Chapter 14

Fluids

Fluids

- Density
- Pressure in a Fluid
- Buoyancy and Archimedes Principle
- Fluids in Motion

Fluid = Gas or Liquid

Densities

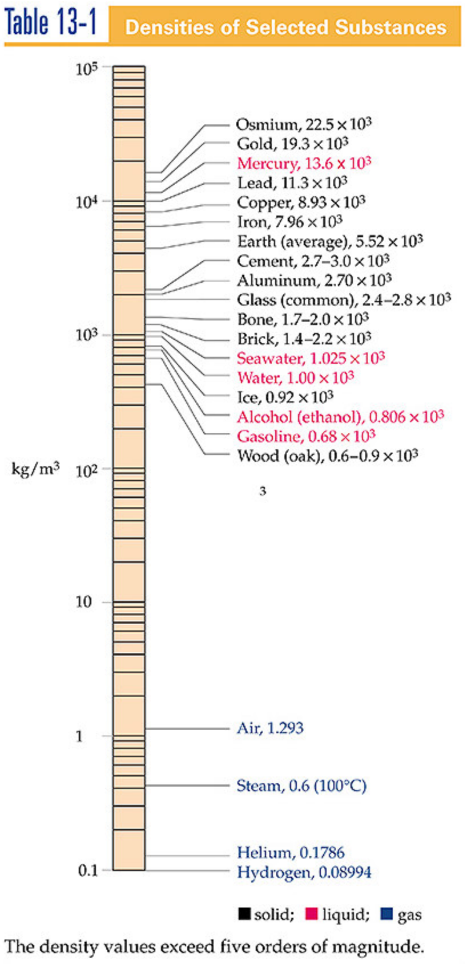
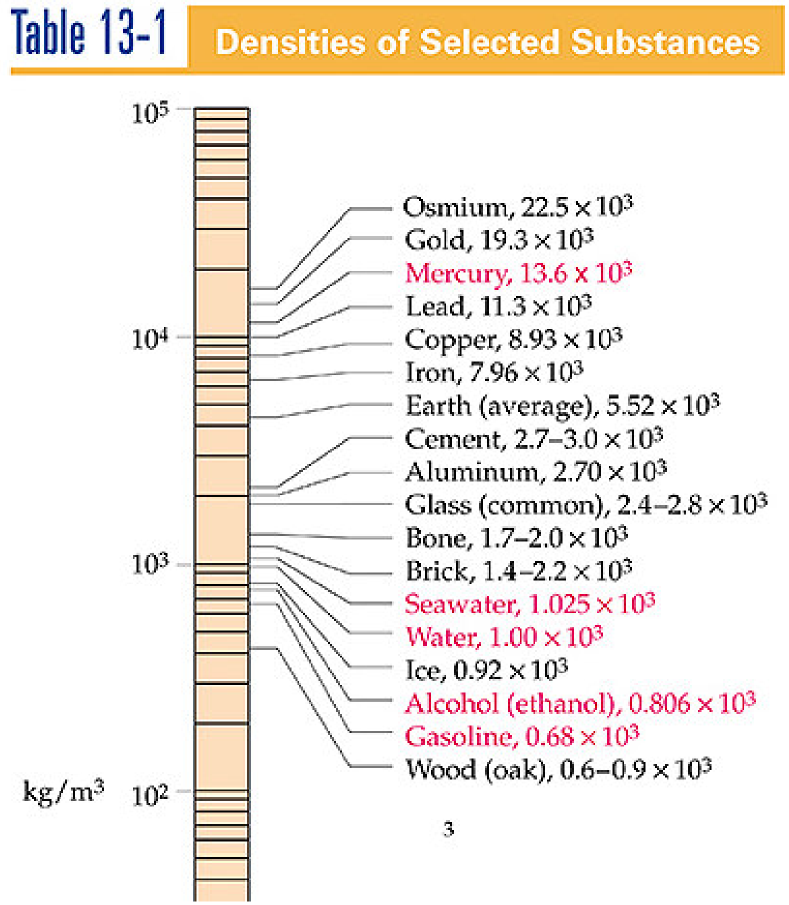


Table 13-1 Densities of Selected Substances
10^5
10^4
10^3
kg/m^3 10^2
10
1
0.1
Osmium, 22.5 × 10^3
Gold, 19.3 × 10^3
Mercury, 13.6 x 10^3
Lead, 11.3 × 10^3
Copper, 8.93 × 10^3
Iron, 7.96 × 10^3
Earth (average), 5.52 × 10^3
Cement, 2.7–3.0 × 10^3
Aluminum, 2.70 × 10^3
Glass (common), 2.4–2.8 × 10^3
Bone, 1.7–2.0 × 10^3
Brick, 1.4–2.2 × 10^3
Seawater, 1.025 × 10^3
Water, 1.00 × 10^3
Ice, 0.92 × 10^3
Alcohol (ethanol), 0.806 × 10^3
Gasoline, 0.68 × 10^3
Wood (oak), 0.6–0.9 × 10^3
3
Air, 1.293
Steam, 0.6 (100°C)
Helium, 0.1786
Hydrogen, 0.08994
solid; liquid; gas
The density values exceed five orders of magnitude.



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Densities

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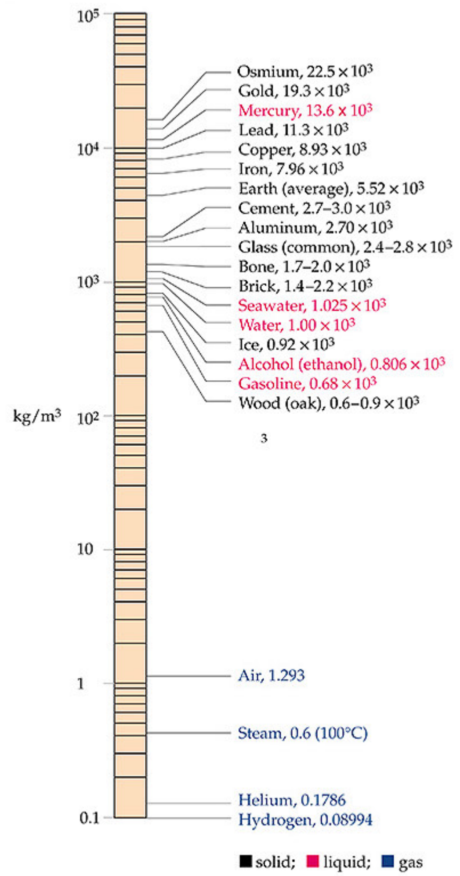

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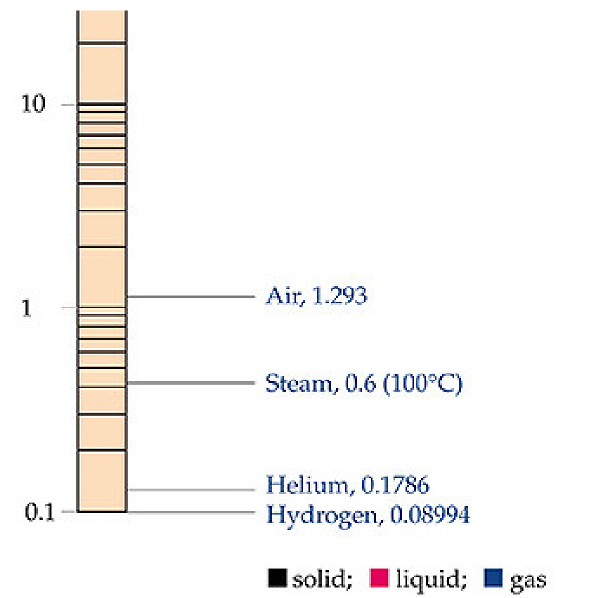


The density values exceed five orders of magnitude.

Densities

$$Mass = Density * Volume$$

$$m = \rho V$$

$$kg = \frac{kg}{m^3} \times m^3$$

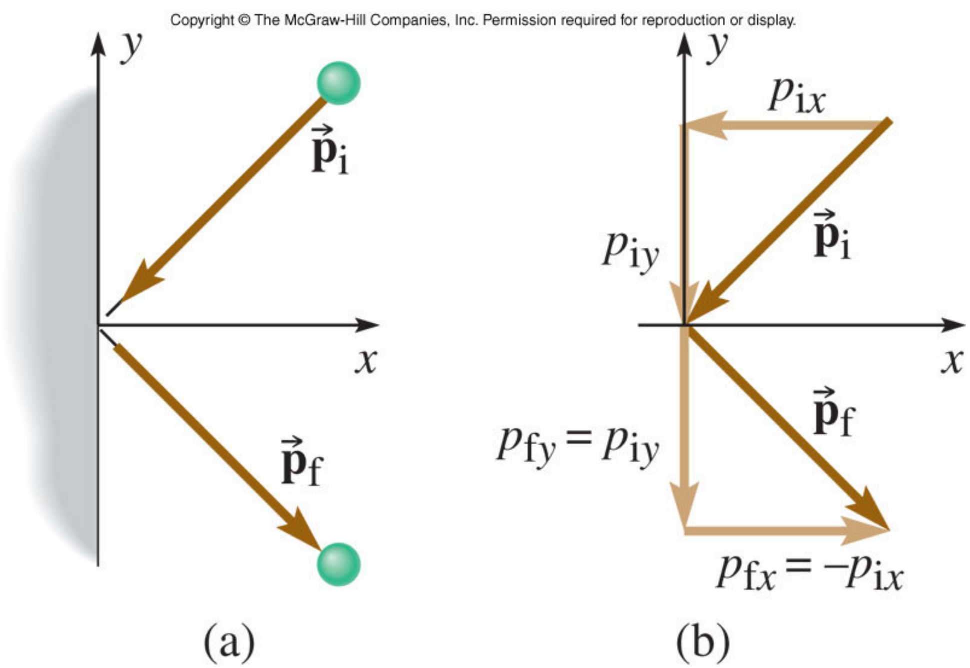
$$1\frac{g}{cm^3} = \frac{g}{cm^3} \times \frac{kg}{1000\,g} \times \left(\frac{10^2\,cm}{m}\right)^3$$

$$1\frac{g}{cm^3} = \frac{g}{cm^3} \times \frac{kg}{1000\,g} \times \frac{10^6\,cm^3}{m^3}$$

$$1\frac{g}{cm^3} = \frac{10^3\,kg}{m^3}$$

Pressure

Pressure arises from the collisions between the particles of a fluid with another object (container walls for example).



