Chapter 8

Conservation of Energy

Conservation of Energy

- Potential Energy
- Conservation of Mechanical Energy
- The Conservation of Energy
- Mass and Energy

Potential Energy

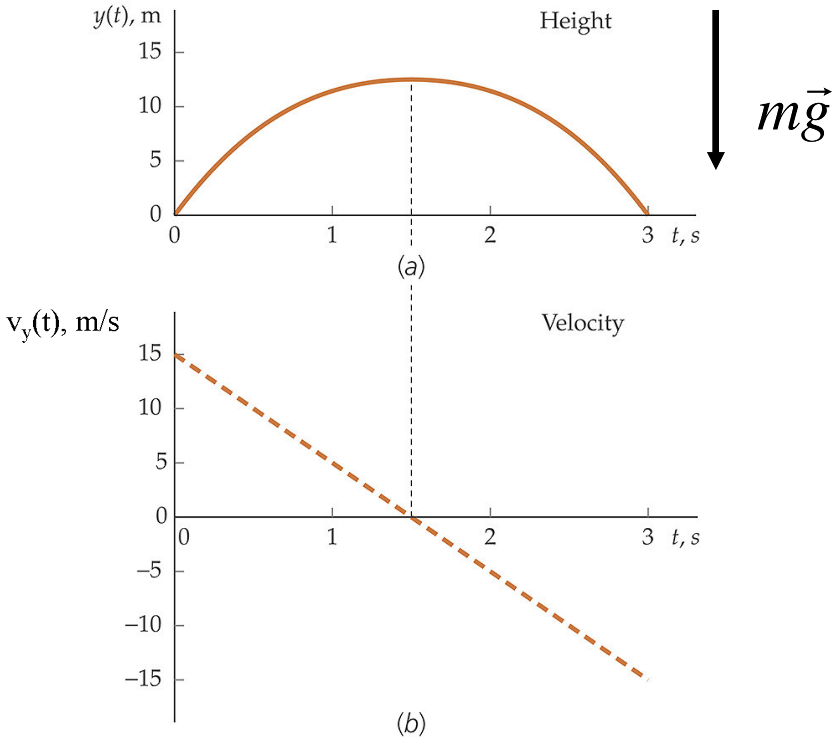
In our work with projectiles we were interacting with a form of potential energy but we didn't mention the energy concept.

In the last chapter we introduced energy in the form of kinetic energy and now we want to introduce potential energies.

Different forces will have different forms of potential energy but kinetic energy will maintain its form:

$$KE = \tfrac{1}{2}mv^2$$

Potential Energy

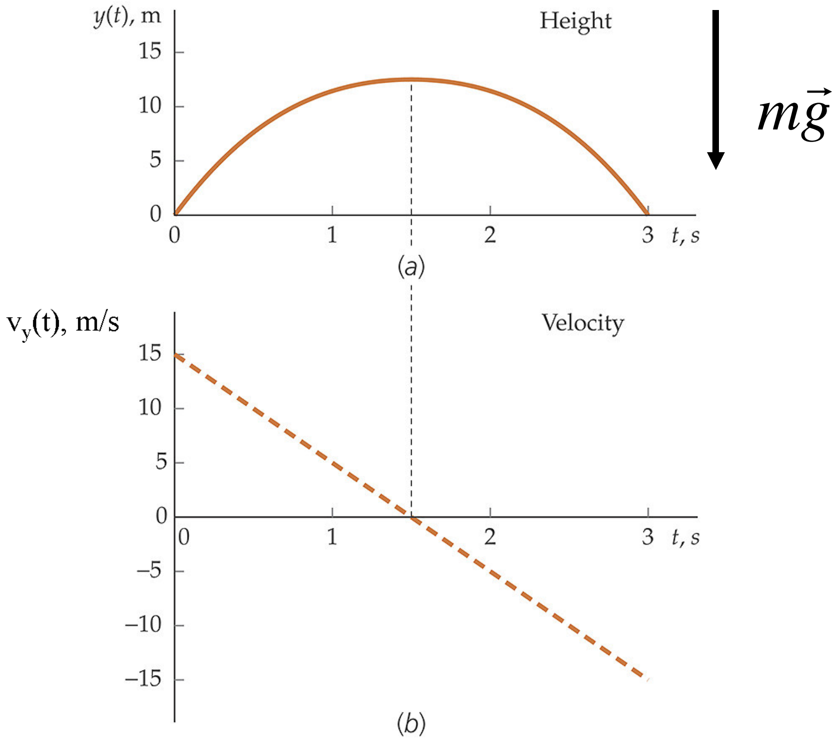


The mass m finds itself in a gravitation force field designated by $m\vec{g}$.

Gravity is a conservative force field which means that it has an associated potential energy.

Upon rising in the gravitational field the mass's KE goes to 0 and then returns to its original value as it reaches the horizontal.

Transfer of Energy Between KE and PE

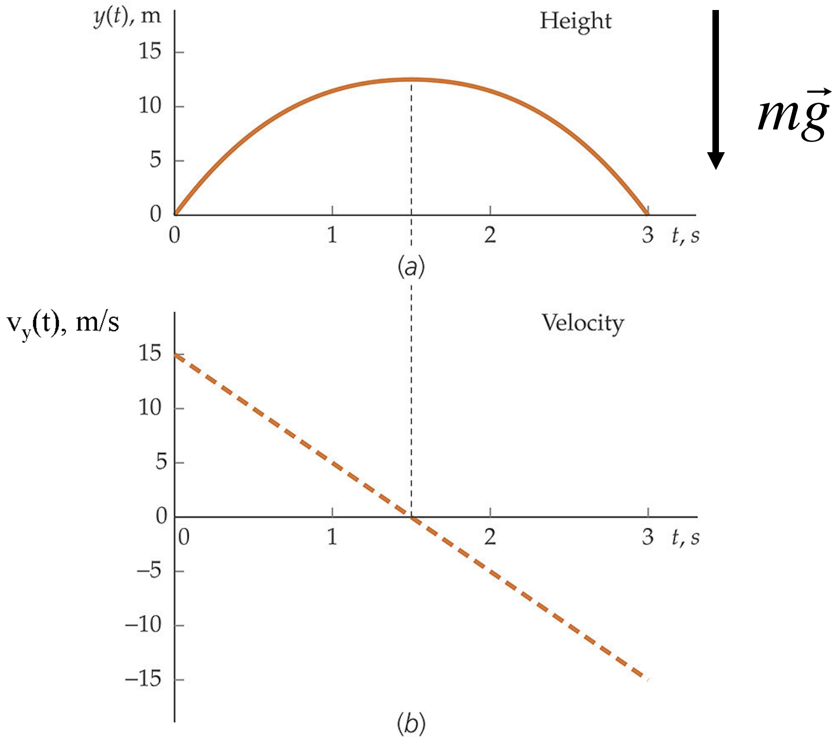


By doing work against gravity the mass's KE was converted into gravitational potential energy.

