calculate their area using the units of the axes.

Work Done by A Spring





$Fx = -kx$ is negative because x is positive.



$Fx = -kx$ is positive because x is negative.

$$F_x = -kx$$

$$W_{byspring} = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} (-kx)dx = -k\int_{x_1}^{x_2} xdx$$

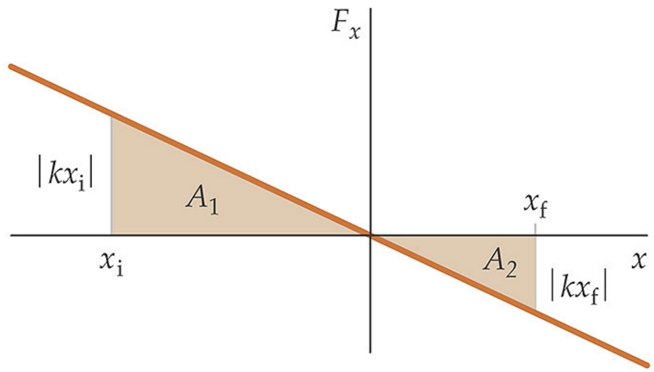
$$W_{byspring} = -k\left(\frac{x_2^2}{2} - \frac{x_1^2}{2}\right) = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$

$U = \dfrac{1}{2}kx^2$ is the Potential Energy (PE)

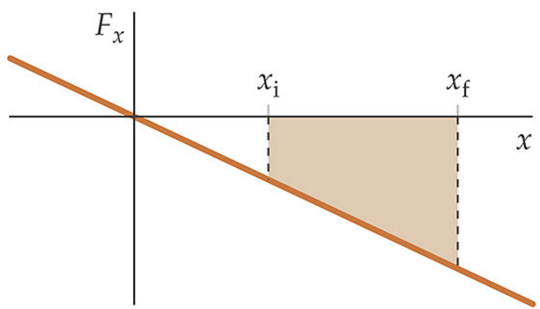
This is an example of a harmonic oscillator (SHO).

A SHO doesn't obey our kinematic equations.

Work As Area - Again



(a)



(b)

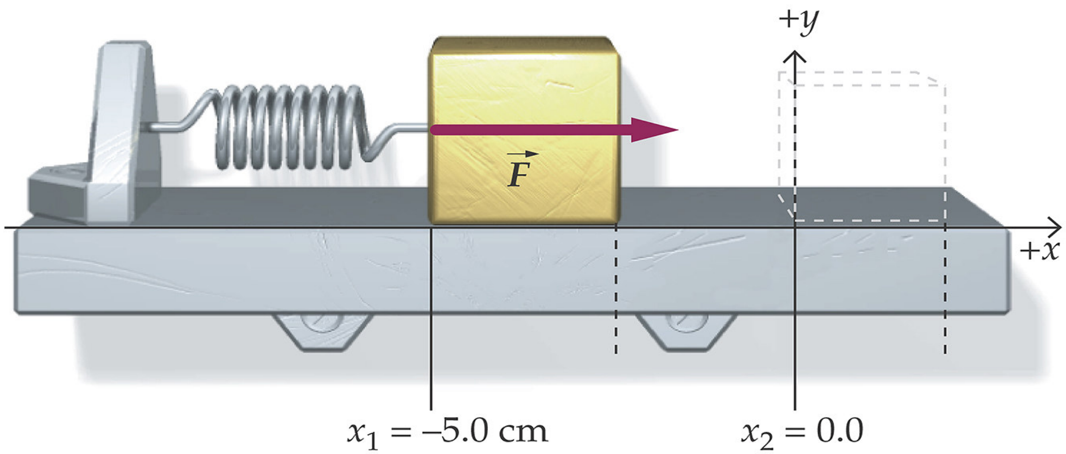
A geometric approach will yield the equations on the previous page

Work Done on a Block by a Spring

Questions: (a.) Calc work done by the spring on the block?

