Chapter 13

Gravity

Gravity

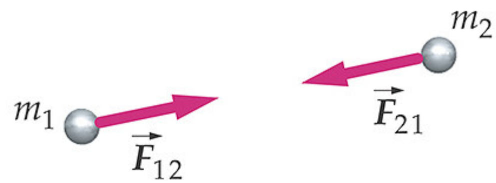
Kepler’s Laws

Newton’s Law of Gravity

Gravitational Potential Energy

The Gravitational Field

Characteristics of Gravity



Acts along a line connecting the center of mass of the two bodies.

It is a central force which implies that it is a conservative force.

It is only attractive. There are no repulsive gravitational forces, i.e. there is no anti-gravity.

Its magnitude is proportional to the two masses

$$F_g \; \alpha \; m_1 m_2$$

Characteristics of Gravity

The strength of the gravitational force decreases with distance $F_g \; \alpha \frac{1}{r^2}$

Therefore $F_g \; \alpha \frac{m_1 m_2}{r_{12}^2}$

The gravitational force is $F_g = G\frac{m_1 m_2}{r_{12}^2}$

The value of G was determined from the data of Henry Cavendish: $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

Characteristics of Gravity

Gravity is the weakest of all the forces. It takes large masses to produce a gravitational force that is noticeable.

Gravity acts most on the grand astronomical scale where large masses are found.

However, it is the exact balance of the positive and negative charges that effectively cancels the long range electric fields that allows gravity to be observed even at these large astronomical distances.

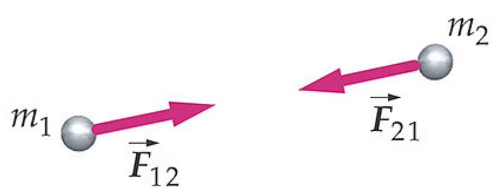
Characteristics of Gravity

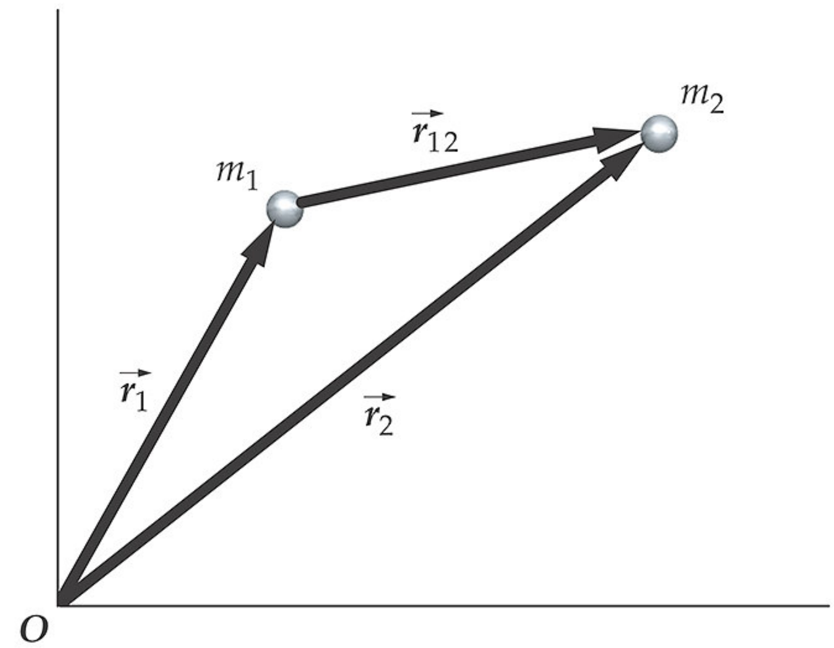
Gravity is a vector quantity

$$\vec{F}_{12} = G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$$

$$\hat{r}_{12} = -\hat{r}_{21}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$





Early Astronomers

Tycho Brahe (1546-1601)

- He gathered observational data over a twenty year period of measuring planet positions.

Johannes Kepler (1571-1630)

- Kepler worked with Brahe
- He inherited Brahe's data when he died.
- Kepler deduced his three laws from this data

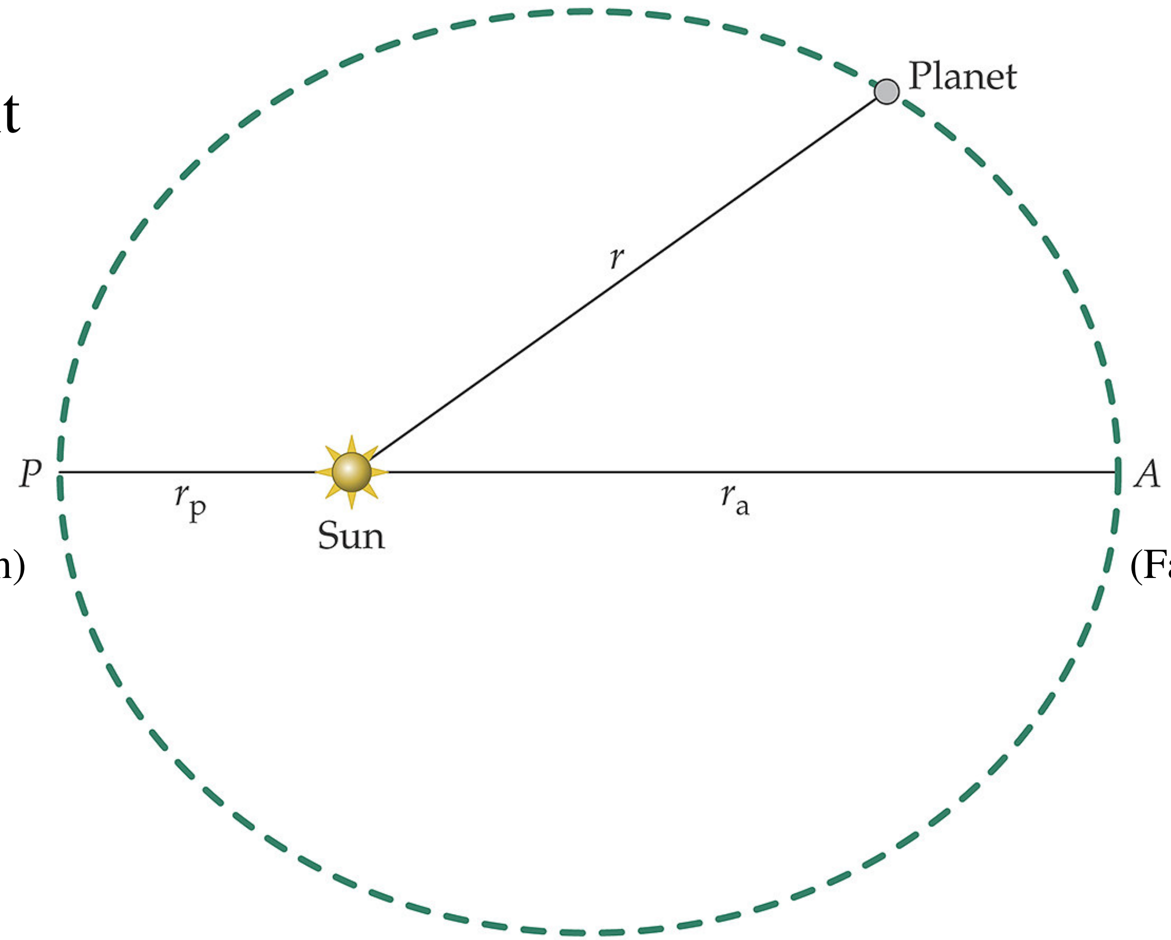
Kepler's Laws

1. Law of Orbits - ellipse
2. Law of Areas - equal area in equal times
3. Law of Periods -

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right) r^3$$

Kepler's 1st Law

Elliptical
shaped orbit



Perigee

(Closest approach)