On the way back down the gravitational field did work on the mass and restored its KE back to its original value.

The value of the total energy of the mass remained constant as it moved from KE to PE and back to KE once more. In between the energy was part KE and part PE with a constant total

Transfer of Energy Between KE and PE



From the kinematic point of view the final velocity is in the opposite direction of the initial velocity, however their magnitudes are identical.

But KE is a scalar

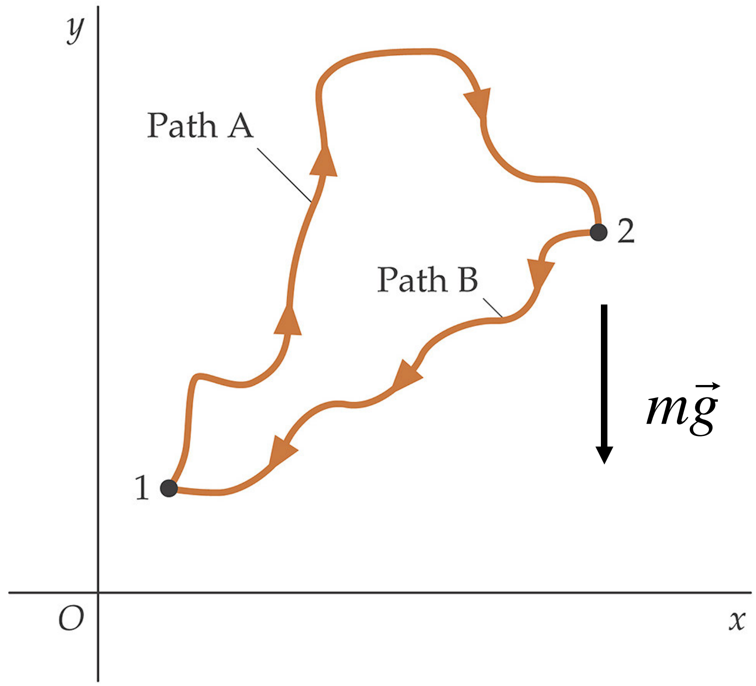
$$KE = \tfrac{1}{2} m v^2$$

and only depends on the magnitude of the velocity.

Potential Energy

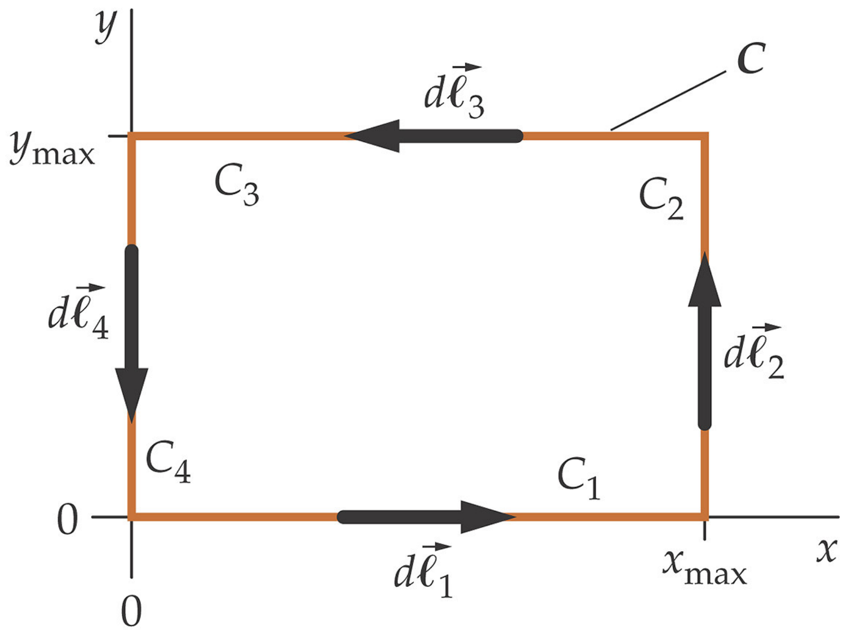
- In addition to possessing a potential energy function a conservative force field has another interesting property.
- The work done on an object, by a conservative force field, in moving the object in the field, from point 1 to point 2, is independent of the pathway between those two points.

Work Done is Independent of the Path



Assume that the pathways above are situated in a uniform and vertical gravitational field. The work done on the object depends on the endpoints, not how the object got there.

Closed Loop Path



If the initial point and the final point of the pathway are identical then the work done is zero. This is another characteristic of a conservative field. The work around a closed path is zero.

Work in a Conservative Field

$$Work = \int_{1}^{2} \vec{F} \bullet d\vec{l} = -\Delta U$$

$$\Delta U = U_2 - U_1 = -\int_{1}^{2} \vec{F} \bullet d\vec{l}$$

$$dU = -\vec{F} \bullet d\vec{l}$$

- For conservative forces it is much easier to use the scalar potential (U) to solve problems that it is to use vector force (***F***).
- *The work done along a path is reduced to evaluating the potential function (U) at the end points of that path*.

Total Mechanical Energy

The Total Mechanical Energy of a system is defined as the sum of the total Kinetic Energy and the total Potential Energy. For the gravitational force, near the surface of the earth, the Total Mechanical Energy is given as follows:

$$E_T = KE + PE = \tfrac{1}{2}mv^2 + mgh$$

In the absence of non-conservative forces the Total Mechanical Energy is conserved, i.e. it remains constant over time.

The Conservation of Total Mechanical Energy

The conservation of total mechanical energy of a system is very useful in problem solving.