(b.) What is the velocity of the block at $x_2 = 0.0$?

k = 400 N/m



$$W_{byspring} = \int_{x_1}^{x_2} F_x dx = -\int_{x_1}^{x_2} kx dx = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$

$$\tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2 = \tfrac{1}{2}400\left(\left(-0.05\right)^2 - 0^2\right) = 0.50J$$

Work Done on a Block by a Spring

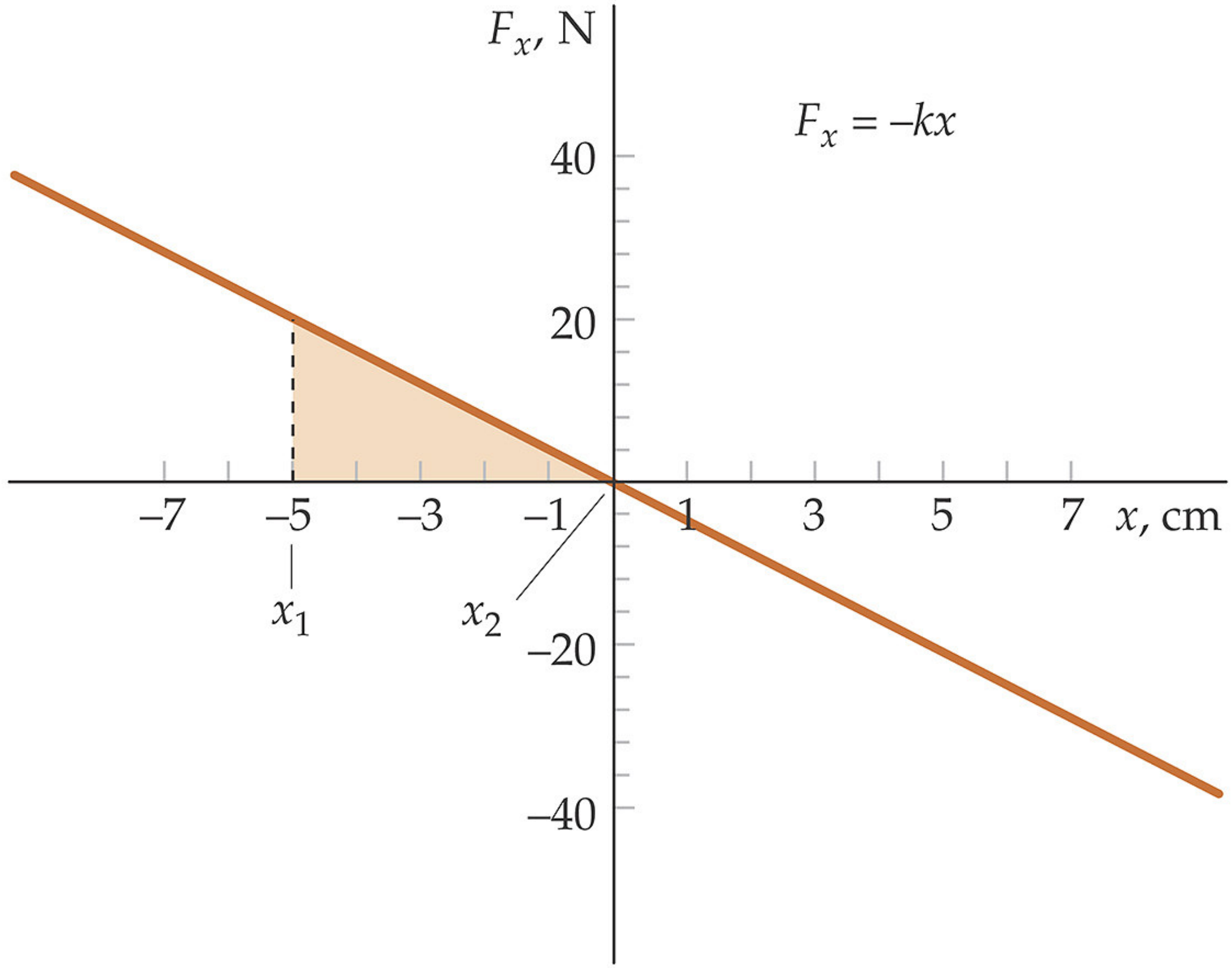
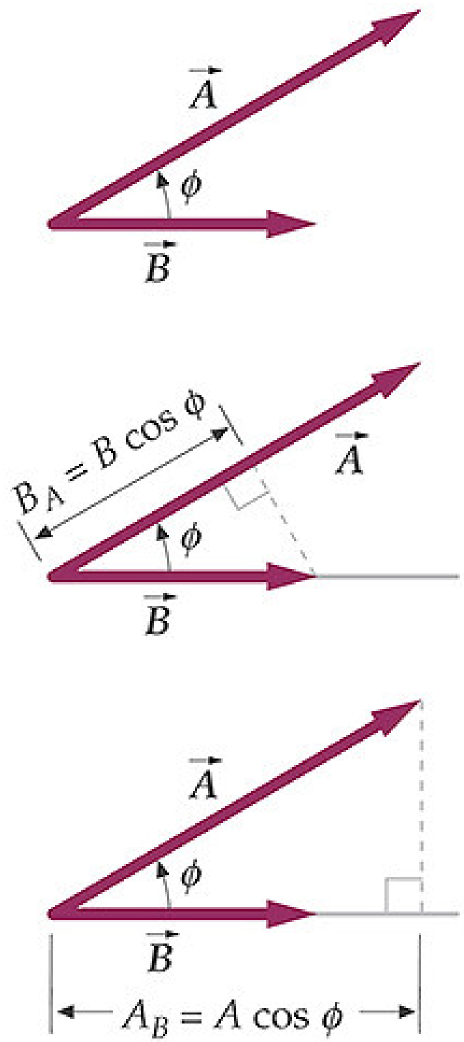