There is a momentum change (impulse) that is away from the container walls. There must be a force exerted on the particle by the wall.

By Newton's 3rd Law, there is a force on the wall due to the particle.

Pressure is defined as $P = \frac{F}{A}$.

The units of pressure are N/m^2 and are called Pascals (Pa).

Note: 1 atmosphere (atm) = 101.3 kPa

Example (text problem 9.1): Someone steps on your toe, exerting a force of 500 N on an area of 1.0 cm^2. What is the average pressure on that area in atmospheres?

$$1.0\ \text{cm}^2\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2 = 1.0\times10^{-4}\ \text{m}^2$$

A 500N person weighs about 113 lbs.

$$P_{\text{av}} = \frac{F}{A} = \frac{500\ \text{N}}{1.0\times10^{-4}\ \text{m}^2}$$

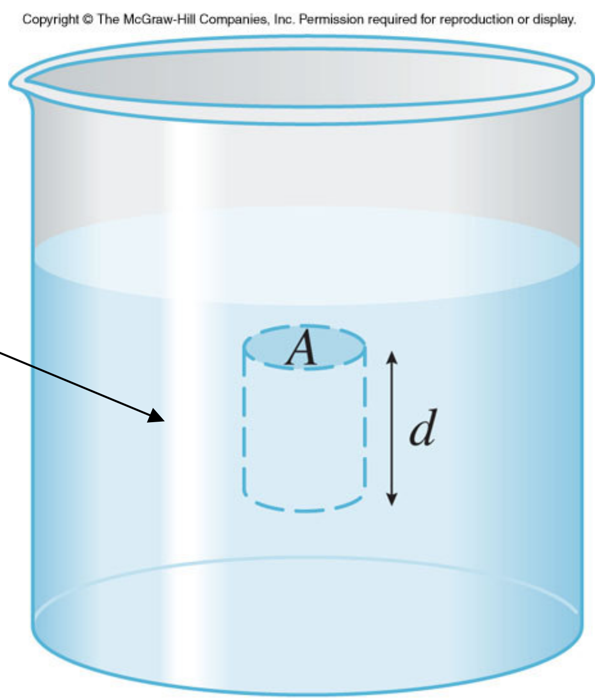
$$= 5.0\times10^6\ \text{N/m}^2\left(\frac{1\ \text{Pa}}{1\ \text{N/m}^2}\right)\left(\frac{1\ \text{atm}}{1.013\times10^5\ \text{Pa}}\right)$$

$$= 49\ \text{atm}$$

Gravity's Effect on Fluid Pressure

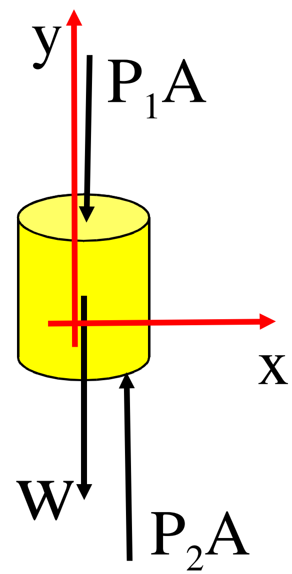
An imaginary cylinder of fluid

Imaginary cylinder can be any size



(a)

FBD for the fluid cylinder



Apply Newton's 2nd Law to the fluid cylinder. Since the fluids isn't moving the net force is zero.

$$\sum F = P_2 A - P_1 A - w = 0$$
$$P_2 A - P_1 A - (\rho A d) g = 0$$
$$P_2 - P_1 - \rho g d = 0$$
$$\therefore P_2 - P_1 = \rho g d$$
$$\text{or} \;\; P_2 = P_1 + \rho g d$$

If P_1 (the pressure at the top of the cylinder) is known, then the above expression can be used to find the variation of pressure with depth in a fluid.

If the top of the fluid column is placed at the surface of the fluid, then $P_1 = P_{atm}$ if the container is open.

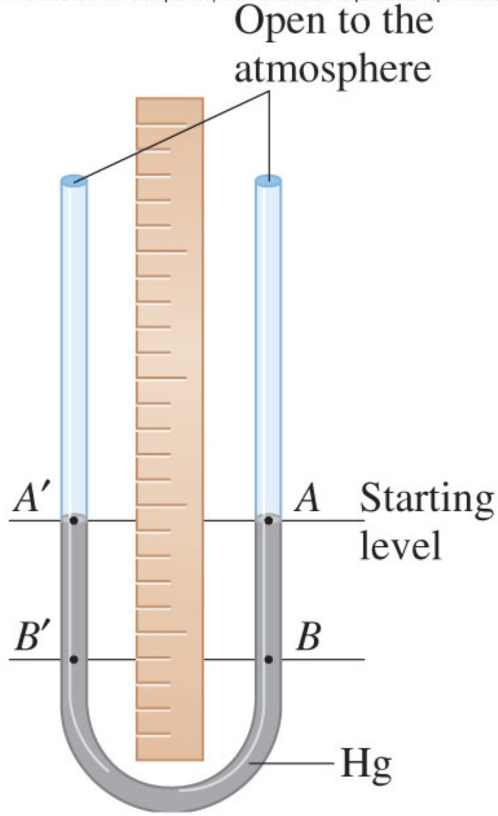
$$P = P_{\text{atm}} + \rho g d$$

You noticed on the previous slide that the areas canceled out. Only the height matters since that is the direction of gravity.

Think of the pressure as a *force density* in N/m^2

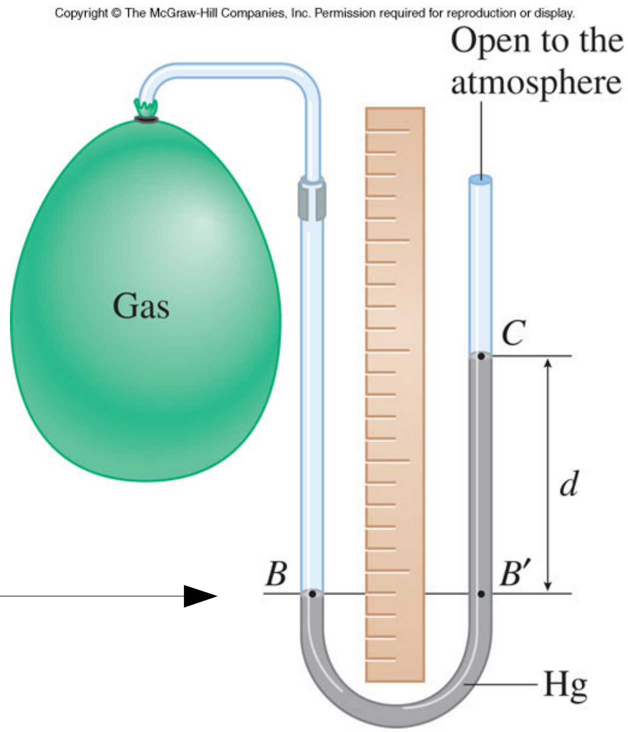
Measuring Pressure

A manometer is a U-shaped tube that is partially filled with liquid, usually Mercury (Hg).



Both ends of the tube are open to the atmosphere.

A container of gas is connected to one end of the U-tube



Equal pressure → ← Equal pressure

If there is a pressure difference between the gas and the atmosphere, a force will be exerted on the fluid in the U-tube. This changes the equilibrium position of the fluid in the tube.

From the figure: At point C $P_c = P_{atm}$

Also $P_B = P_{B'}$

The pressure at point B is the pressure of the gas.