In kinematics we tended to follow a system in time. With conservation of energy we observe a system before and after an event and are still be able to determine information about the system

$$E_T^{Before} = E_T^{After}$$

$$KE_i + PE_i = KE_f + PE_f$$

$$\tfrac{1}{2}mv_i^2 + mgh_i = \tfrac{1}{2}mv_f^2 + mgh_f$$

The Conservation of Total Mechanical Energy

By rearranging the symbols we can write the same relationships as follows:

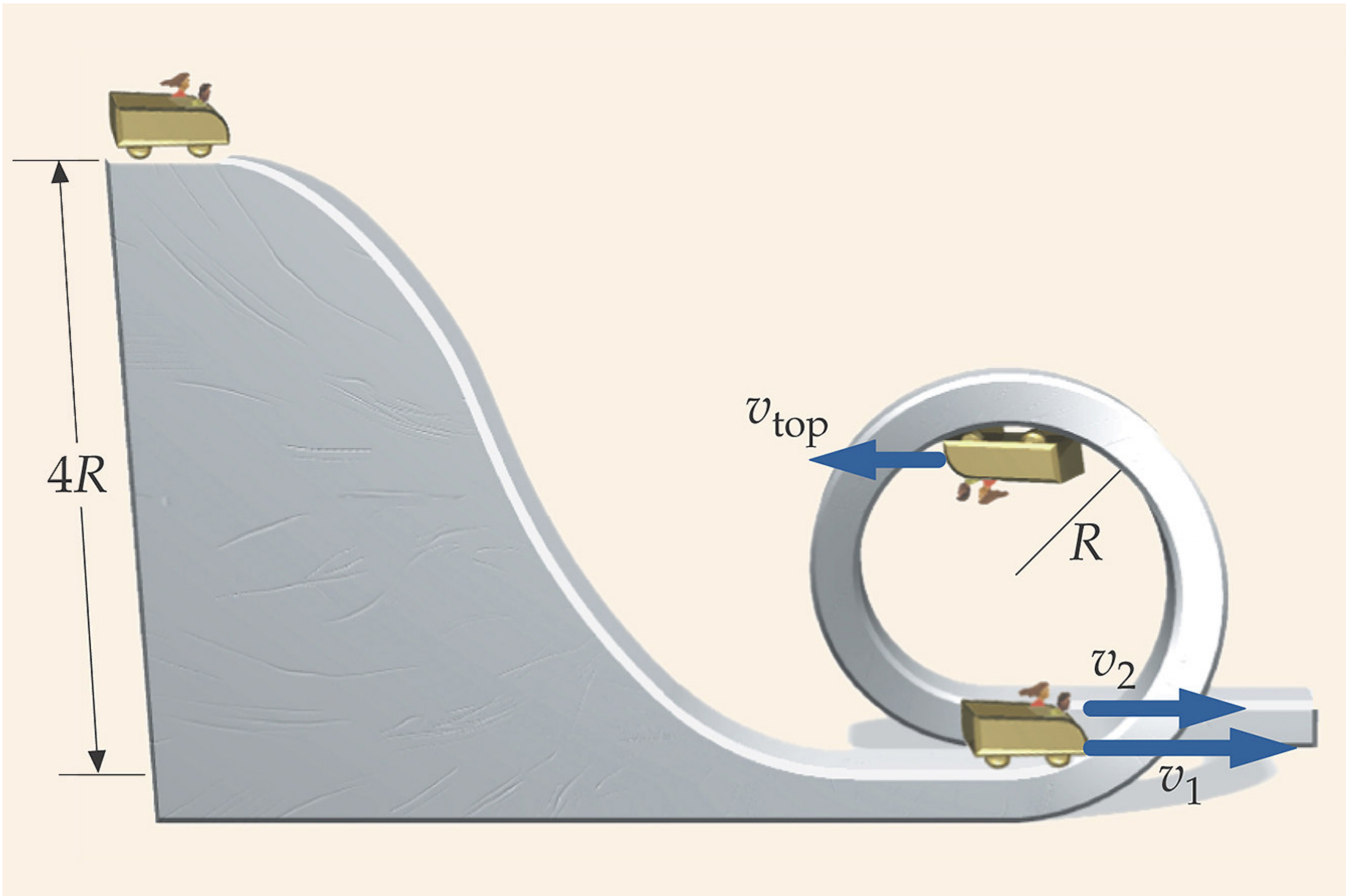
$$\Delta E_T = E_T^{After} - E_T^{Before} = 0$$

$$\Delta E_T = \Delta KE + \Delta PE = \left(KE_f - KE_i\right) + \left(PE_f - PE_i\right) = 0$$

$$\Delta E_T = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2 + mgh_f - mgh_i = 0$$

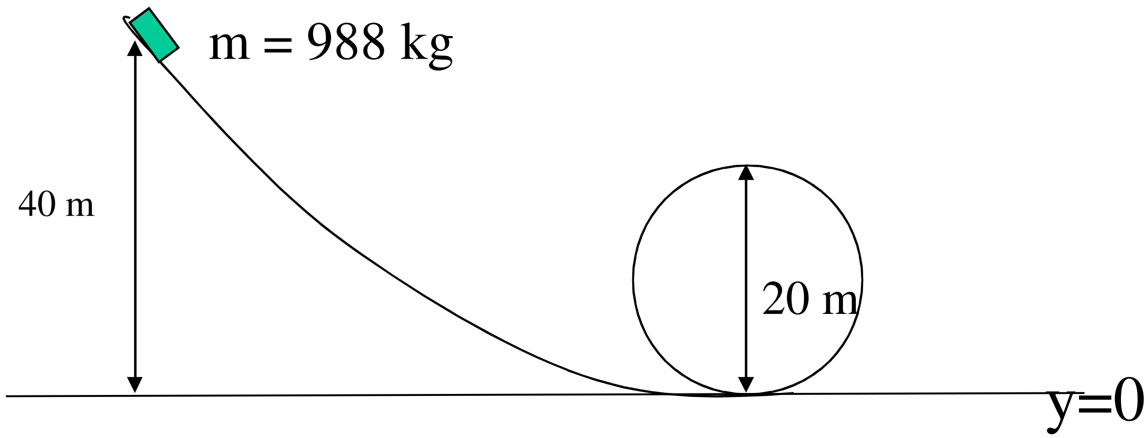
The first formulation is saying that conservation means Before = After while the second formulation is saying the change (“the Δ”) is zero - Both mean the same thing.

Roller Coaster Problem



Roller Coaster Problem

A roller coaster car is about to roll down a track. Ignore friction and air resistance.