Apogee

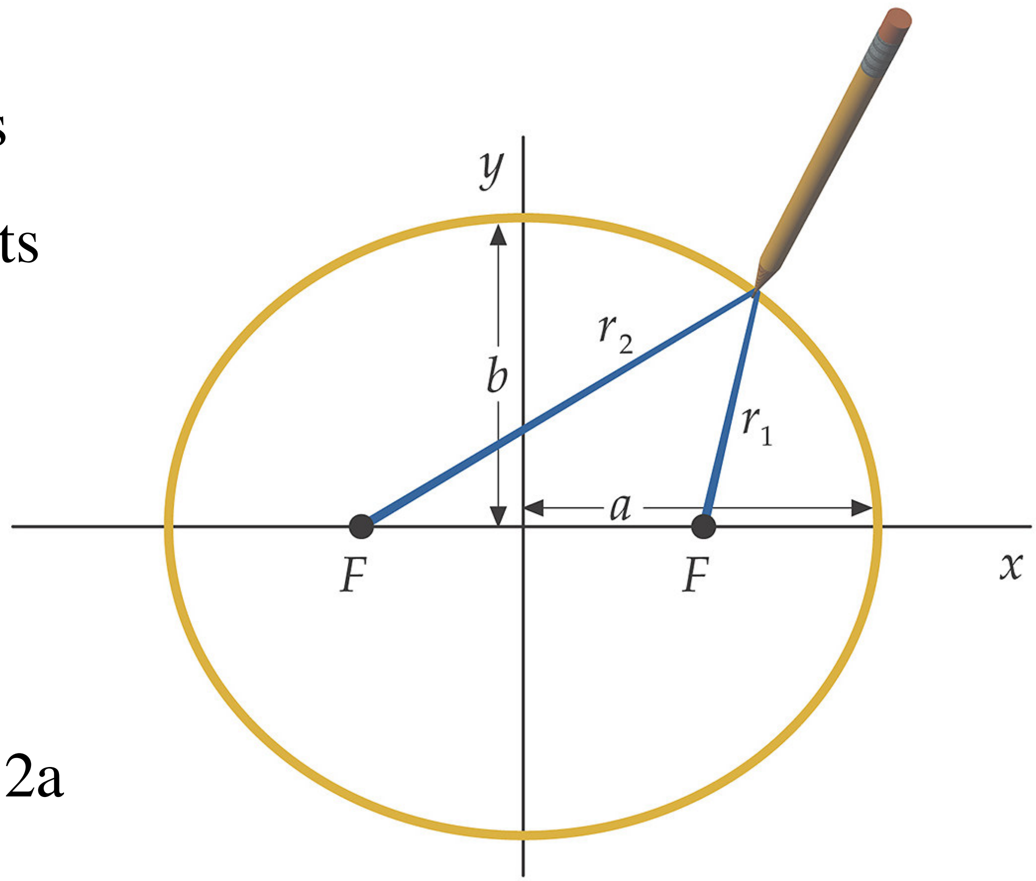
(Farthest approach)

