Chapter 9
Rotation
Homework Solutions

PHY 2425

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CHAP 9 - #60

$\omega_0 = 0$

$\tau_A = 50.0$ N·m

$\Delta t = 20.0$ s

$f = 600$ rev/min

TORQUE REMOVED

$\omega \to 0$ IN 120s.

QUES:

(a.) $I = ?$

(b.) $\tau_{FRIC} = ?$

(a.) $\Sigma\tau = I\alpha_1$

$f = 600 \frac{rev}{min} \times \frac{1 \, min}{60 s} = 10 \frac{rev}{s}$

$\omega_f = \omega_0 + \alpha_1 \Delta t$

$\frac{\omega_f - \omega_0}{\Delta t} = \alpha_1$

$\omega = 2\pi f = 2\pi(10) = 20\pi$ rad/s

$\alpha_1 = \frac{20\pi - 0}{20} = \pi$ rad/s^2

THERE ARE TWO TORQUES BEING APPLIED TO THE GRINDING WHEEL

$\tau_A = 50.0$ N·m AND τ_f IN PART (a.) THE OBSERVED

MOTION IS THE RESULT OF BOTH THESE TORQUES

$\Sigma\tau_1 = \tau_A - \tau_f = I\alpha_1$ (1) UNKNOWNS: τ_f, I

(b.) $\omega \to 0$ in $\Delta t = 120$ s.

$\omega = 0 = \omega_f + \alpha_2 \Delta t$

$\alpha_2 = \frac{-\omega_f}{\Delta t} = \frac{-20\pi}{120} = -\frac{\pi}{6}$

$\Sigma\tau = \tau_f = I\alpha_2$ (2)

DIVIDE EQN (1) BY EQN (2)

$\frac{\alpha_1}{\alpha_2} = \frac{\tau_A - \tau_f}{\tau_f} = \frac{\tau_A}{\tau_f} - 1$

$\frac{\tau_A}{\tau_f} = \frac{\alpha_1}{\alpha_2} + 1$

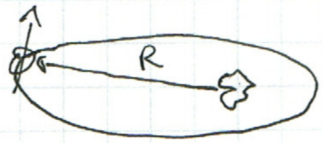
$\tau_f = \frac{\tau_A}{1 + \frac{\alpha_1}{\alpha_2}} = \frac{50}{1 + \frac{\pi}{\pi/6}} = \frac{50}{7}$

$\boxed{\tau_f = 7.14 \text{ N·m}}$

$I = \frac{\tau_f}{\alpha_2} = \frac{7.14}{\pi/6} = \boxed{13.6 \text{ kg·m}^2}$

Ans on HMWK Answers are found by using $\alpha_2 = -\pi/6$ above. These are incorrect. We are dealing in magnitudes with the sign difference explicitly shown in Eqn (1). Magnitudes are always positive.

CHAP 9 #66



EARTH

QUES: $\frac{KE_s}{KE_o} = ?$

$KE_s = \frac{1}{2} I_s w_s^2$ $\qquad$ $KE_o = \frac{1}{2} I_o w_o^2$

$I_s = \frac{2}{5} M r^2$ $\qquad$ $I_o = M R^2$

$w_s = \frac{2\pi}{T_s}$ $\qquad$ $w_o = \frac{2\pi}{T_o}$

$$\frac{KE_s}{KE_o} = \frac{\frac{1}{2} I_s w_s^2}{\frac{1}{2} I_o w_o^2} = \frac{I_s}{I_o}\left(\frac{w_s}{w_o}\right)^2$$

$$\frac{w_s}{w_o} = \frac{\frac{2\pi}{T_s}}{\frac{2\pi}{T_o}} = \frac{T_o}{T_s}$$

$$\frac{KE_s}{KE_o} = \frac{2}{5}\left(\frac{r}{R}\right)^2\left(\frac{T_o}{T_s}\right)^2$$

$$\frac{I_s}{I_o} = \frac{\frac{2}{5} M r^2}{M R^2} = \frac{2}{5}\left(\frac{r}{R}\right)^2$$

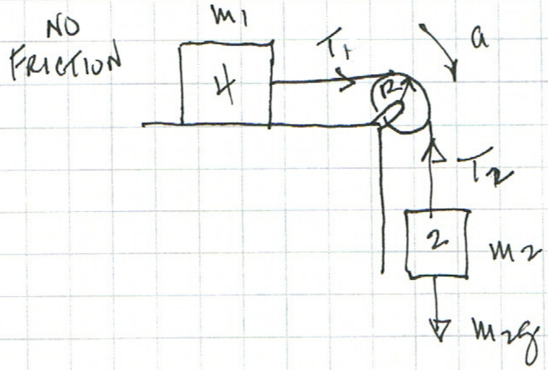
$$\frac{KE_s}{KE_o} = \frac{2}{5}\left(\frac{6.4 \times 10^6}{1.5 \times 10^{11}}\right)^2\left(\frac{365}{1}\right)^2 = \frac{2}{5}\left(\frac{6.4}{1.5}\right)^2 10^{-10} \cdot 1.33 \times 10^5$$

$$\frac{KE_s}{KE_o} = \frac{2}{5} \cdot 18.2\,(1.33) \times 10^{-5}$$

$$\frac{KE_s}{KE_o} = 9.6 \times 10^{-5}$$

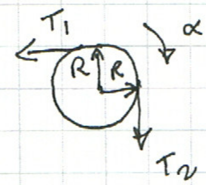
$$\therefore \frac{KE_o}{KE_s} = \frac{1}{9.6 \times 10^{-5}} = 10,417$$