(a) At what speed does the car reach the top of the loop?

$$E_i = E_f$$

$$U_i + K_i = U_f + K_f$$

$$mgy_i + 0 = mgy_f + \frac{1}{2}mv_f^2$$

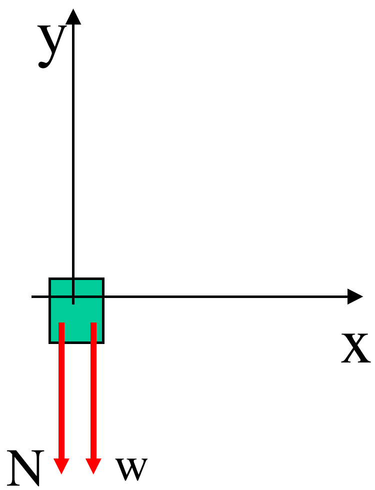
$$v_f = \sqrt{2g\left(y_i - y_f\right)} = 19.8\ \text{m/s}$$

Roller Coaster Problem

Example continued:

(b) What is the force exerted on the car by the track at the top of the loop?

FBD for the car:



Apply Newton's Second Law:

$$\sum F_y = -N - w = -ma_r = -m\frac{v^2}{r}$$

$$N + w = m\frac{v^2}{r}$$

$$N = m\frac{v^2}{r} - mg = 2.9\times10^4 \text{ N}$$

Roller Coaster Problem

Example continued:

(c) From what minimum height above the bottom of the track can the car be released so that it does not lose contact with the track at the top of the loop?

Using conservation of mechanical energy:

$$E_i = E_f$$

$$U_i + K_i = U_f + K_f$$

$$mgy_i + 0 = mgy_f + \frac{1}{2} m v_{\min}^2$$

Solve for the starting height $$y_i = y_f + \frac{v_{\min}^2}{2g}$$

Roller Coaster Problem

Example continued:

What is v_{min}? $v = v_{min}$ when N = 0. This means that

$$N + w = m\frac{v^2}{r}$$

$$w = mg = m\frac{v_{\min}^2}{r}$$

$$v_{\min} = \sqrt{gr}$$

The initial height must be

$$y_i = y_f + \frac{v_{\min}^2}{2g} = 2r + \frac{gr}{2g} = 2.5r = 25.0\ \mathrm{m}$$