Construction of an Ellipse

a = semi major axis

b = semi minor axis

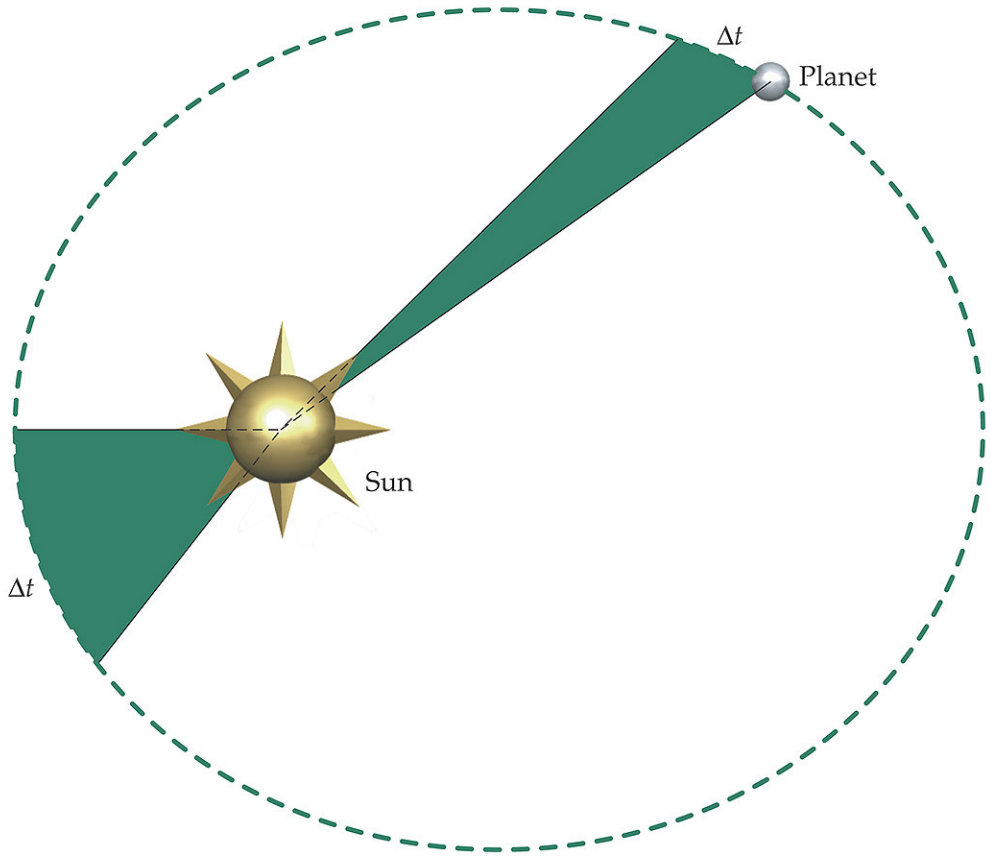
F are the focal points



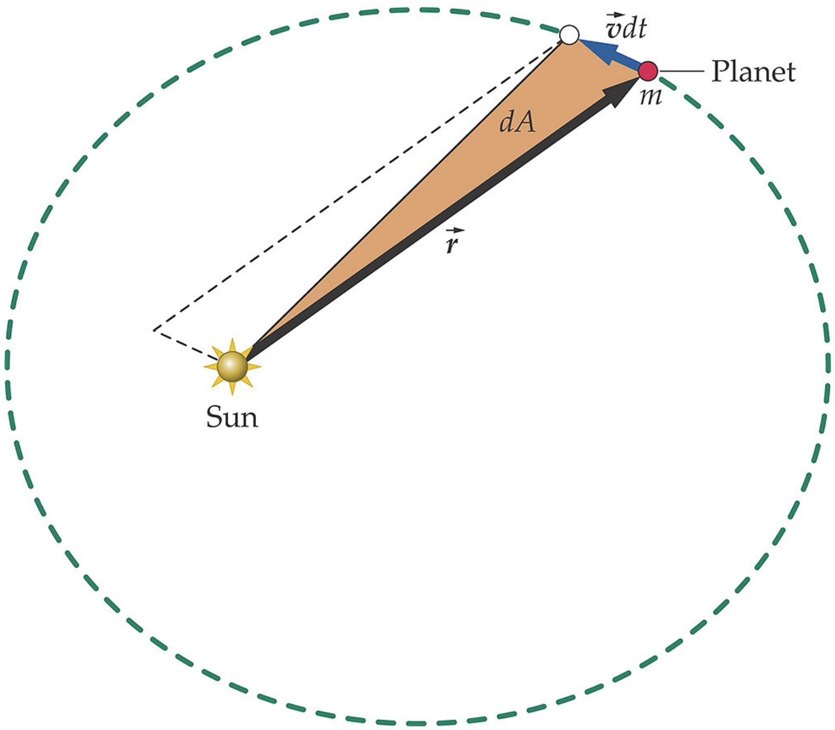
$r_1 + r_2 = 2a$

Kepler's 2nd Law

Equal areas are swept out in equal times



Derivation of Kepler's 2nd Law



$$dA = \tfrac{1}{2}\left|\vec{r} \times \vec{v}dt\right|$$

$$dA = \frac{\left|\vec{r} \times m\vec{v}\right|}{2m}dt$$

$$\frac{dA}{dt} = \frac{L}{2m}$$

F is a central force so L is conserved and therefore

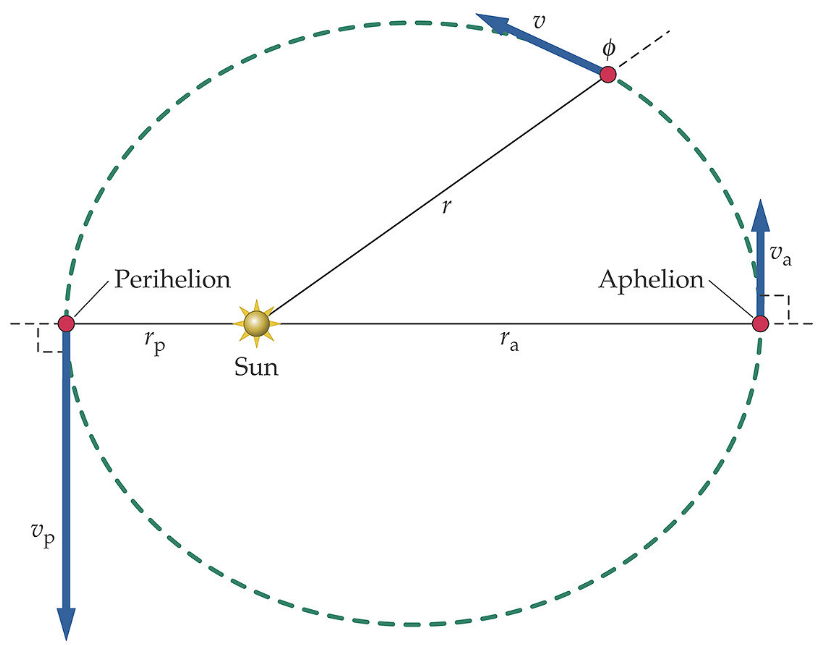
$$\frac{dA}{dt} = constant$$

Conservation of Angular Momentum

L = mrvsinφ = constant, therefore rvsinφ is constant with r, v and φ all changing but the product of the three remaining constant in value

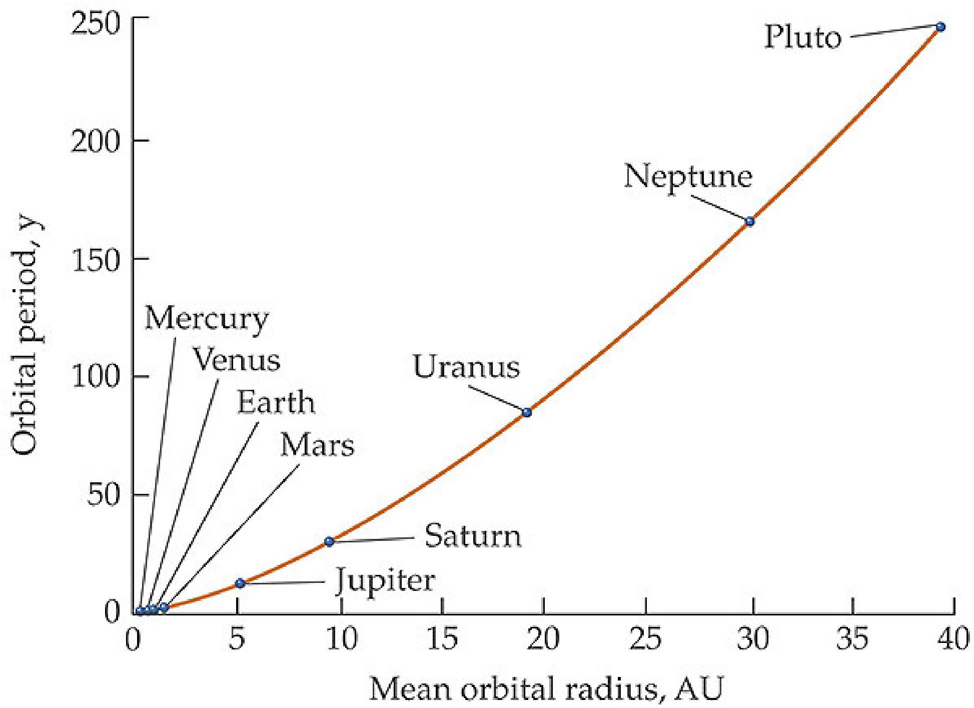
Evaluate the expression at $\varphi = 90^0$ where sinφ = 1, at apogee and perigee