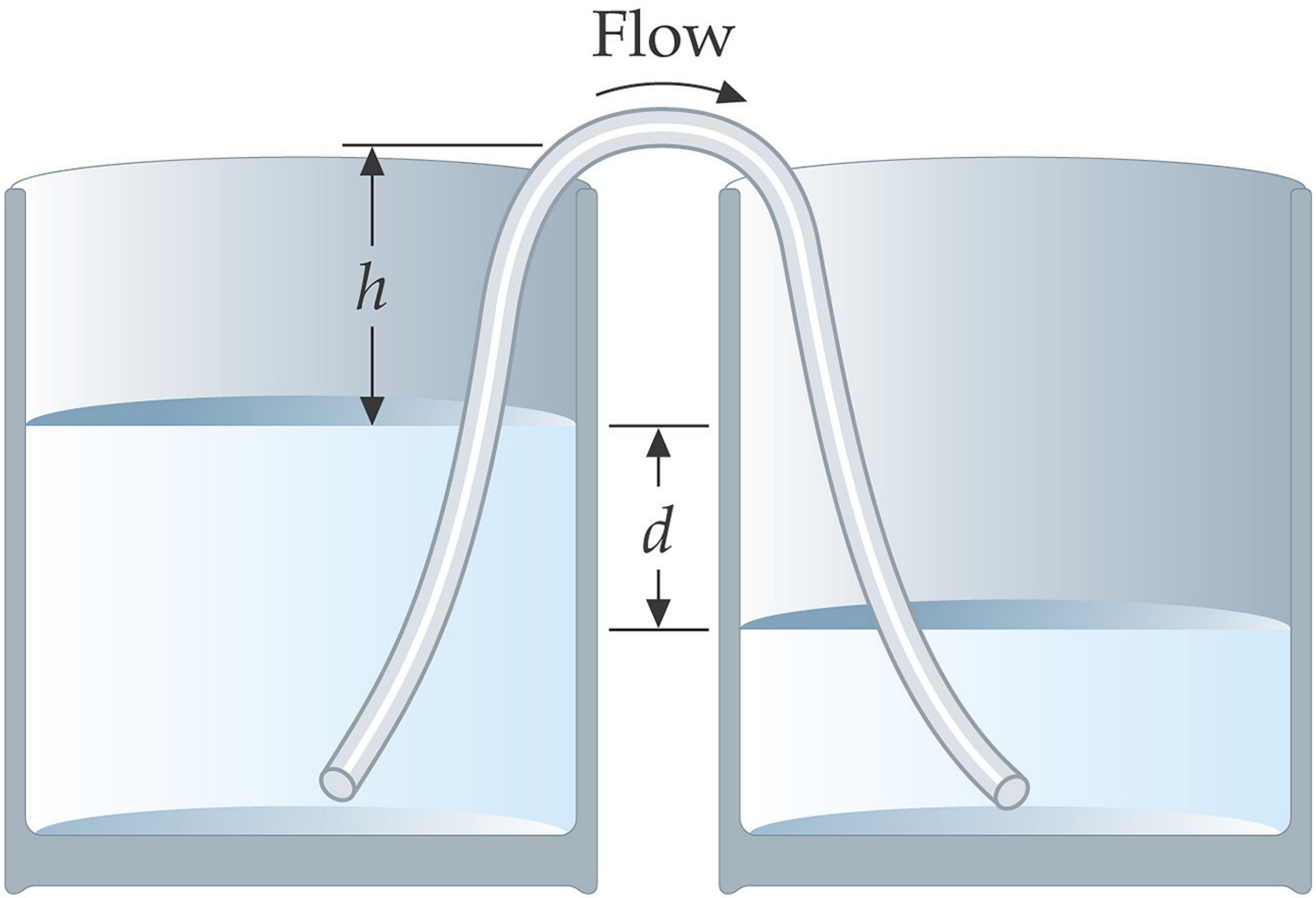
$$P_B = P_{B'} = P_C + \rho g d$$

$$P_B - P_C = P_B - P_{atm} = \rho g d$$

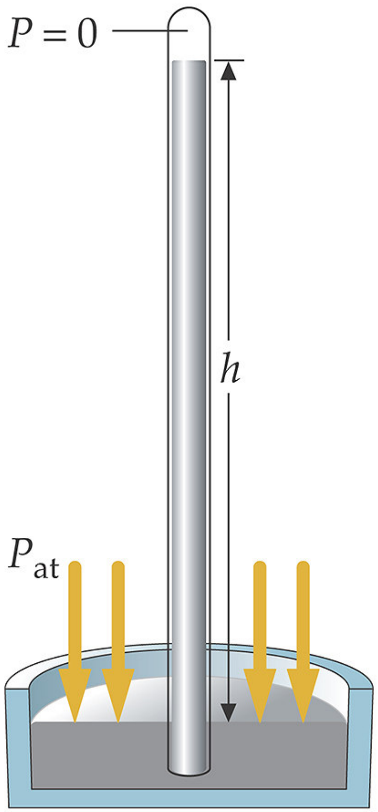
$$P_{gauge} = \rho g d$$

$$P_{gauge} = P_{meas} - P_{atm}$$

Siphoning



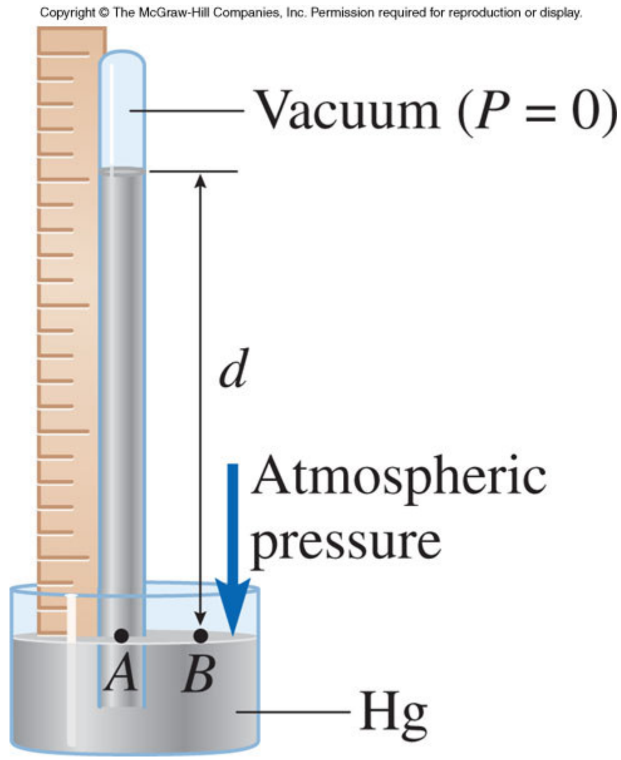
Atmospheric Pressure Will Support a Column of Fluid



The column is sealed at one end, filled with the fluid and then inverted into a container of the same fluid.

The difference in pressure between the two ends of the column makes the process work

A Barometer



The atmosphere pushes on the container of mercury which forces mercury up the closed, inverted tube. The distance d is called the **barometric pressure**.

From the figure $P_{\mathrm{A}} = P_{\mathrm{B}} = P_{\mathrm{atm}}$

and $P_A = \rho g d$

Atmospheric pressure is equivalent to a column of mercury 76.0 cm tall.

The Many Units of Pressure

1 ATM equals

1.013×10^5 N/m^2

14.7 lbs/in^2

1.013 bar

76 cm Hg

760 mm Hg

760 Torr

34 ft H_2O

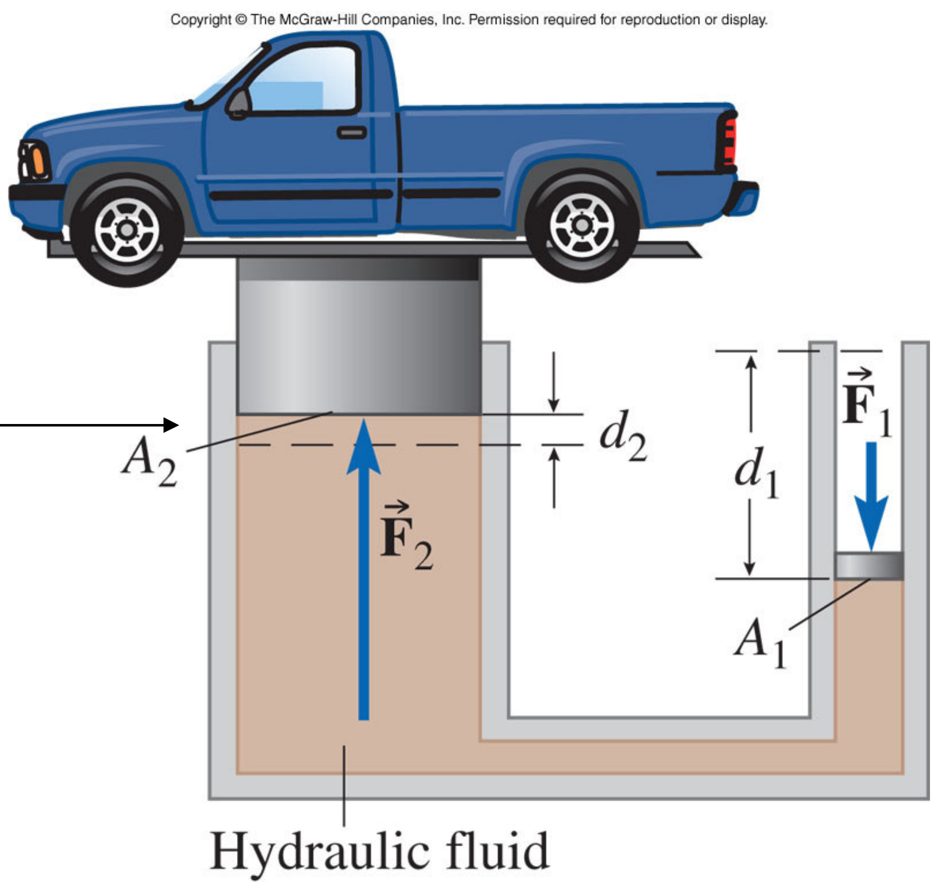
29.9 in Hg

Pascal's Principle

A change in pressure at any point in a confined fluid is transmitted everywhere throughout the fluid. (This is useful in making a hydraulic lift.)