Roller Coaster Problem

For a clearer approach stay at the v^2 level

$$Bottom \quad v_f^2 = 2gh$$

$$Top \quad \frac{mv^2}{R} - mg = 0$$

$$v_{min}^2 = gR$$

$$Min\ at\ Top$$

$$v_f^2 = 2gD + v_{min}^2$$

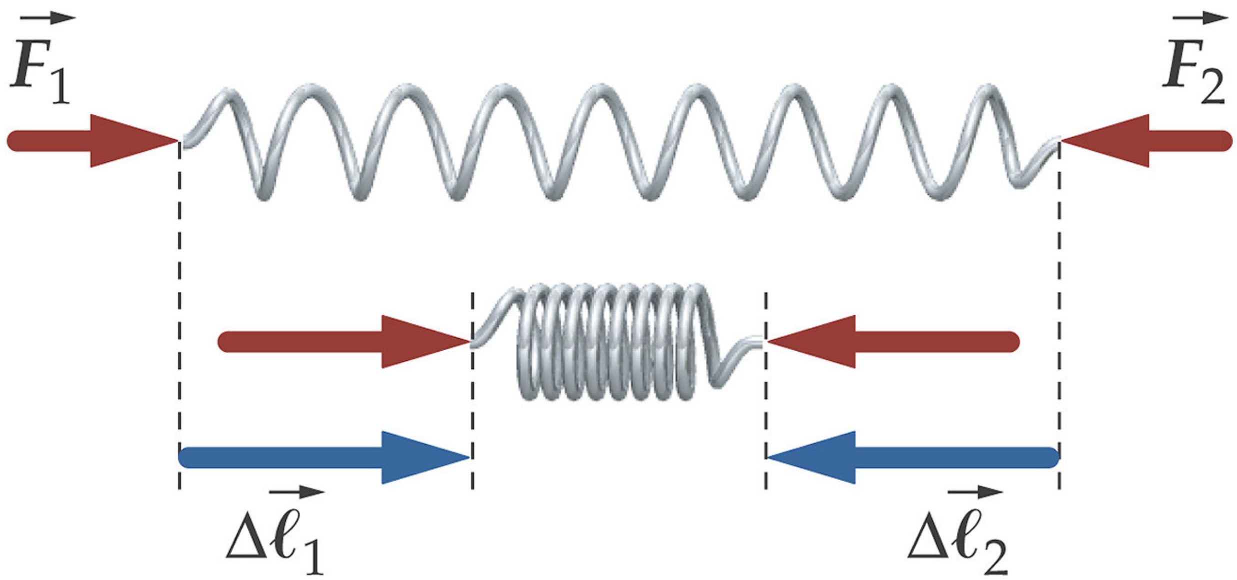
$$2gh = 2gD + gR$$

$$2gh = 4gR + gR = 5gR$$

$$h = \tfrac{5}{2}R$$

The Spring Force is also a Conservative Force

The Spring Also Has a Potential Function

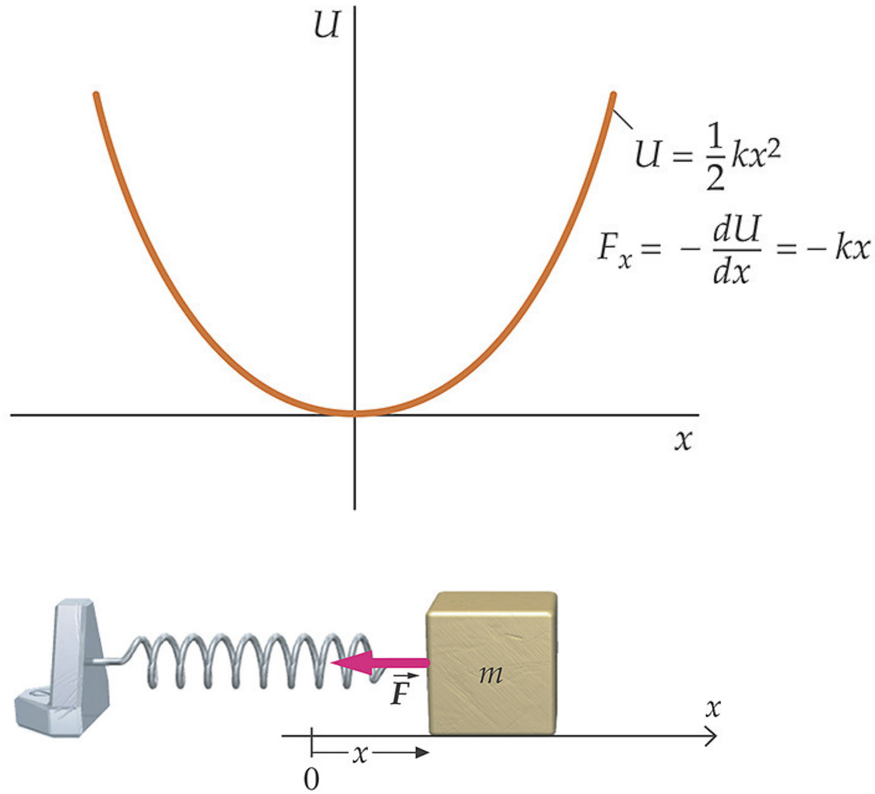


Spring Potential Energy Function

$$\vec{F} = -\vec{\nabla} U = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

The scalar potential function is very useful and easier to deal with than its associated vector force.

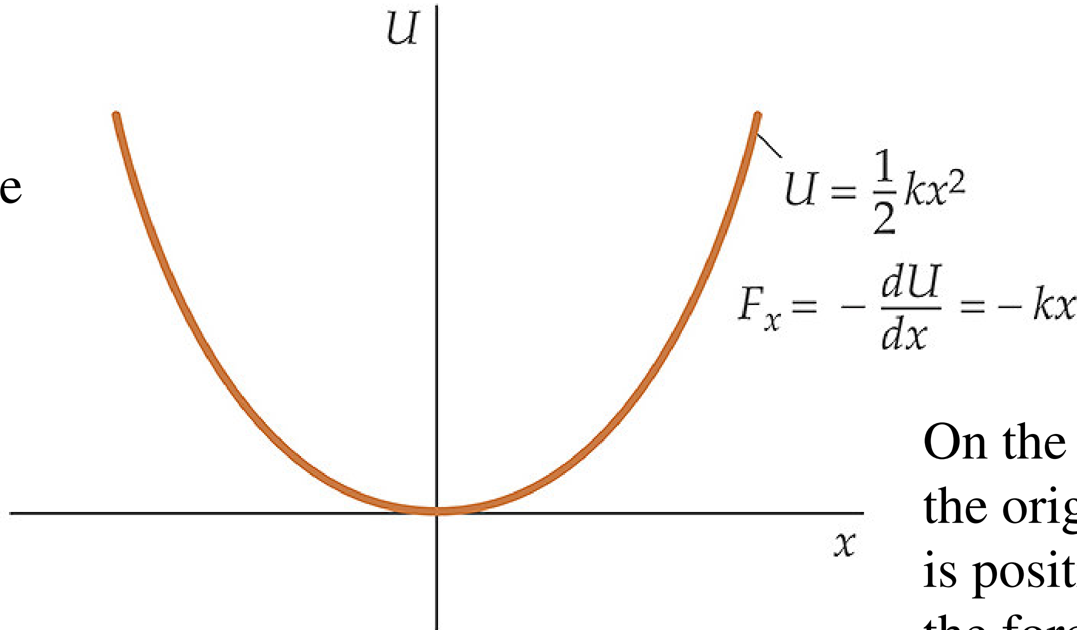
In most advanced work potentials are preferred to forces.



Spring Potential Energy Function

The force is the negative of the slope of the potential energy function

On the left side of the origin the slope is negative and so the force points in the +x-direction.

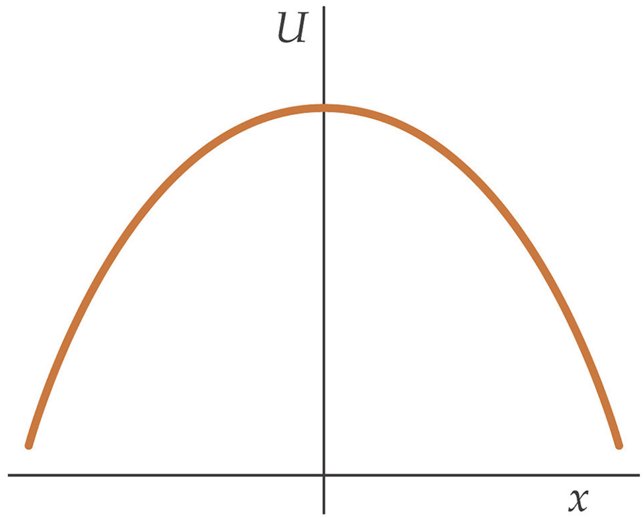


On the right side of the origin the slope is positive and so the force points in the -x-direction.

The system behaves like it wants to minimize its potential energy. The shape of potential curve represents a ***stable equilibrium***

Unstable Equilibrium

On the left side of the origin the slope is positive and so the force points in the -x-direction.

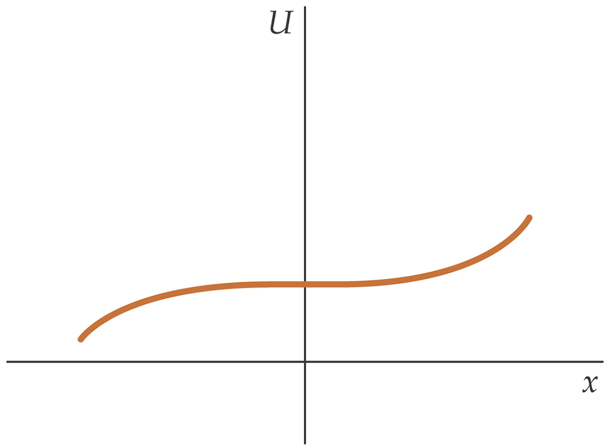


On the right side of the origin the slope is negative and so the force points in the +x-direction.

This system also behaves like it wants to minimize its potential energy but that takes it away from the origin. The shape of potential curve represents an ***unstable equilibrium***

Neutral Equilibrium?

On the left side of the origin the slope is positive and so the force points in the -x-direction.



On the right side of the origin the slope is positive and so the force points in the -x-direction.

- The neutral labeling of this system depends on the center section of the potential curve being flat and the systems motion relative to this flat section.
- Systems always want to minimize their potential energy and ultimately that leads to the left. If released at the right a ball would roll right through the origin and keep on going.
- To be truly neutral the curve needs to be flat everywhere.