$$r_a v_a = r_p v_p$$



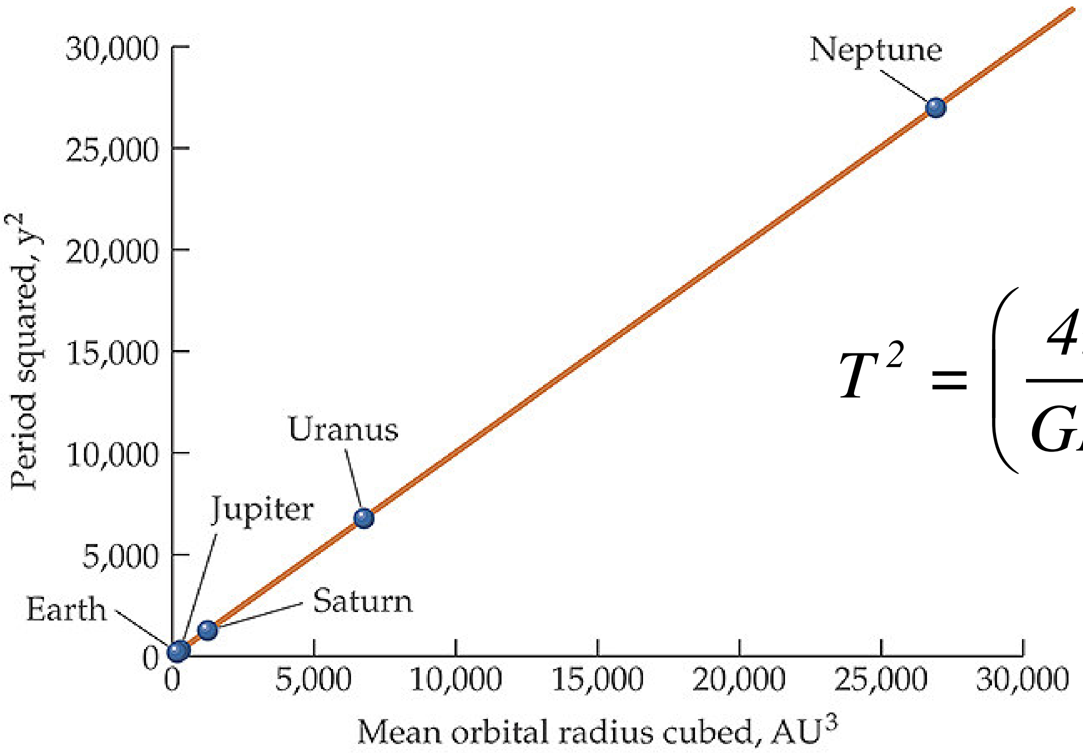
Kepler's Laws

Variation of periods with mean orbital radius



Kepler's 3rd Law

The Period Equation



$$T^2 = \left(\frac{4\pi^2}{GM_s}\right) r^3$$

Kepler's 3rd Law

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)r^3$$

Astronomical Unit = Mean radius of earth orbit around sun

1 AU = 1.50 x 10^{-11} m = 93.0 x 10^6 mi

For orbits around the **Sun** $T^2(Years) = R^3(AU)$

For other applications replace M_s in the equation above with the mass of the body that the object is orbiting around.

The Determination of G

The Determination of G

According to legend, i.e. one text book author copying the historical aspects of physics from the authors that came before him, Henry Cavendish made the first accurate measurement of G in 1745.

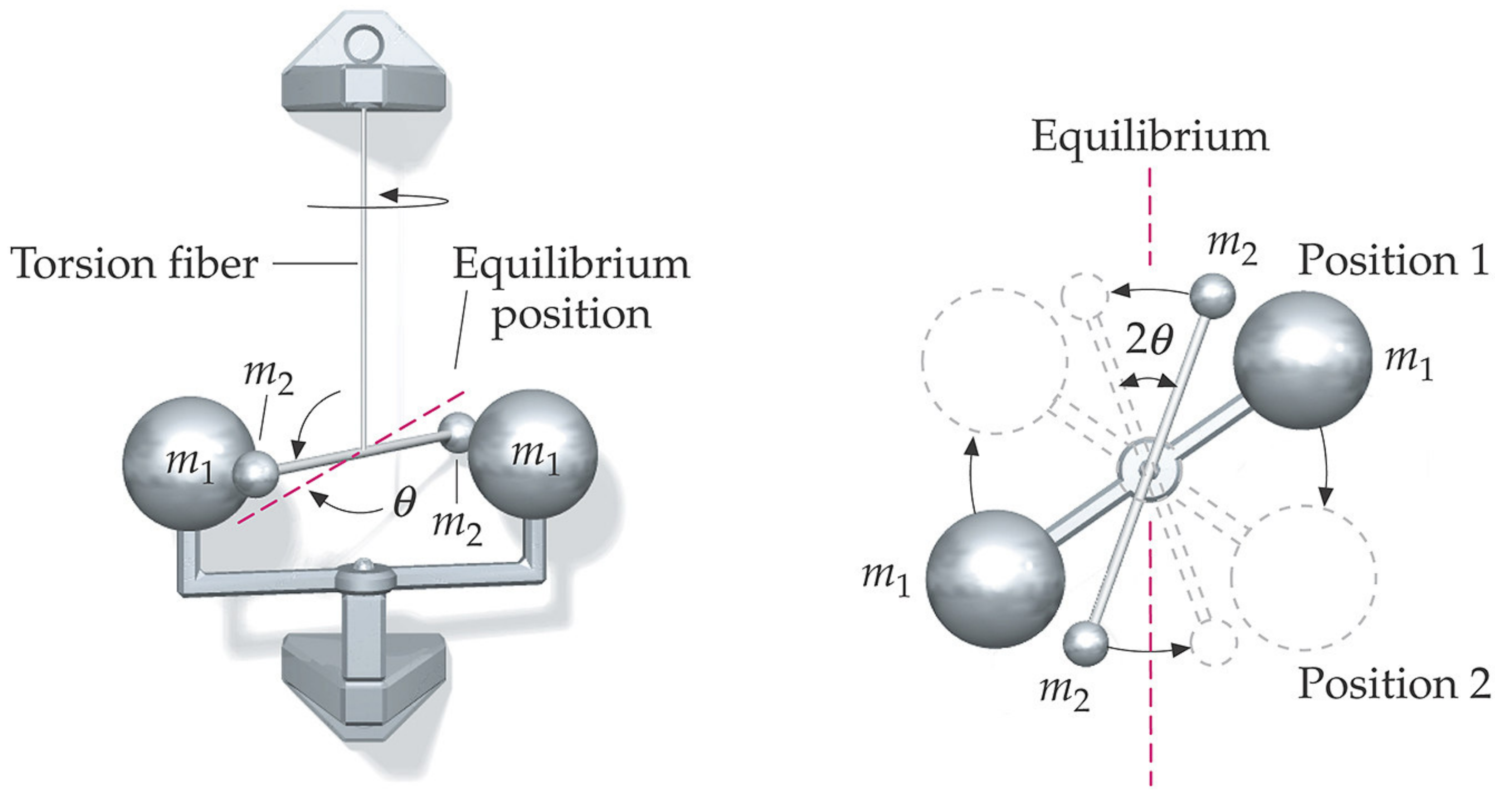
It turns out that the legend is incorrect. In fact, Cavendish wasn't even interested in measuring the value of G. Henry Cavendish wanted to measure the density of the earth.

75 years later another researcher was able to squeeze a value of G out of Cavendish's old data.

Moral of the story - take good notes while you're alive and you might still be making discoveries after you're dead.

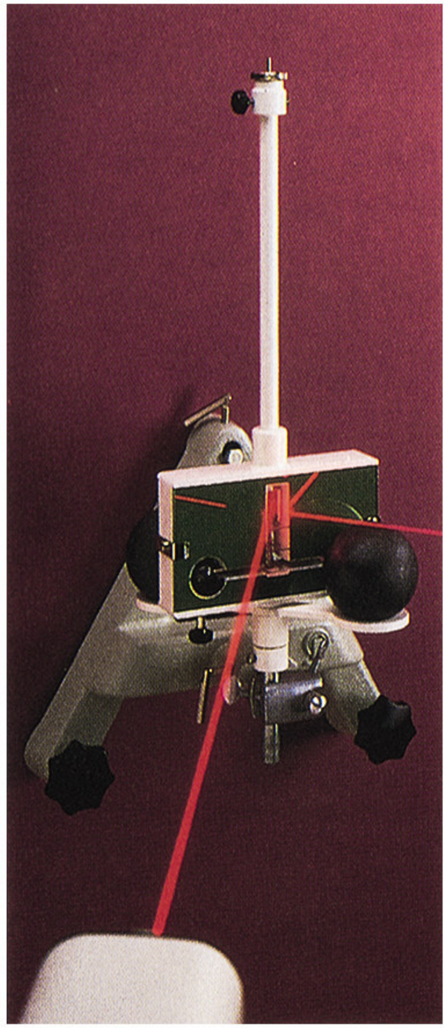
The Cavendish Experiment

(minus the environmental shielding)



(a) (b)