The applied force is transmitted to the piston of cross-sectional area A_2 here.

Apply a force F_1 here to a piston of cross-sectional area A_1.



In these problems neglect pressure due to columns of fluid.

Mathematically,

$$\Delta P \text{ at point } 1 = \Delta P \text{ at point } 2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

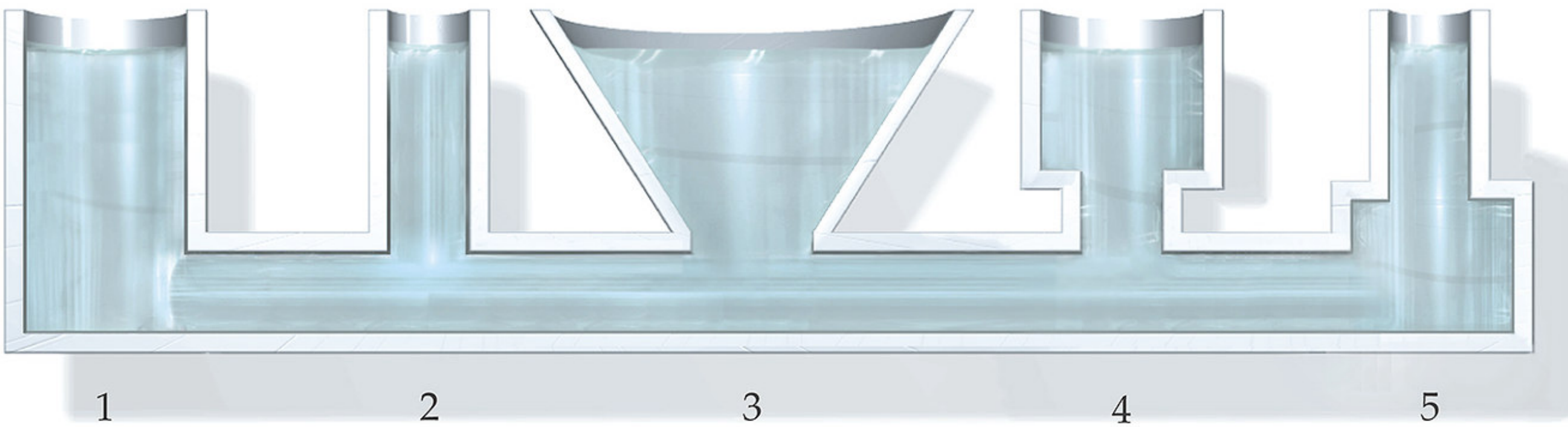
$$F_2 = \left(\frac{A_2}{A_1}\right) F_1$$

Example: Assume that a force of 500 N (about 110 lbs) is applied to the smaller piston in the previous figure. For each case, compute the force on the larger piston if the ratio of the piston areas (A_2/A_1) are 1, 10, and 100.

Using Pascal's Principle:

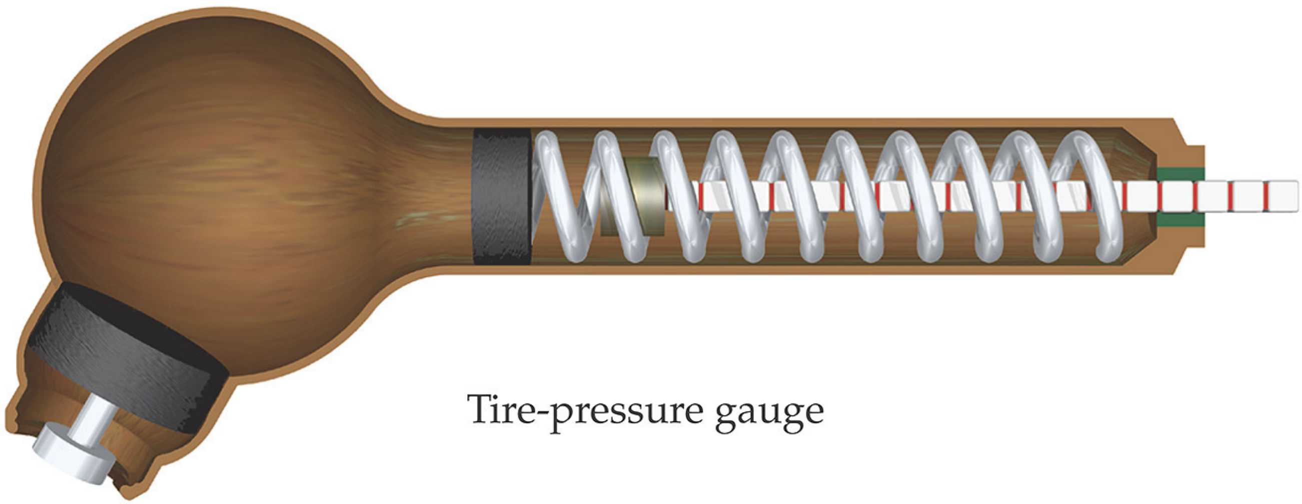
A_2/A_1	F_2
1	500 N
10	5000 N
100	50,000 N

Pressure Depends Only on the Vertical Height



"Pressure depends only on the depth of the fluid, not on the shape of the container. So the pressure is the same for all parts of the container that are at the same depth."

Pressure Gauge

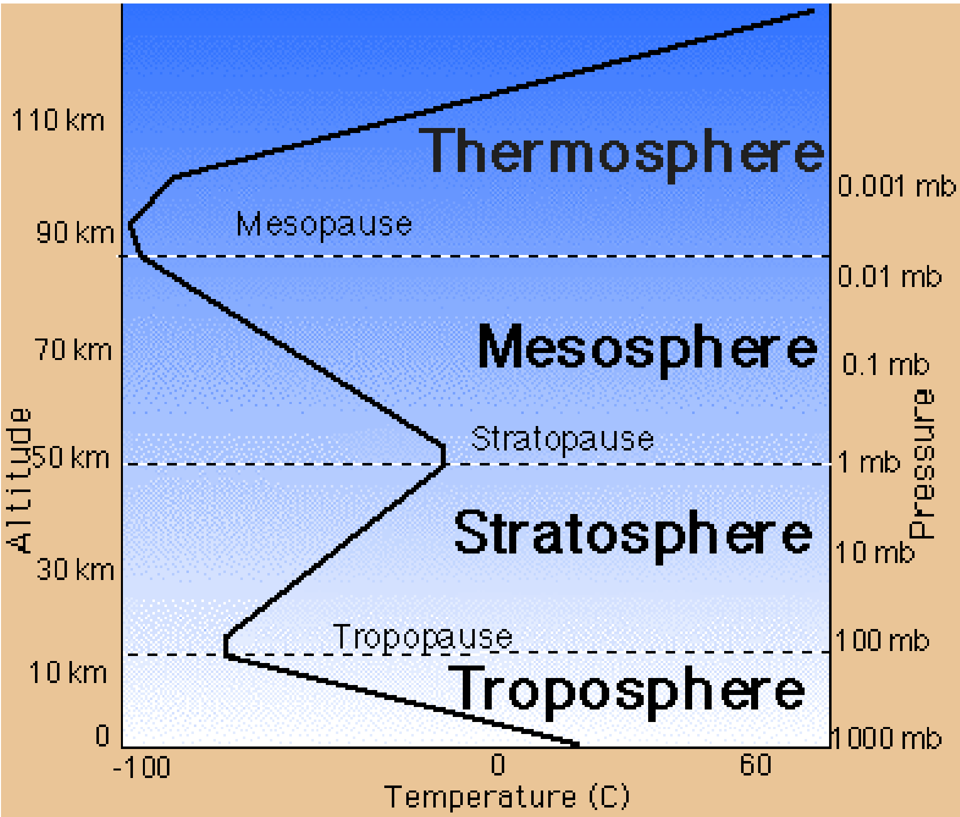


Tire-pressure gauge

$$P_{gauge} = P - P_{atm}$$

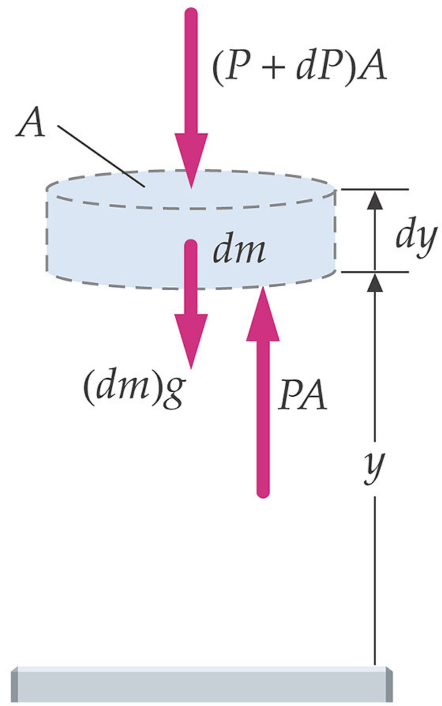
When the tire is flat the pressure inside the tire is atmospheric pressure.