Conservation of Energy Examples

Conservation of Energy Problems

- Vertical and Horizontal Springs
- Pendulum
- Springs on Incline Planes
- Springs, Incline Planes & Friction
- Free Fall onto a Spring Platform

The Bungee Problem

The primary goal is to find the acceleration on the jumper when he is at the lowest position. That is straight forward.

The problem comes in when you discover they haven't given you the value of the spring constant (K). That is what all the rest of the problem is about.

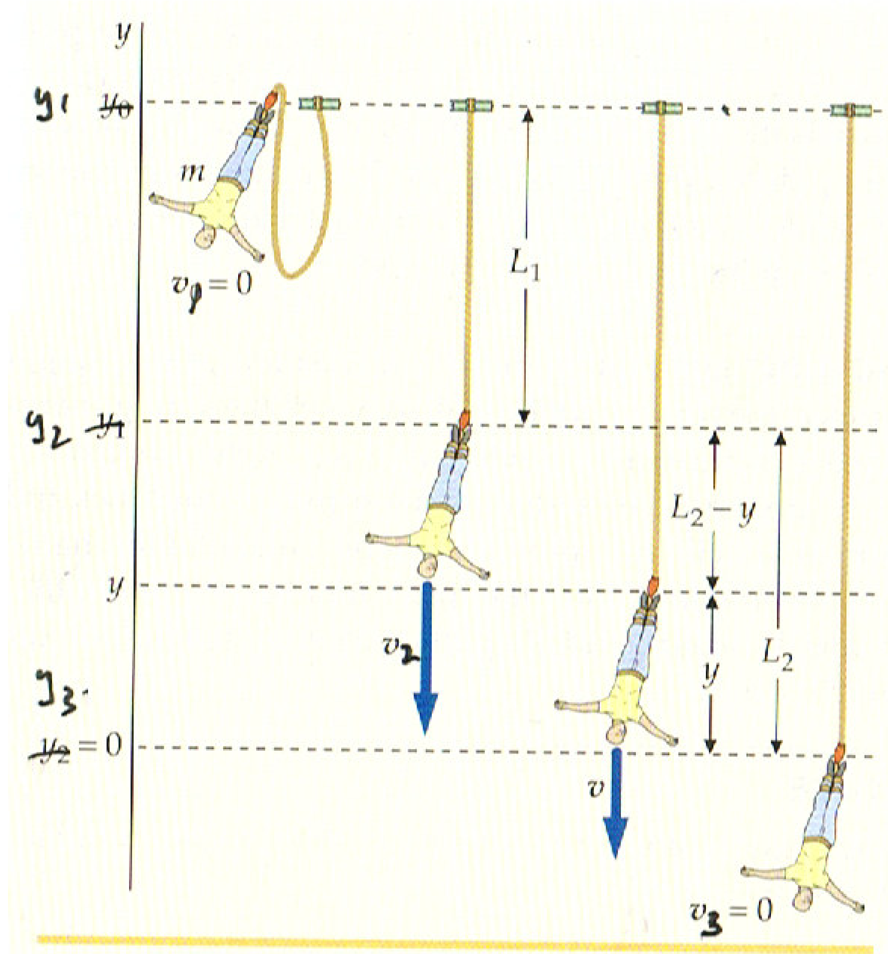


FIGURE 7-14

The Bungee Problem

You can find the spring constant by comparing level 1 with level 3, where in both cases the KE is zero.

The loss of GPE is converted into spring PE.

$$E_1 = E_3$$

$$mg(L_1 + L_2) = \tfrac{1}{2}m(0)^2 + mg(0) + \tfrac{1}{2}kL_2^2$$

$$mg(L_1 + L_2) = \tfrac{1}{2}kL_2^2$$

$$k = \frac{2mg(L_1 + L_2)}{L_2^2}$$

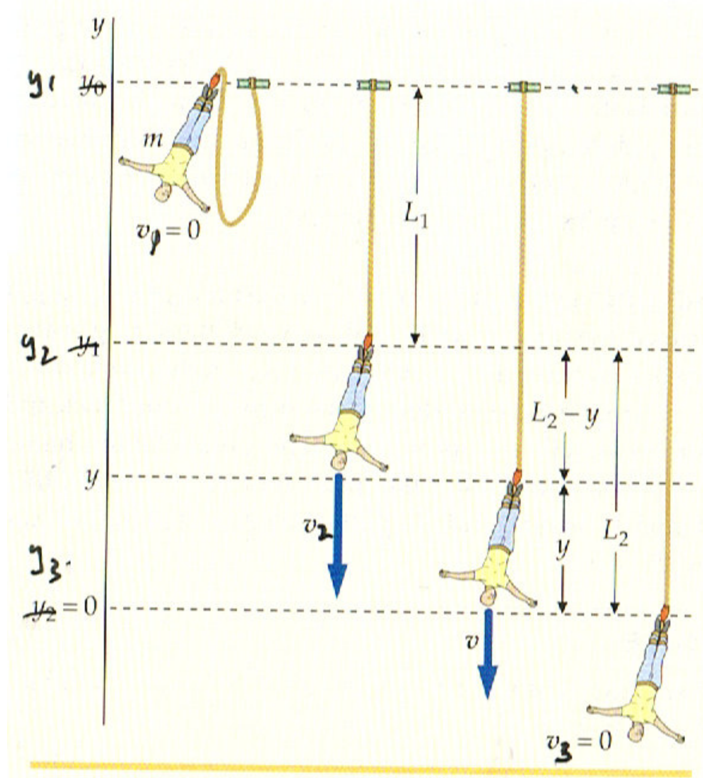


FIGURE 7-14

The Bungee Problem

You can find the net acceleration on the jumper with a simple force diagram. At level 3 there are only two forces acting: the bungee cord (spring) and gravity.