The Cavendish Experiment

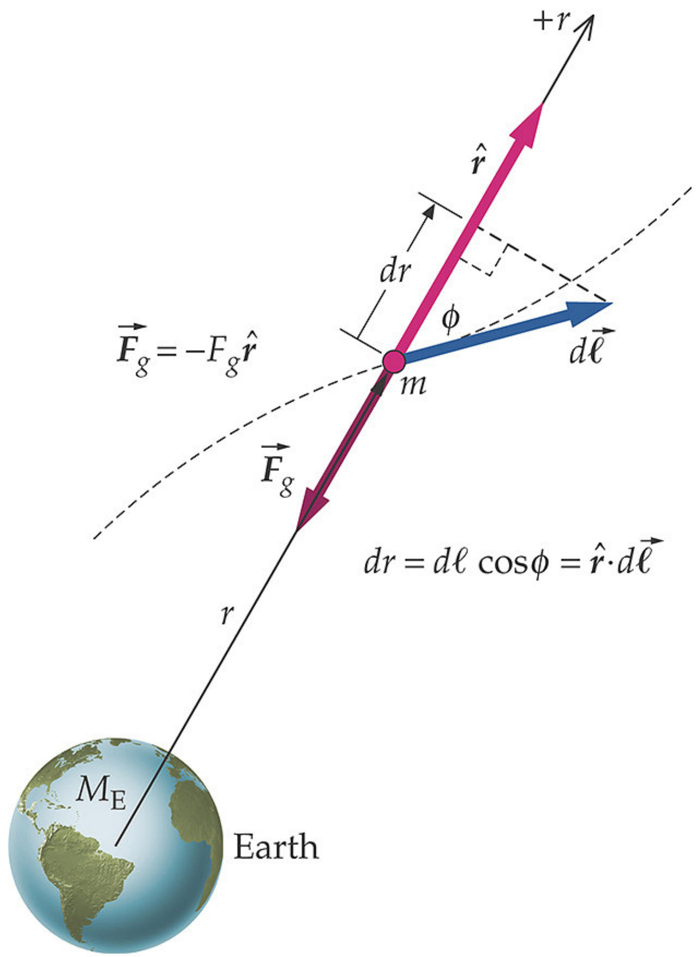


Actual Cavendish Experiment Data

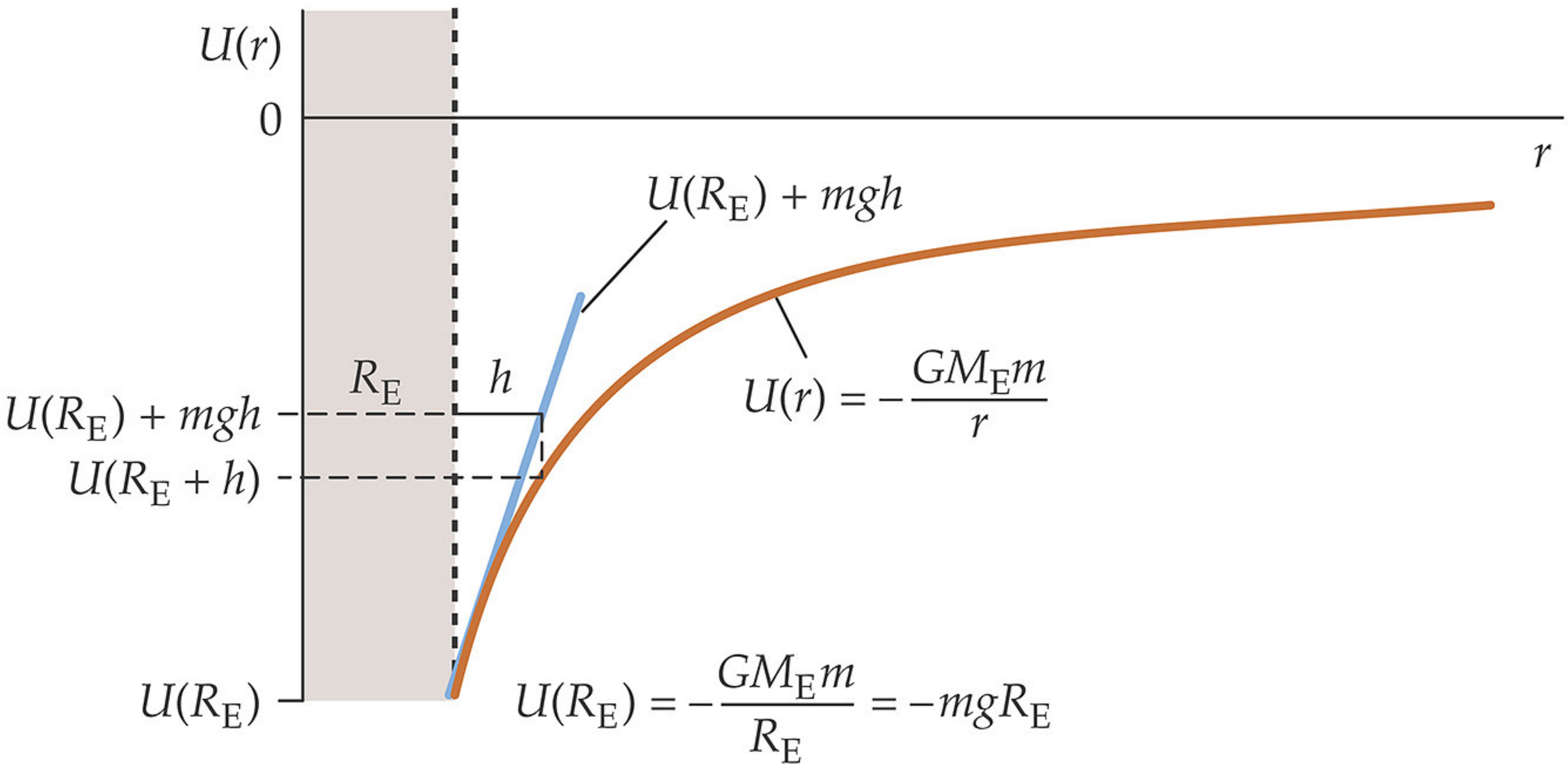
Screen Deflection - 1.5m away

Each tiny square is 1 minute.