Law of Atmosphere

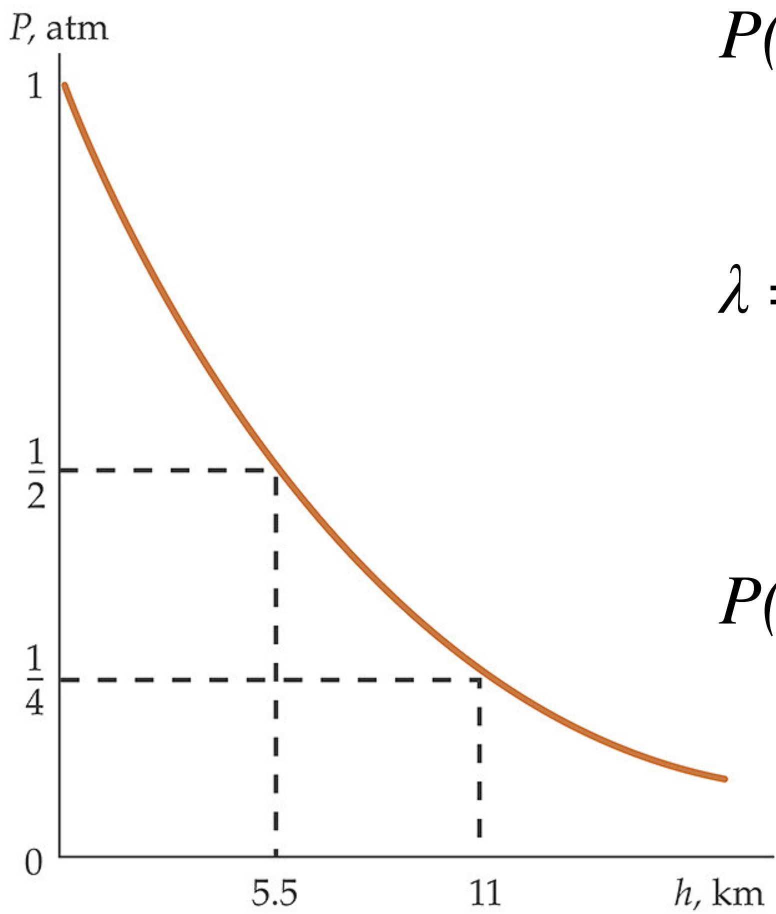


$$\frac{dP}{dy} = -\lambda P$$

$$\int \frac{dP}{P} = -\int \lambda dy$$



Pressure Decrease with Height



$$P(y) = P_0 e^{-\lambda y}$$

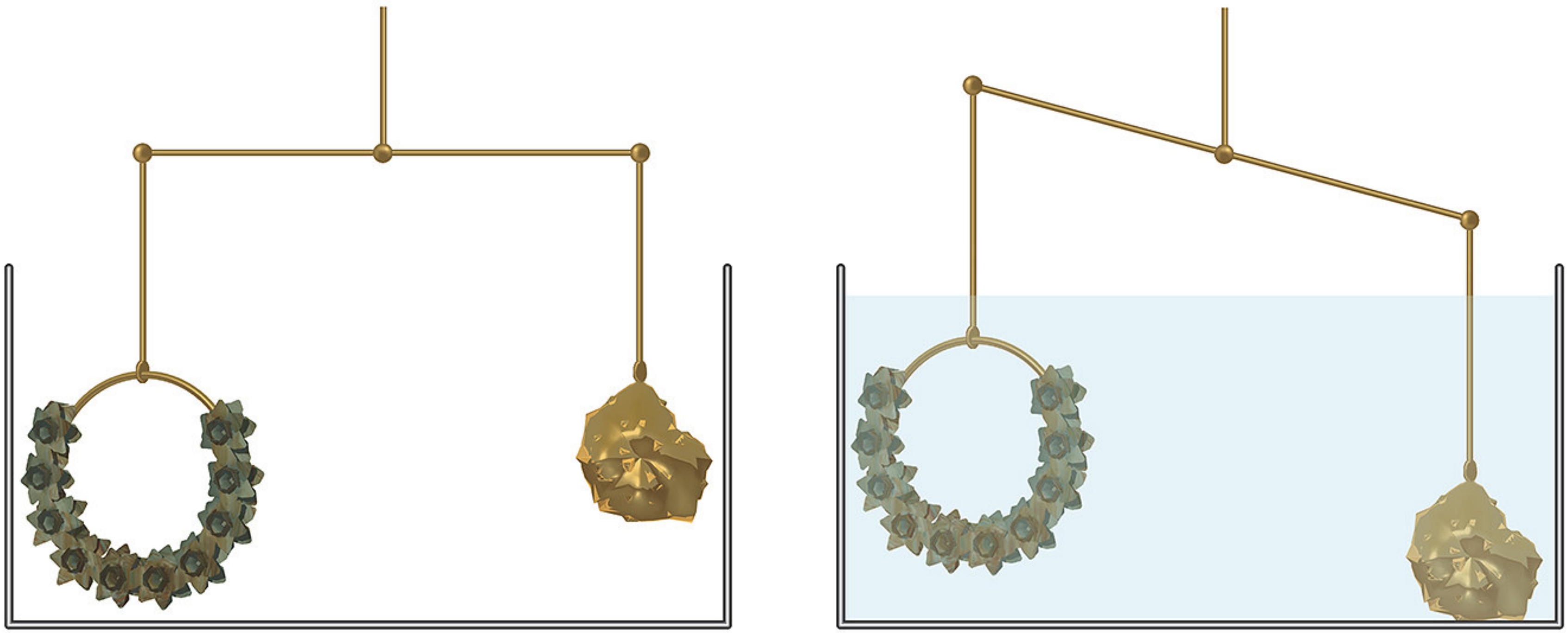
$$\lambda = \frac{ln(2)}{5.5(km)}$$

$$P(y) = P_0 exp\left[-\frac{y(km)}{5.5} ln(2) \right]$$

Archimedes's Principle

"A body wholly or partially submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced."

Archimedes's Principle



(*a*) Crown and gold nugget have equal weight.

(*b*) Crown displaces more water than does the gold nugget.

Archimedes's Principle

(a) Crown and gold nugget have equal weight.

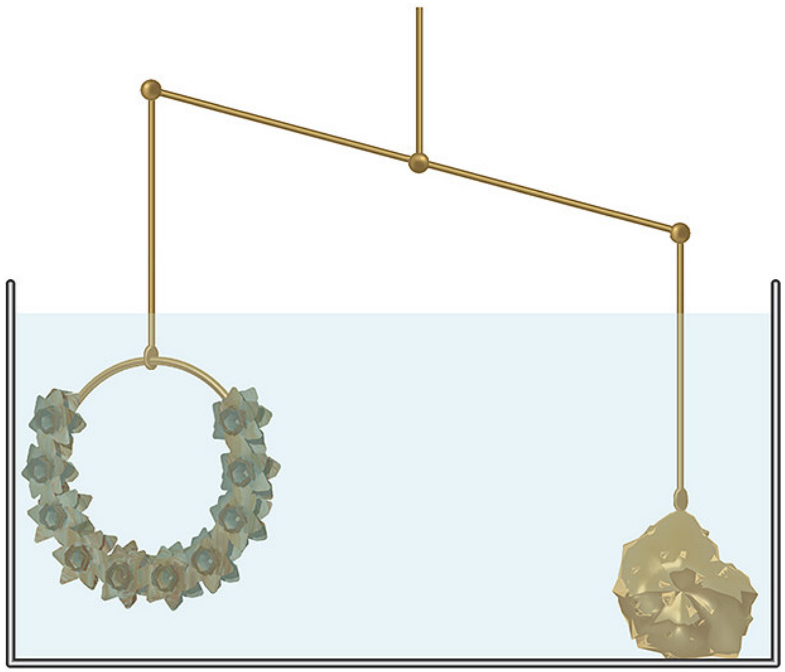
$$m_1 g = m_2 g$$

$$m_1 = m_2$$

$$\rho_1 V_1 = \rho_2 V_2$$

The density and volume are tied together

Archimedes's Principle



(b) Crown displaces more water than does the gold nugget.