Table 6-1 Properties of Scalar Products

If	Then
$\vec{A}$ and $\vec{B}$ are perpendicular,	$\vec{A} \cdot \vec{B} = 0$ (because $\phi = 90°$, $\cos\phi = 0$)
$\vec{A}$ and $\vec{B}$ are parallel,	$\vec{A} \cdot \vec{B} = AB$ (because $\phi = 0°$, $\cos\phi = 1$)
$\vec{A} \cdot \vec{B} = 0$,	Either $\vec{A} = 0$ or $\vec{B} = 0$ or $\vec{A} \perp \vec{B}$
Furthermore,	
$\vec{A} \cdot \vec{A} = A^2$	Because $\vec{A}$ is parallel to itself
$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$	Commutative rule of multiplication
$(\vec{A} + \vec{B}) \cdot \vec{C} = \vec{A} \cdot \vec{C} + \vec{B} \cdot \vec{C}$	Distributive rule of multiplication

Scalar Product = Dot Product



Vectors with Rectangular Unit Vectors

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

Dot Product - Scalar

The dot product multiplies the portion of A that is *parallel* to B with B

$$\hat{i} \bullet \hat{i} = 1$$

$$\hat{i} \bullet \hat{j} = 0$$

$$\hat{i} \bullet \hat{k} = 0$$

	$\hat{i}$	$\hat{j}$	$\hat{k}$
$\hat{i}$	1	0	0
$\hat{j}$	0	1	0
$\hat{k}$	0	0	1

Dot Product - Scalar

In 2 dimensions

$$\vec{A} \bullet \vec{B} = A\,B\cos(\Theta)$$

In any number of dimensions

$$\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

The dot product multiplies the portion of A that is *parallel* to B with B

Kinetic Energy

$$KE = \frac{1}{2}mv^2$$

The units of kinetic energy are Joules

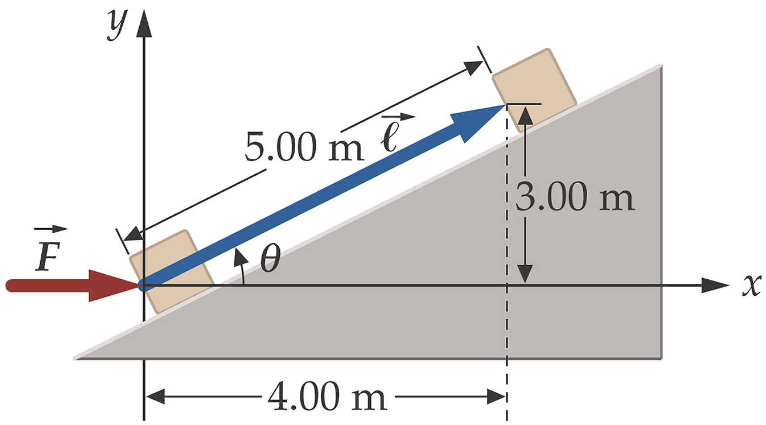
Work-Kinetic Energy Theorem

Work = F x D = ΔKE

Work > 0   Energy ==> Into System

Work < 0   Energy <== Out of System

An Incomplete Example



"Pushing A Box" - This is an incomplete problem because it only examines the amount of work done - not where it goes.