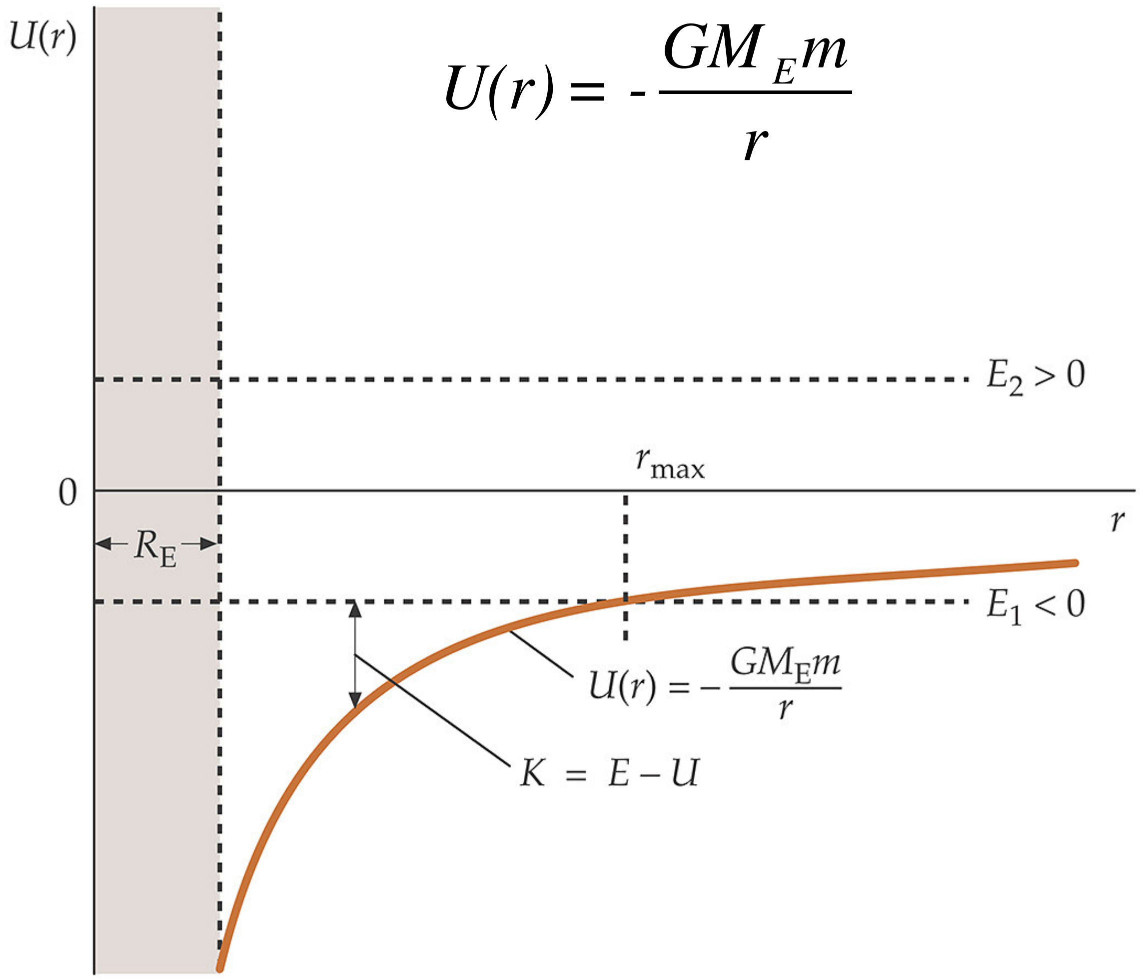
Gravitational Potential Energy



Bound State - Unbound State

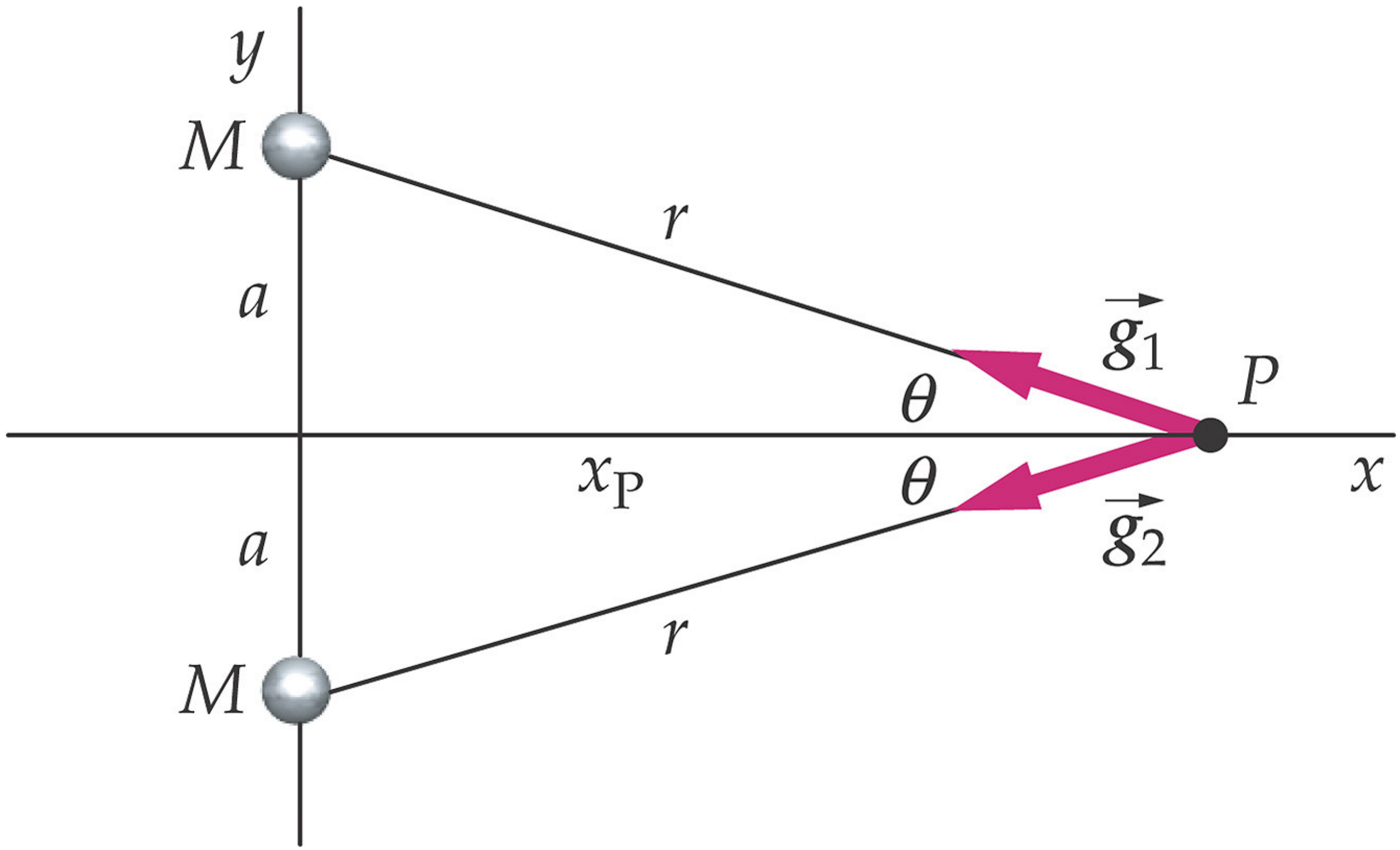


Escape Speed of a Projectile

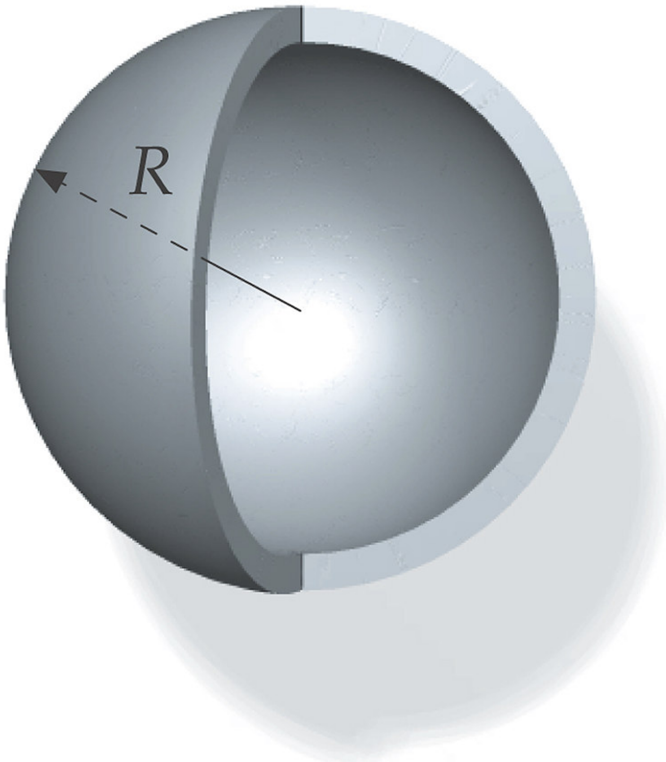


Gravitational Field

Preview of Electric Field Calc.-EPII



g-Field of a Thin Spherical Shell



$$\vec{g} = -\frac{GM}{r^2}\hat{r};\ r > R$$

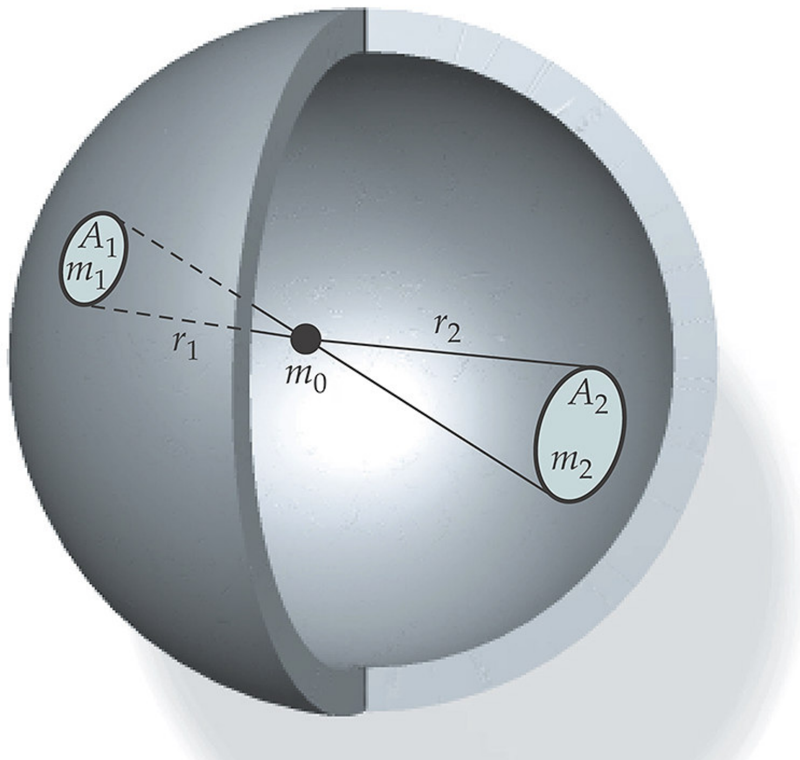
$$\vec{g} = 0;\ r < R$$

g outside the mass distribution is the same as if all the mass was concentrated at the CM.

Requires uniform mass distribution or a mass distribution that only varies with r.