$$BF_1 > BF_2$$

$$\rho_L V_1 g > \rho_L V_2 g$$

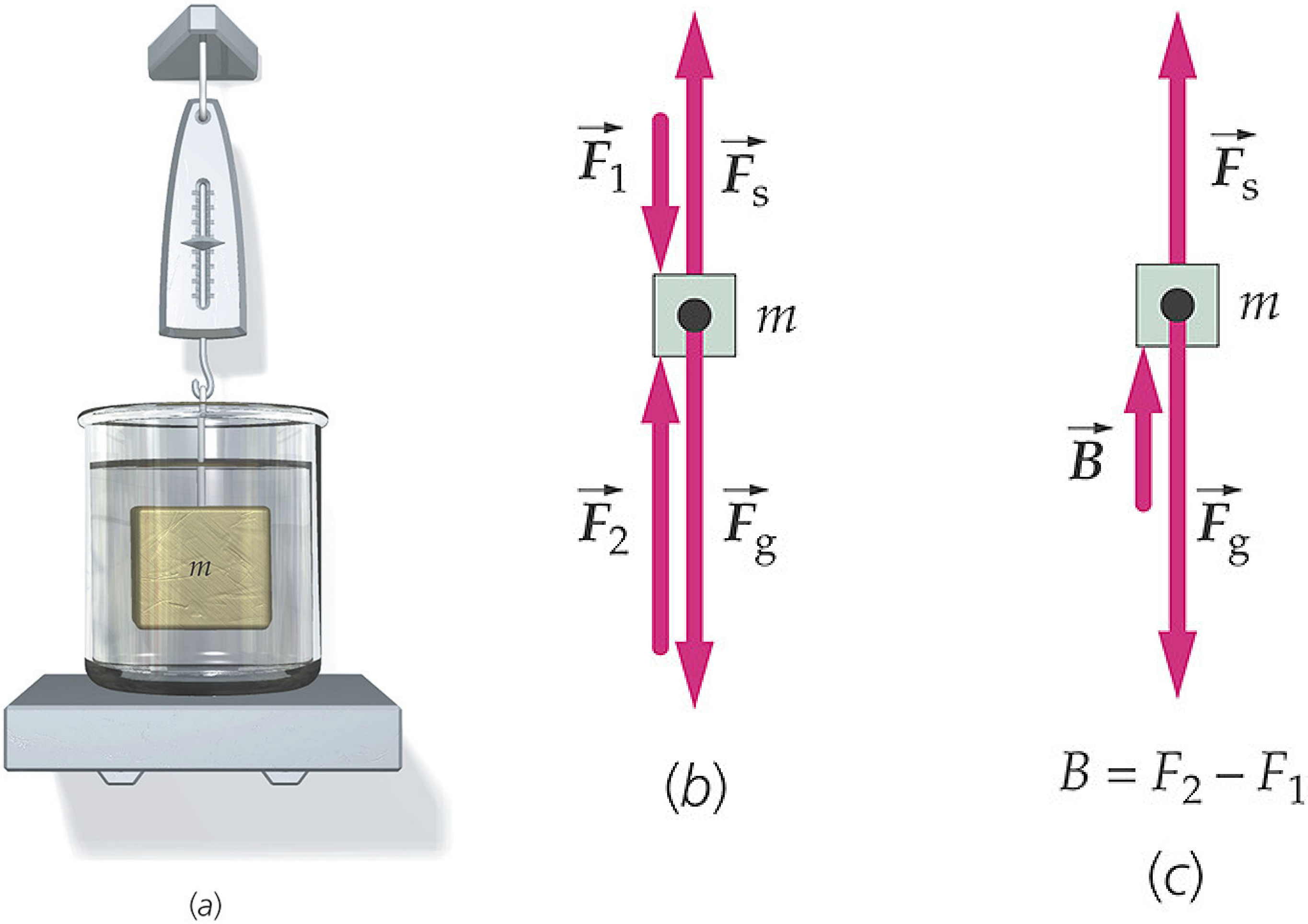
$$V_1 > V_2$$

$$Since\ \ \rho_1 V_1 = \rho_2 V_2$$

$$\rho_1 < \rho_2$$

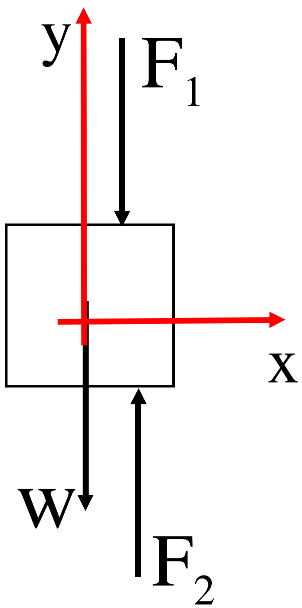
The crown isn't 100% gold.

Archimedes’s Principle



Archimedes' Principle

An FBD for an object floating submerged in a fluid.



The total force on the block due to the fluid is called the buoyant force.

$$\mathbf{F}_B = \mathbf{F}_2 - \mathbf{F}_1$$
$$\text{where } |\mathbf{F}_2| > |\mathbf{F}_1|$$

Buoyant force = the weight of the fluid displaced

The magnitude of the buoyant force is:

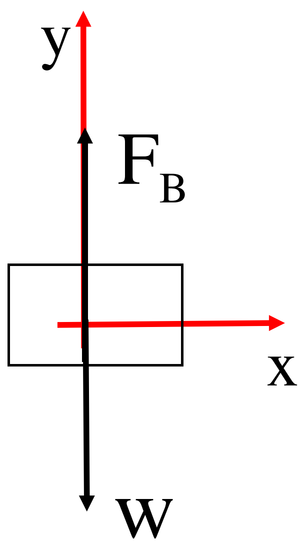
$$\begin{aligned} F_B &= F_2 - F_1 \\ &= P_2 A - P_1 A \\ &= (P_2 - P_1) A \end{aligned}$$

From before: $P_2 - P_1 = \rho g d$

The result is $F_B = \rho g d A = \rho g V$

Example: A flat-bottomed barge loaded with coal has a mass of 3.0×10^5 kg. The barge is 20.0 m long and 10.0 m wide. It floats in fresh water. What is the depth of the barge below the waterline?

FBD for the barge



Apply Newton's 2nd Law to the barge:

$$\sum F = F_B - w = 0$$

$$F_B = w$$

$$m_w g = (\rho_w V_w) g = m_b g$$

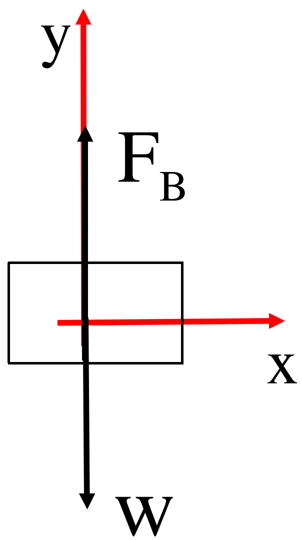
$$\rho_w V_w = m_b$$

$$\rho_w (Ad) = m_b$$

$$d = \frac{m_b}{\rho_w A} = \frac{3.0\times10^5\ \text{kg}}{(1000\ \text{kg/m}^3)(20.0\ \text{m} * 10.0\ \text{m})} = 1.5\ \text{m}$$

Example: A piece of metal is released under water. The volume of the metal is 50.0 cm^3 and its specific gravity is 5.0. What is its initial acceleration? (Note: when v = 0, there is no drag force.)

FBD for the metal



Apply Newton's 2nd Law to the piece of metal:

$$\sum F = F_B - w = ma$$

The magnitude of the buoyant force equals the weight of the fluid displaced by the metal.

$$F_B = \rho_{\text{water}} V g$$

Solve for a:

$$a = \frac{F_B}{m} - g = \frac{\rho_{\text{water}} V g}{\rho_{\text{object}} V_{\text{object}}} - g = g\left(\frac{\rho_{\text{water}} V}{\rho_{\text{object}} V_{\text{object}}} - 1\right)$$

Example continued:

Since the object is completely submerged $V = V_{object.}$

$$\text{specific gravity} = \frac{\rho}{\rho_{\text{water}}}$$

where $\rho_{water} = 1000\ \text{kg/m}^3$ is the density of water at 4 °C.

Given $$\text{specific gravity} = \frac{\rho_{\text{object}}}{\rho_{\text{water}}} = 5.0$$

$$a = g\left(\frac{\rho_{\text{water}} V}{\rho_{\text{object}} V_{\text{object}}} - 1\right) = g\left(\frac{1}{S.G.} - 1\right) = g\left(\frac{1}{5.0} - 1\right) = -7.8\ \text{m/s}^2$$

The sign is minus because gravity acts down. BF causes $a < g$.

Fluid Flow

A moving fluid will exert forces parallel to the surface over which it moves, unlike a static fluid. This gives rise to a viscous force that impedes the forward motion of the fluid.

A **steady flow** is one where the velocity at a given point in a fluid is constant.

$V_1 =$ constant

$V_2 =$ constant

$V_1 \neq V_2$

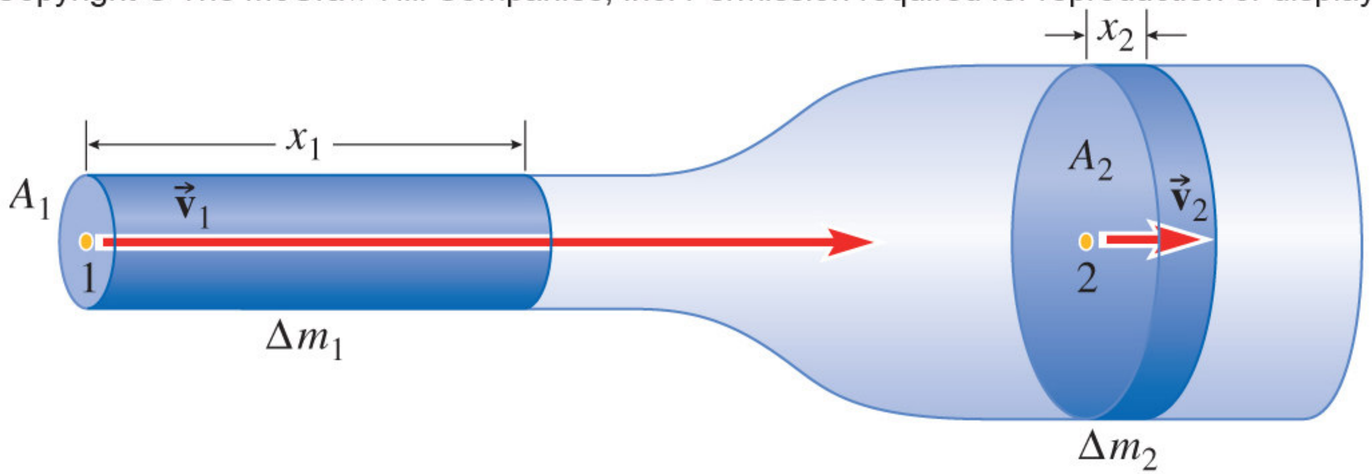
Steady flow is laminar; the fluid flows in layers. The path that the fluid in these layers takes is called a streamline.