CHAP 9 #71



GIVEN:
$m_1 = 4kg$
$m_2 = 2kg$
$R = 8cm = 0.08m$
$M = 0.60kg$

QUES:
$a = ?$
$T_1 = ?$
$T_2 = ?$



#1 $T_1 = m_1 a$

#2 $m_2 g - T_2 = m_2 a$

PULLEY $T_2 R - T_1 R = I\alpha$; NO SLIP CONDITION $\rightarrow$ $a = R\alpha$

$T_2 - T_1 = \frac{I\alpha}{R}$

SOLVE #2 FOR T_2

$T_2 = m_2 g - m_2 a$

$T_1 - T_2 = m_1 a - (m_2 g - m_2 a) = -\frac{I\alpha}{R}$ $\qquad I = \frac{1}{2}MR^2$

$(m_1 + m_2)a - m_2 g = -\frac{I\alpha}{R} = -\frac{\frac{1}{2}MR^2}{R} \cdot \frac{a}{R} = -\frac{1}{2}Ma$

$(m_1 + m_2 + \frac{M}{2})a = m_2 g$

$a = \frac{m_2 g}{m_1 + m_2 + \frac{M}{2}} = \frac{2(9.8)}{4 + 2 + \frac{0.60}{2}} = \frac{19.6}{6.30}$

$a = 3.11 \ m/s^2$

$T_1 = m_1 a = 4(3.11) = 12.4N$

$T_2 = m_2(g - a) \ 2(9.8 - 3.11)$

$T_2 = 13.4N$

CHAP 9 #79 (CONTINUED)

$$\alpha = \frac{(m_2 R_2 - m_1' R_1)\, g}{I + m_1' R_1^2 + m_2 R_2^2} = \frac{\text{Net}\,\tau}{I_T}$$

$$\alpha = \frac{[72(0.4) - 36(1.2)]\, 9.8}{40 + 36(1.2)^2 + 72(0.4)^2}$$

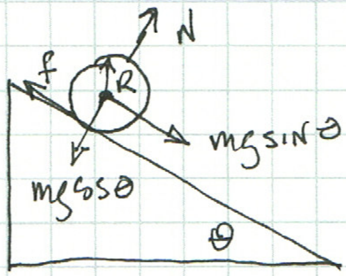
$$\alpha = \frac{-14.4(9.8)}{40 + 51.8 + 11.52} = \frac{-141.1}{103.3}$$

$$\boxed{\alpha = -1.37\ \text{rad/s}^2}$$

$$T_1 = m_1' g + m_1' R_1 \alpha$$
$$= m_1'(g + R_1 \alpha)$$
$$= 36(9.8 + 1.2(-1.37))$$
$$\boxed{T_1 = 293.8\ N}$$

$$T_2 = m_2 g - m_2 R_2 \alpha$$
$$= m_2(g - R_2 \alpha)$$
$$= 72(9.8 - 0.4(-1.37))$$
$$\boxed{T_2 = 745.1\ N}$$

CHAP 9 #88



GIVEN: $I = \frac{2}{5}mR^2$
$a = 0.20g$

QUES: $\theta = ?$

$f = \mu N \qquad \Sigma F_y = N - mg\cos\theta = 0 \longrightarrow N = mg\cos\theta$

$f = \mu mg\cos\theta$

$\Sigma F_x = mg\sin\theta - f = ma$

$mg\sin\theta - \mu mg\cos\theta = ma$

$(\sin\theta - \mu\cos\theta)g = a$ EQN (1)

$\Sigma \tau = I\alpha$

$fR = I\alpha = \frac{Ia}{R}$

$\mu mg\cos\theta R = \frac{2}{5}mR^2 \cdot \frac{a}{R}$

$\mu g\cos\theta = \frac{2}{5}a$ EQN (2)

TWO EQNS
TWO UNKNOWN
θ, μ

SUB (2) INTO (1)

$\sin\theta\, g - \frac{2}{5}a = a$

$\sin\theta = \frac{a + \frac{2}{5}a}{g} = \frac{7}{5}\frac{a}{g} = \frac{7}{5}\frac{0.20g}{g} = \frac{7}{5}(0.2)$

$\sin\theta = 0.28$

$\theta = \sin^{-1}(0.28) = 16.3°$