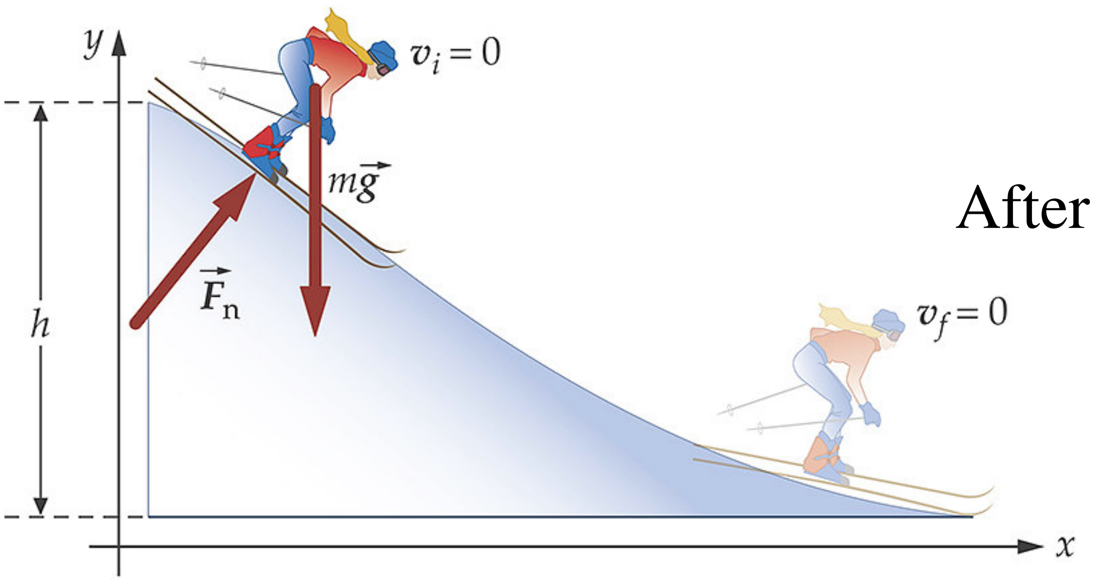
Once you leave the horizontal you do work against gravity.

In Chapter 8

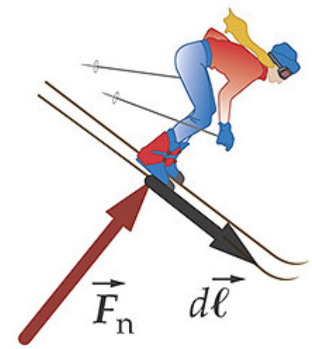
$$mgh = \tfrac{1}{2}mv^2$$

Before

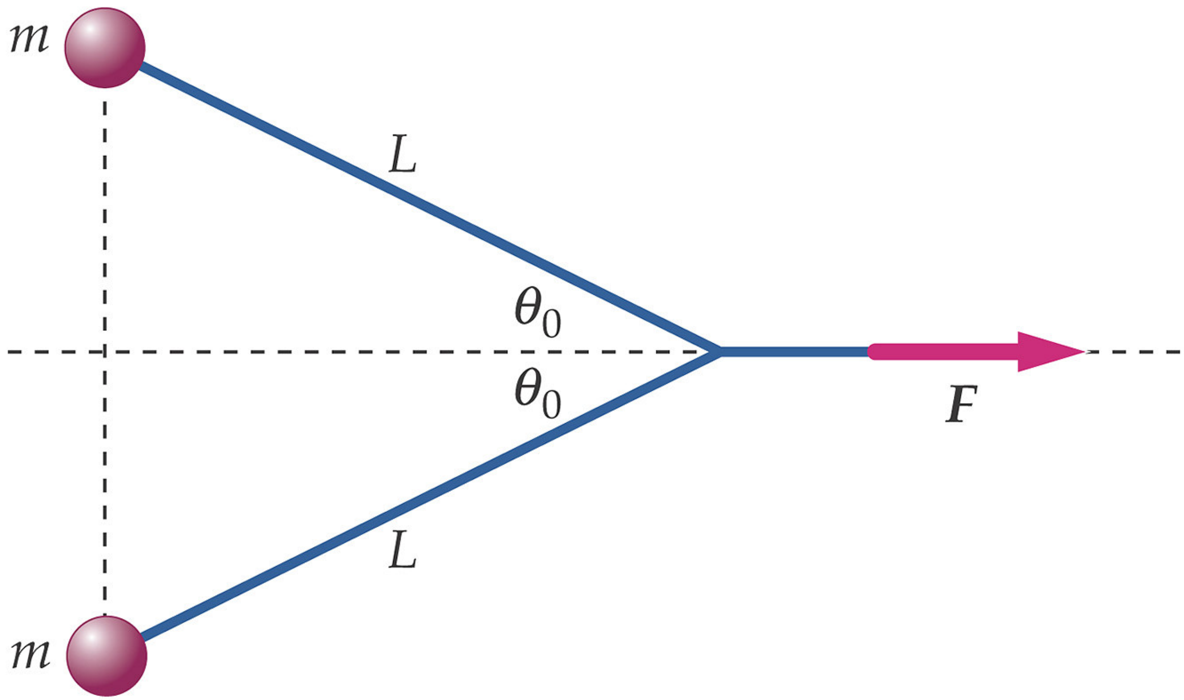
will save you much time.