Streamlines do not cross.

Crossing streamlines would indicate a volume of fluid with two different velocities at the same time.

An **ideal fluid** is incompressible, undergoes laminar flow, and has no viscosity.

The Continuity Equation—Conservation of Mass



Faster Slower

The amount of mass that flows though the cross-sectional area A_1 is the same as the mass that flows through cross-sectional area A_2.

$$\frac{\Delta m}{\Delta t} = \rho A v$$ is the mass flow rate (units kg/s)

$$\frac{\Delta V}{\Delta t} = Av = I_V = Q$$ is the volumetric flow rate (m^3/s)

In general the *continuity equation* is

$$I_{M1} - I_{M2} = \frac{dm_{12}}{dt}$$

If $\frac{dm_{12}}{dt} = 0$ then $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

If the fluid is incompressible, then $\rho_1 = \rho_2$.

Example: A garden hose of inner radius 1.0 cm carries water at 2.0 m/s. The nozzle at the end has radius 0.20 cm. How fast does the water move through the constriction?

Simple ratios

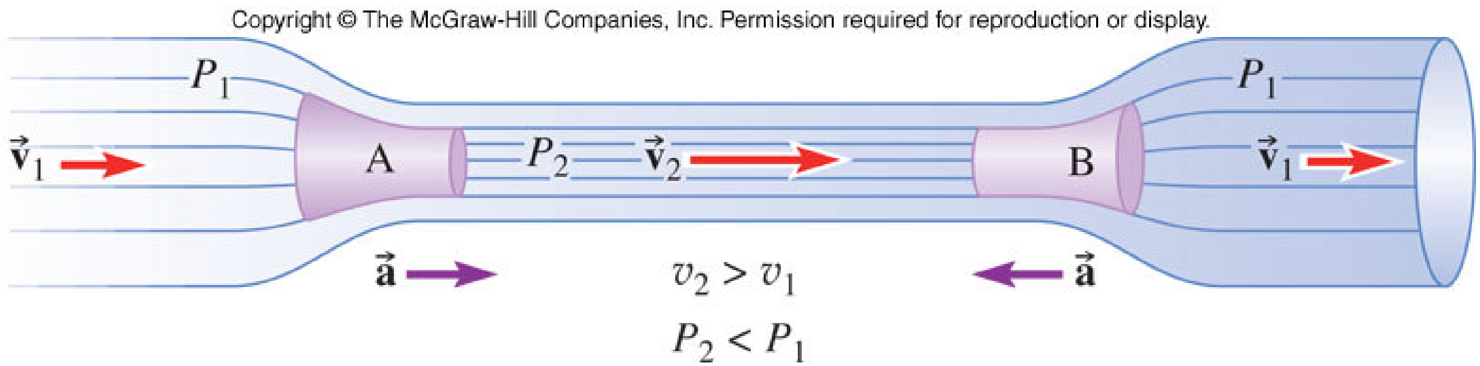
$$A_1 v_1 = A_2 v_2$$

$$v_2 = \left(\frac{A_1}{A_2}\right) v_1 = \left(\frac{\pi r_1^2}{\pi r_2^2}\right) v_1$$

$$= \left(\frac{1.0\ cm}{0.20\ cm}\right)^2 \left(2.0\ m/s\right) = 50\ m/s$$

Bernoulli's Equation

Bernoulli's equation is a statement of energy conservation.



This is the most general equation

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

Work per unit volume done by the fluid

Potential energy per unit volume

Kinetic energy per unit volume

Points 1 and 2 must be on the same streamline

Torricelli's Law

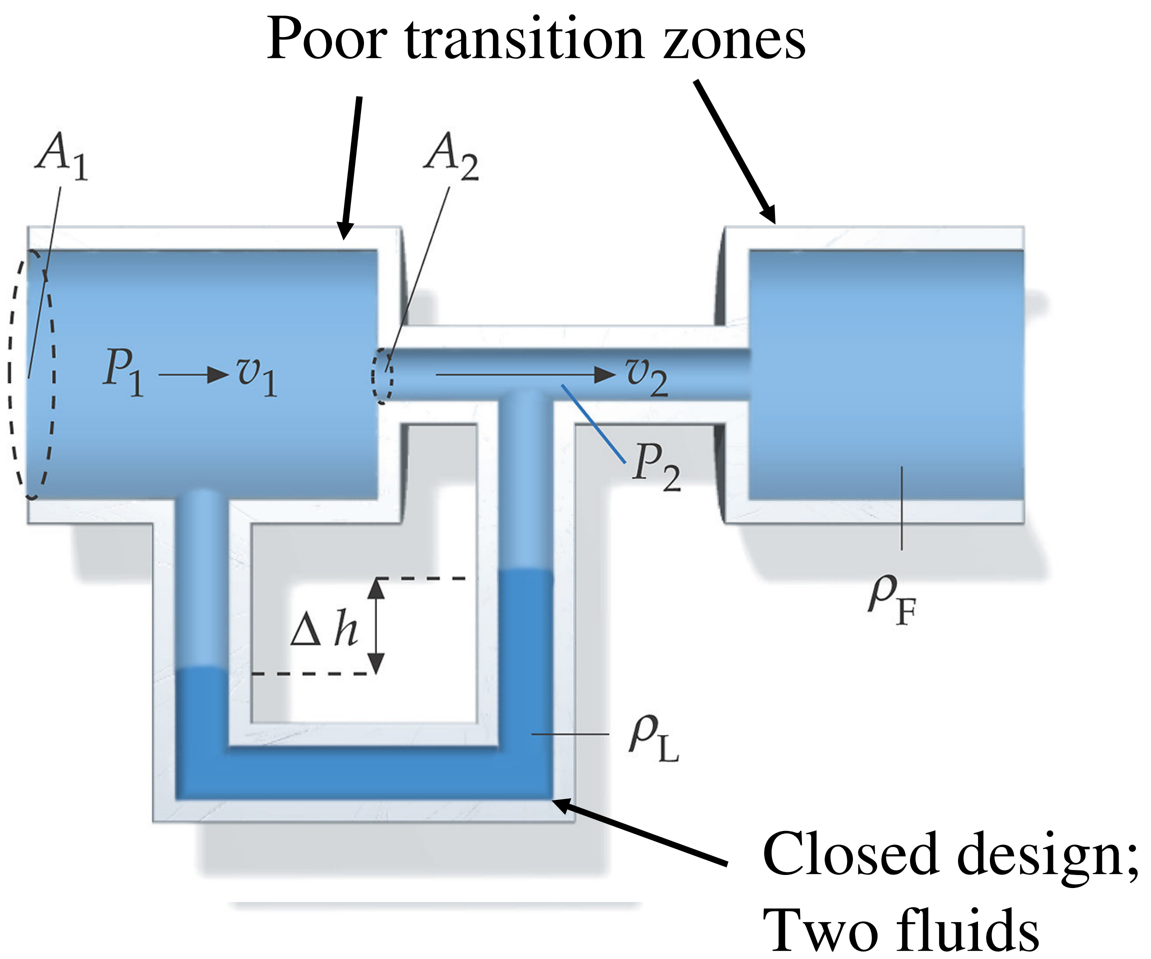
Application of Bernoulli's Law

Torricelli's Law states that the water exiting the hole in the side of the beaker has a speed equal to that it would have had after falling a verticle distance Δh.

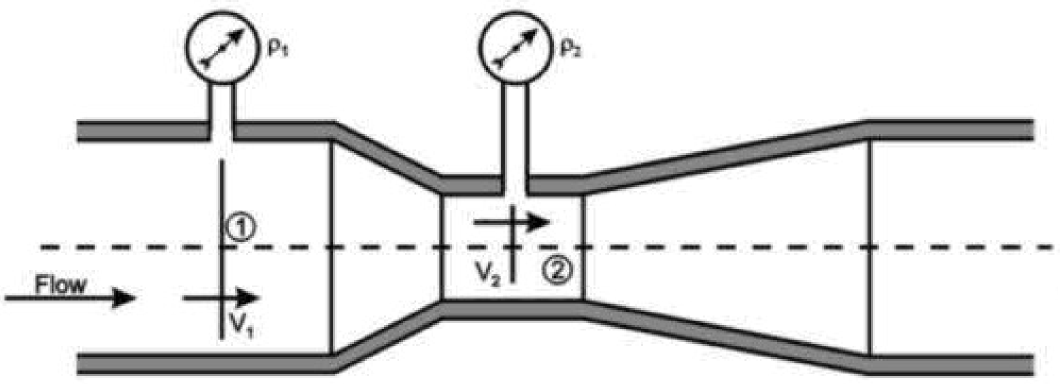
$$\rho g h_a = \rho g h_b + \tfrac{1}{2}\rho v_b^2$$