$$\sum F_y = kL_2 - mg = ma$$

$$a = -g + k\frac{L_2}{m}$$

$$a = -g + \left(\frac{2mg(L_1 + L_2)}{L_2^2}\right)\frac{L_2}{m}$$

$$a = -g + \left(\frac{2g(L_1 + L_2)}{L_2}\right)$$

$$a = g\left(1 + 2\frac{L_1}{L_2}\right)$$

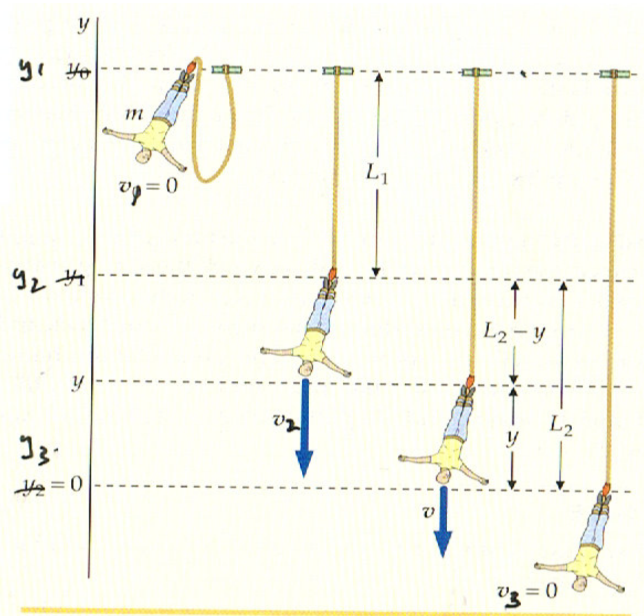
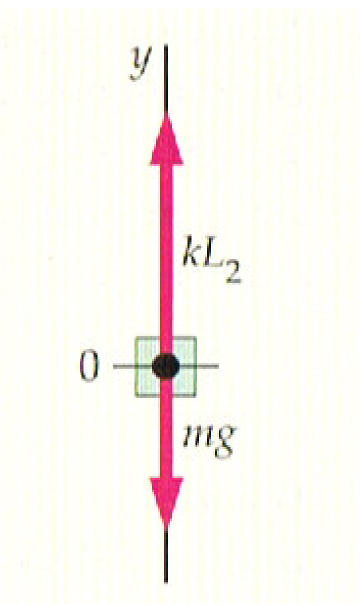
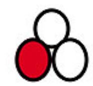
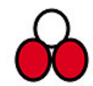
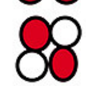


FIGURE 7-14



Mass and Energy

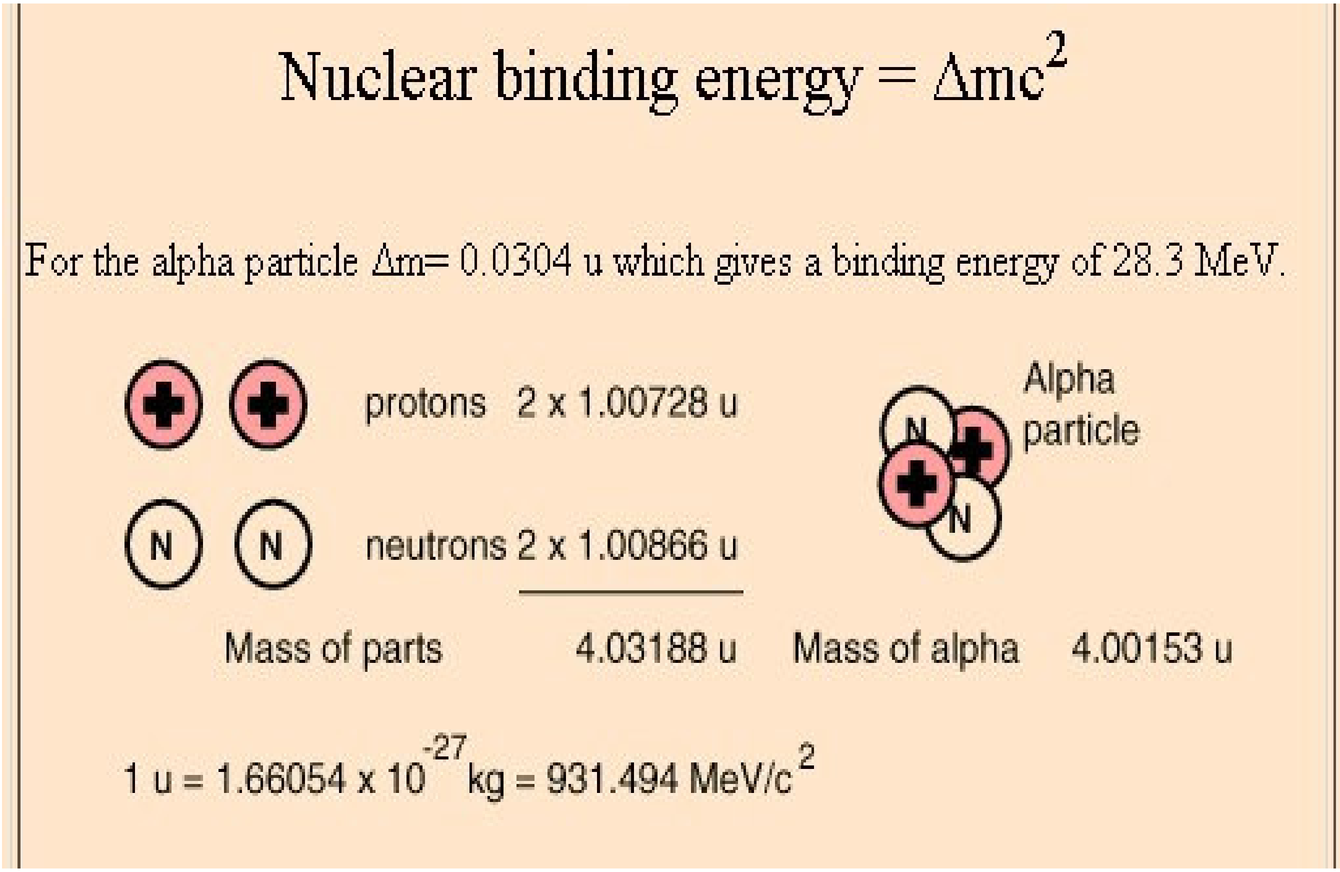
Table 7-1 Rest Energies* of Some Elementary Particles and Light Nuclei†

Particle	Symbol	Rest Energy (MeV)	
Electron	e^-	0.5110	
Positron	e^+	0.5110	
Proton	p	938.272	
Neutron	n	939.565	
Deuteron	d	1875.613	
Triton	t	2808.921	
Helion	h	2808.391	
Alpha particle	α	3727.379	

* Table values are from 2002 CODATA (except for the value for the triton).