Gravitational Field Inside a Hollow Sphere

$$\vec{F}_g = 0$$

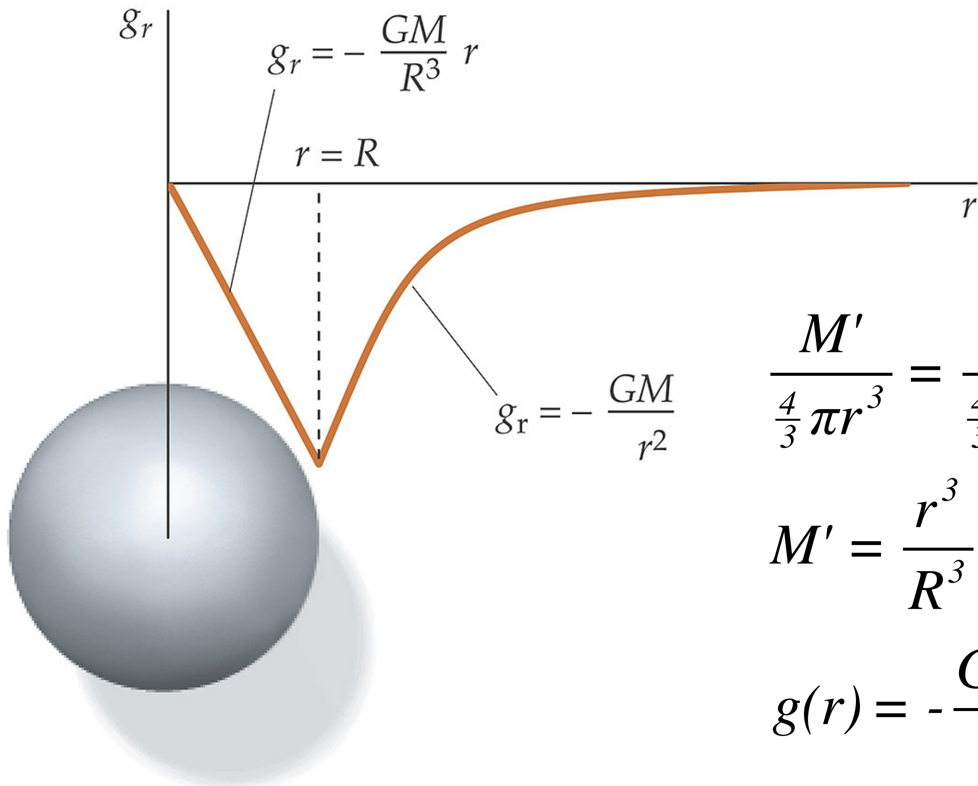


$$\frac{m_1}{m_2} = \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$

$$\frac{m_1}{r_1^2} = \frac{m_2}{r_2^2}$$

$$g = \frac{Gm_1}{r_1^2}$$

g-Field Distribution - Solid Sphere



$$\frac{M'}{\frac{4}{3}\pi r^3} = \frac{M}{\frac{4}{3}\pi R^3}$$

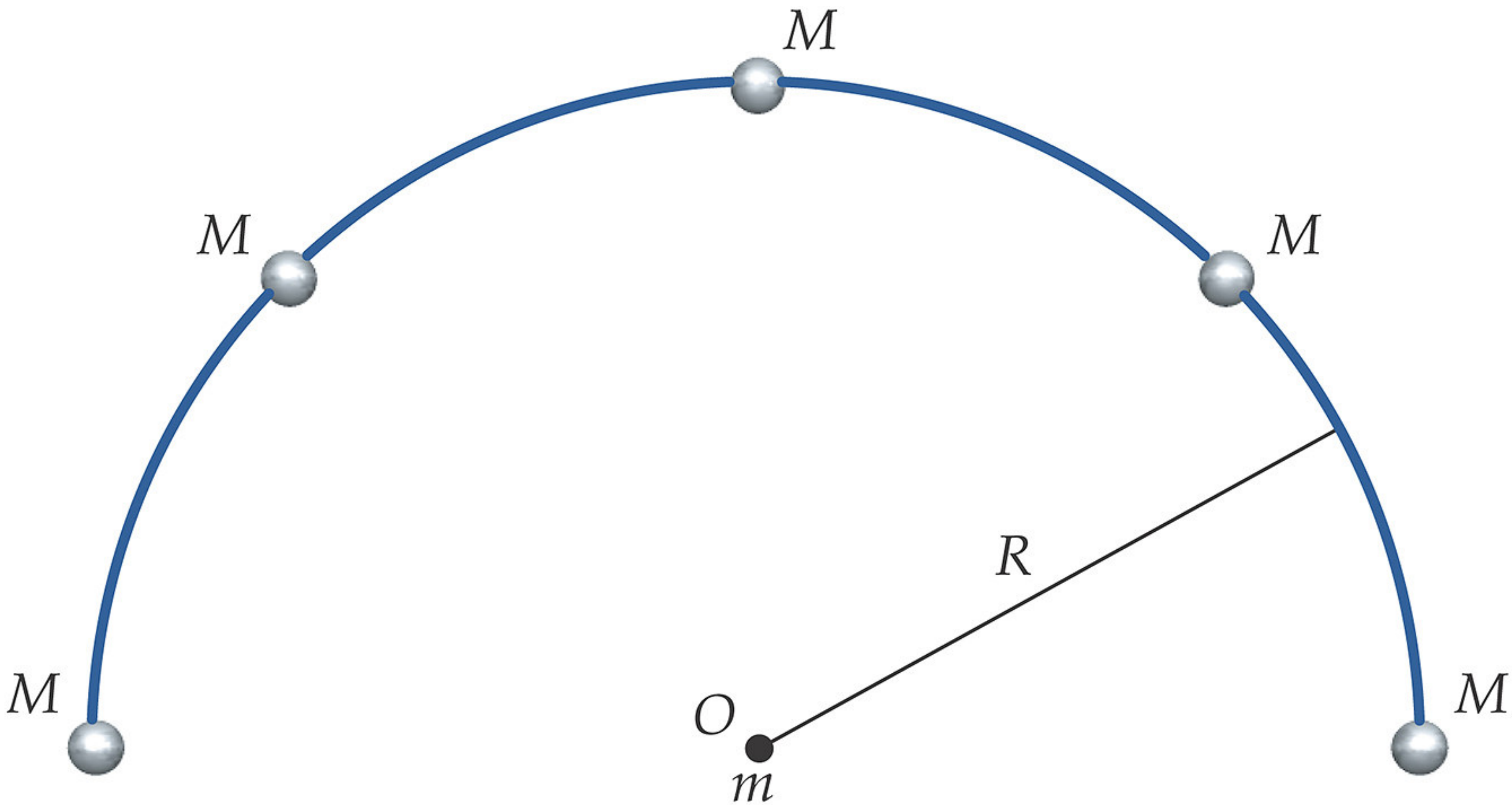
$$M' = \frac{r^3}{R^3}M$$

$$g(r) = -\frac{GM'}{r^2} = -\frac{GM}{r^2}\frac{r^3}{R^3} = -\frac{GM}{R^3}r$$

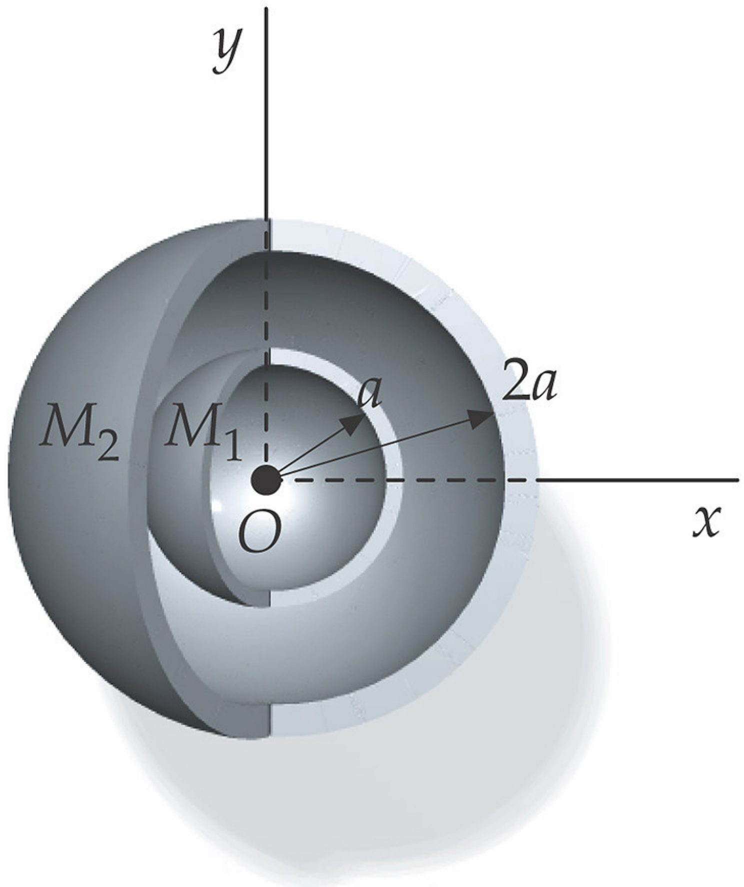
$$for\ r \leq R$$