$$v_b = \sqrt{2g\Delta h}$$

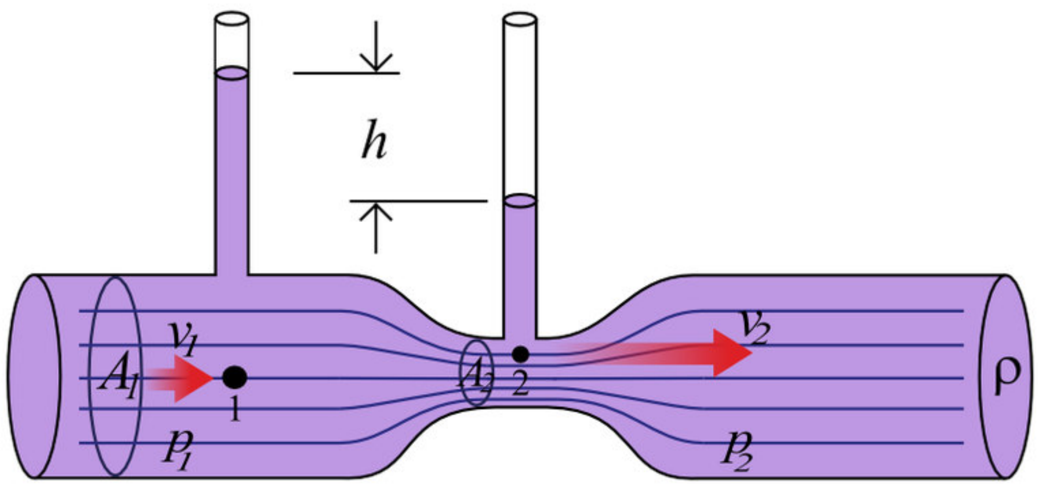
Venturi Meter



Venturi Meters - Good Transitions

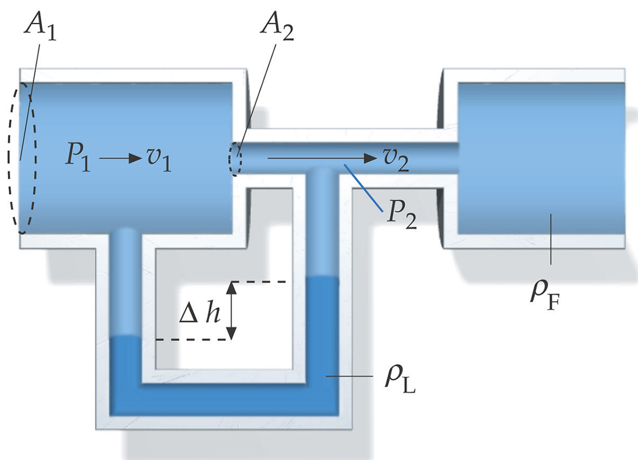


Closed design;
One fluid



Open design;
One fluid

Venturi Meter



$$P_1 + \tfrac{1}{2}\rho_F v_1^2 = P_2 + \tfrac{1}{2}\rho_F v_2^2$$

$$v_2 = \frac{A_1}{A_2} v_1 = r v_1$$

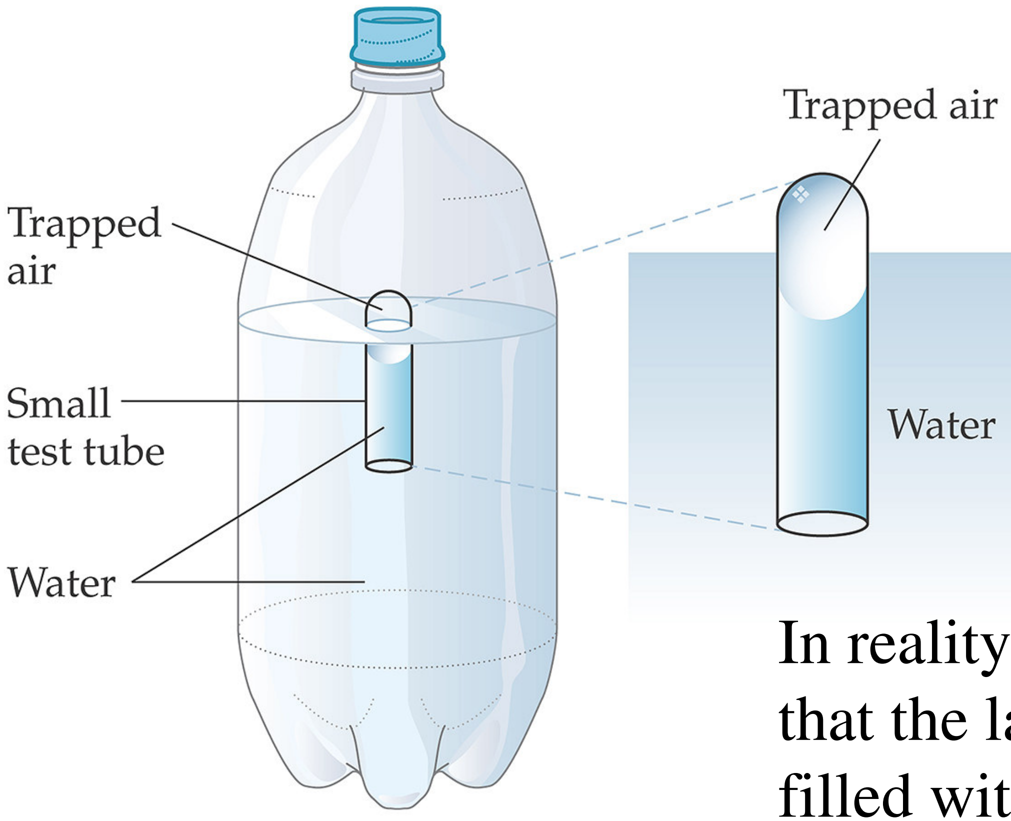
$$P_1 - P_2 = \rho_L g \Delta h - \rho_F g \Delta h$$

$$v_1 = \sqrt{\frac{2(\rho_L - \rho_F)g\Delta h}{\rho_F (r^2 - 1)}}$$

What about the rest of the fluid in the column?

End of Chapter Problems

Cartesian Diver



In reality it is essential that the large bottle be filled with water.

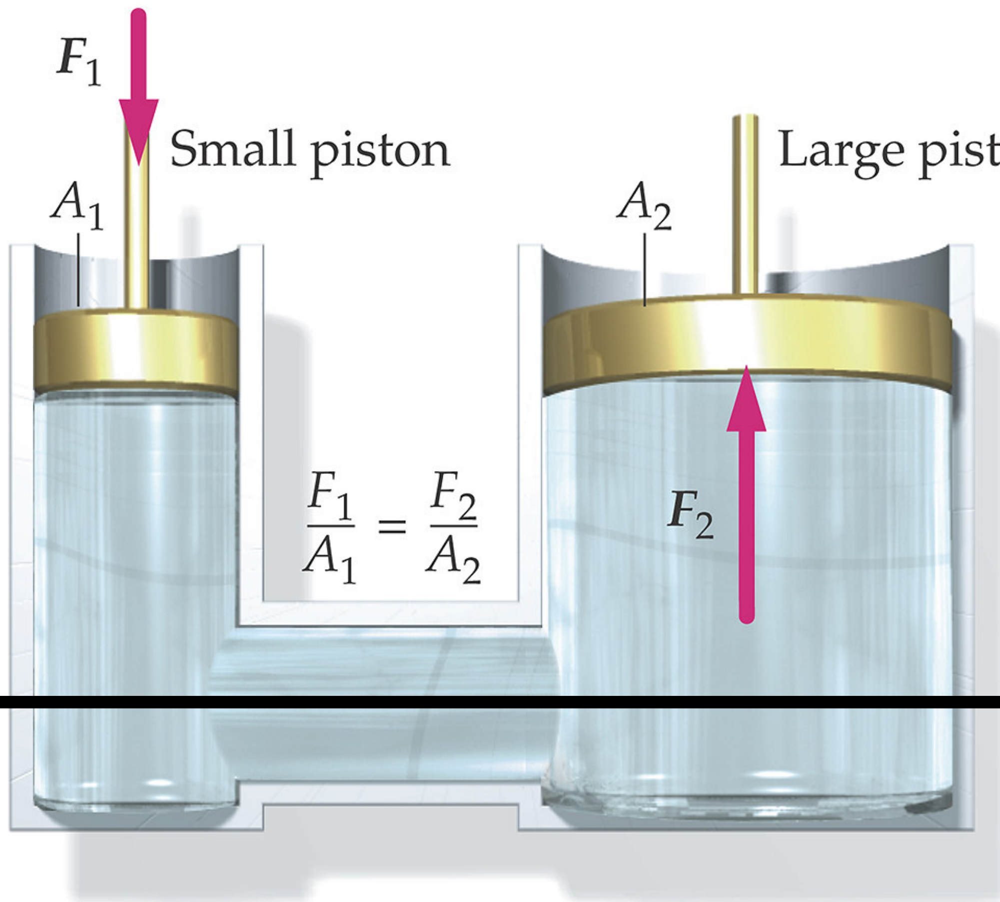
http://www.lon-capa.org/~mmp/applist/f/f.htm

Extra Slides

Low pressures such as natural gas lines are sometimes specified in inches of water, typically written as w.c. (water column) or W.G. (inches water gauge). A typical gas using residential appliance is rated for a maximum of 14 w.c. which is approximately 0.034 atmosphere.

In the United States the accepted unit of pressure measurement for the HVAC industry is inches of water column.

Application of Pascal's Principle



Point 1

Point 2