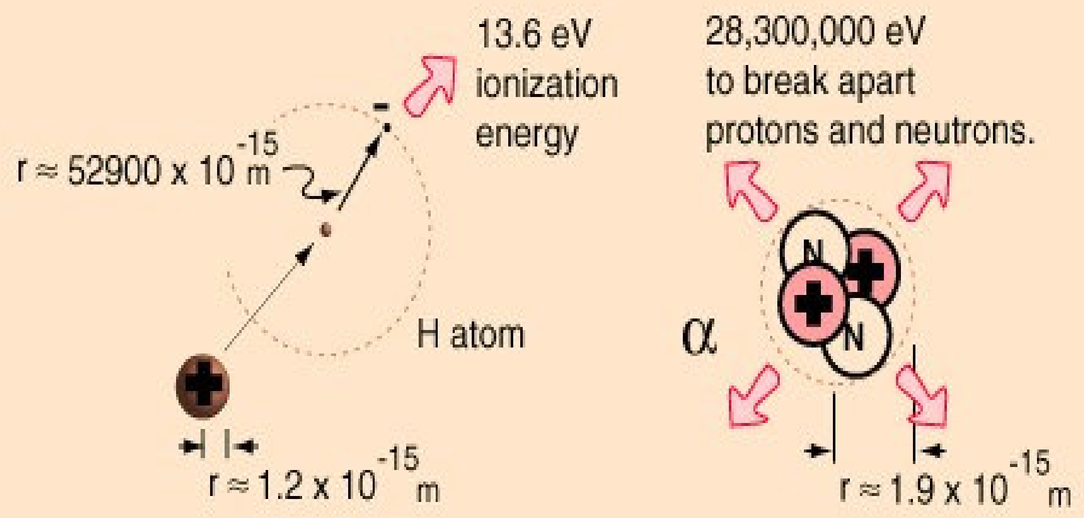
† The proton, deuteron and triton are identical with the nuclei of ^{1}H, ^{2}H, and ^{3}H, respectively, and the helion and alpha particle are identical with the nuclei of ^{3}He and ^{4}He, respectively.

Nuclear Binding Energy



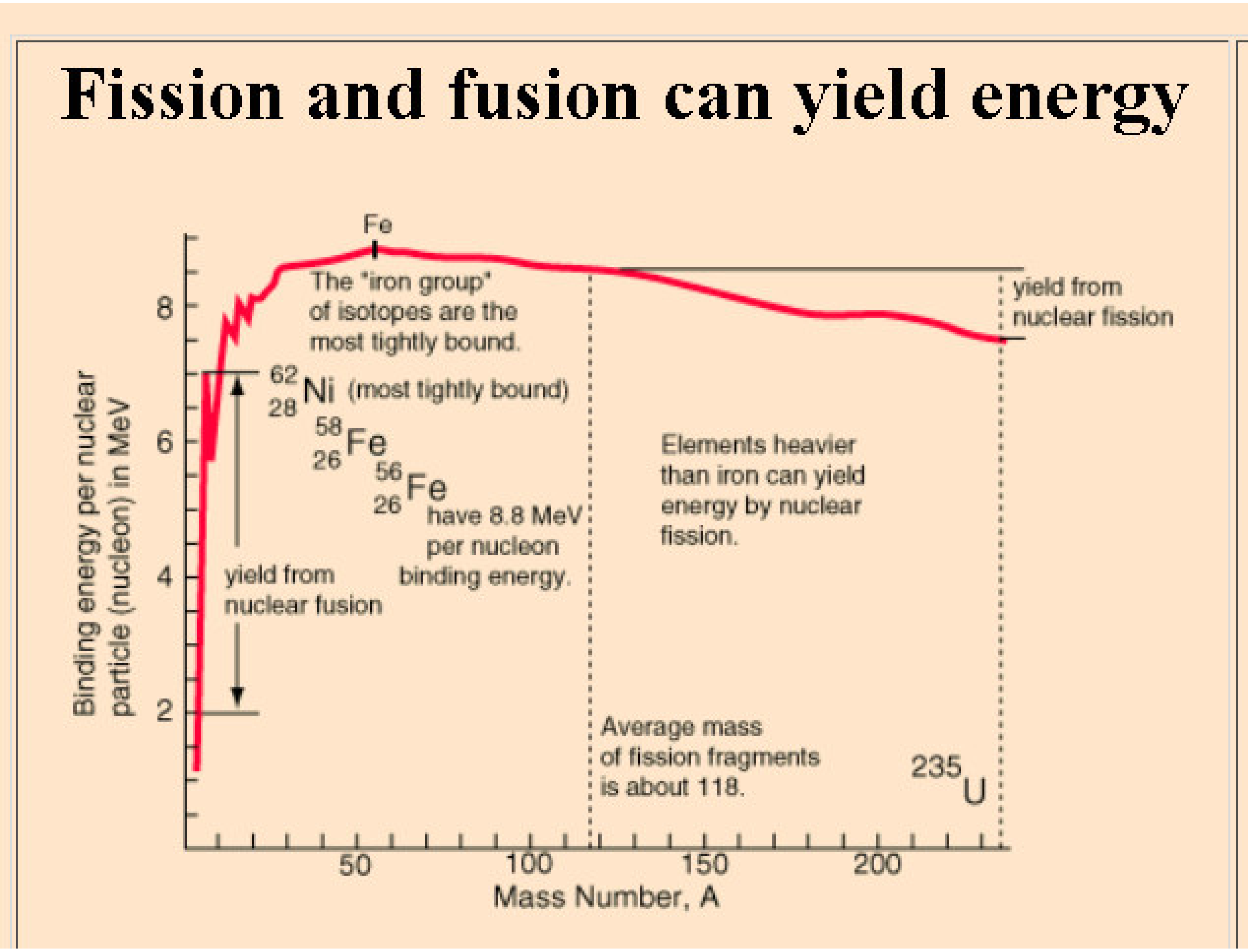
Nuclear Binding Energy

The enormity of the nuclear binding energy can perhaps be better appreciated by comparing it to the binding energy of an electron in an atom. The comparison of the alpha particle binding energy with the binding energy of the electron in a hydrogen atom is shown below. The nuclear binding energies are on the order of a million times greater than the electron binding energies of atoms.



Comparison of atomic and nuclear scales and binding energy

Nuclear Binding Energy Curve



A useful rule of thumb: the average nuclear binding energy is

8 Mev per nucleon.

Extra Slides