Homework Problems

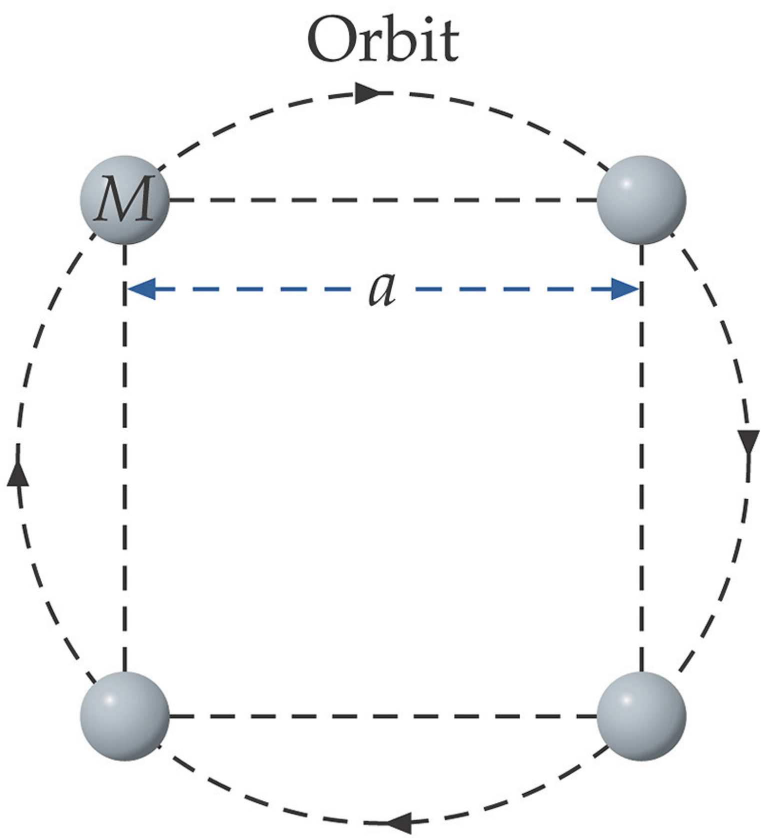
Gravitational force on m and G field when m is gone



Force on mass m at distances: 3a, 1.9a and 0.9 a



What must their speed be if they are to orbit their common center under their mutual gravitational attraction?



Extra Slides

Table 11-1 Mean Orbital Radii and Orbital Periods for the Planets

Planet	Mean Radius r ($\times 10^{10}$ m)	Period T (y)
Mercury	5.79	0.241
Venus	10.8	0.615
Earth	15.0	1.00
Mars	22.8	1.88
Jupiter	77.8	11.9
Saturn	143	29.5
Uranus	287	84
Neptune	450	165
Pluto	590	248