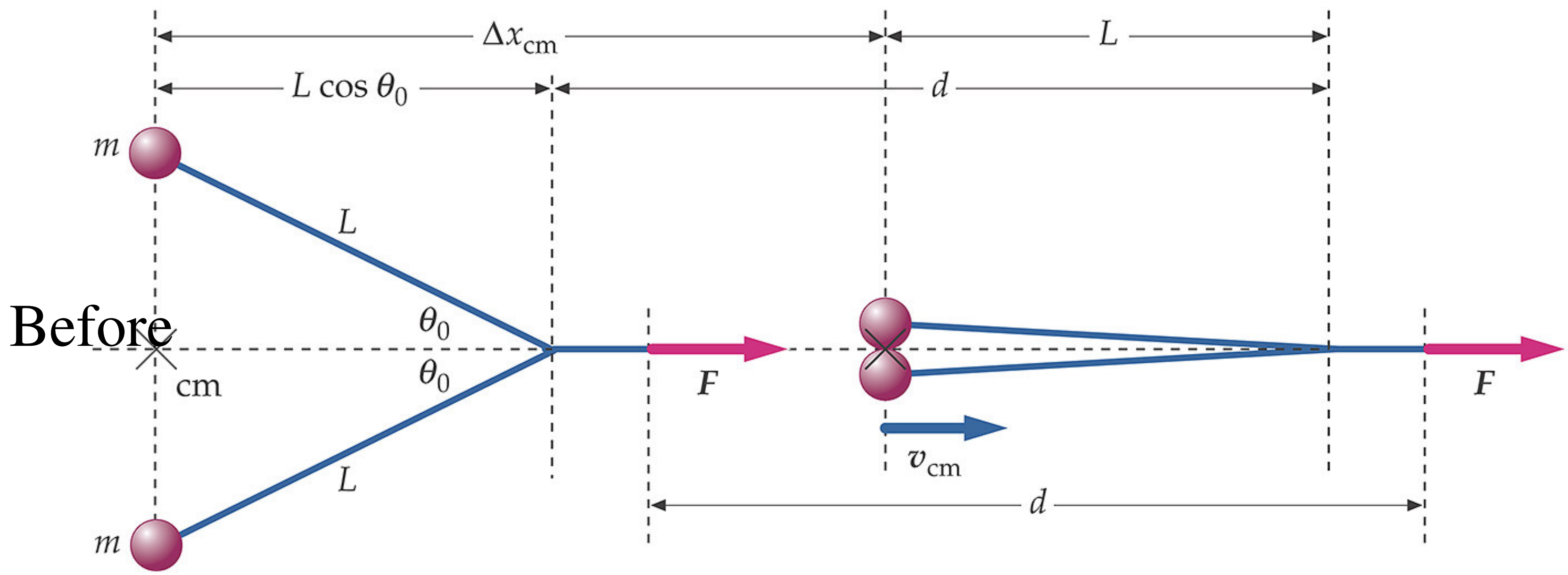


After



Finally! An Application that Needs the Center of Mass Treatment





$$v_{cm} = \sqrt{\frac{F\left[d - L\left(1 - cos\theta_0\right)\right]}{m}}$$

$$mv_{cm}^2 = F\left[d - L\left(1 - cos\theta_0\right)\right]$$

d - L(1-cosθ_o) is the net distance along the x-direction that the CM moved to the right.

L(1-cosθ_o) is the distance that the CM moves to the left as the balls come together.