# Gravitation Problems & Solutions

Dr. Michael F. McGraw

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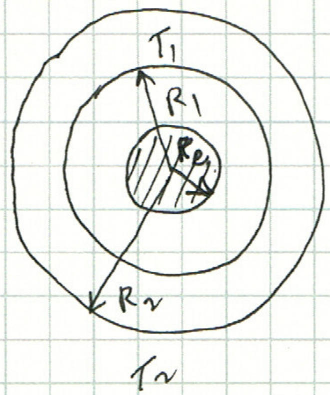
CHAP 11, #44, p. 10 — Ans: 0.51




$T_2 = 24$ hr
$T_1 = 12$ hr

$M_e = 5.98 \times 10^{24}$ kg
$R_e = 6.38 \times 10^6$ m

TWO SATELLITES ORBIT EARTH

QUES:

$$\frac{R_2 - R_1}{R_e} = ?$$

$$T^2 \propto r \quad \therefore \quad r \propto T^{2/3}$$

$$\frac{R_2}{R_1} = \left(\frac{T_2}{T_1}\right)^{2/3} = \left(\frac{24}{12}\right)^{2/3} = 2^{2/3} = 4^{1/3}$$

$$R_2 = 4^{1/3} R_1$$

$$R_2 - R_1 = (4^{1/3} - 1) R_1$$

$$\frac{R_2 - R_1}{R_e} = (4^{1/3} - 1)\frac{R_1}{R_e}$$

$$\frac{R_2 - R_1}{R_e} = 0.587 \frac{R_1}{R_e}$$

$$\frac{R_2 - R_1}{R_e} = 0.587\,(4.17)$$

$$\frac{R_2 - R_1}{R_e} = 2.45$$

$$T^2 = \frac{4\pi^2}{GM_e} r^3 \; ; \; GM_e = 9.8 R_e^2$$

$$T^2 = \frac{4\pi^2}{9.8 R_e^2} r^3 = \frac{4\pi^2 R_e}{9.8}\left(\frac{r}{R_e}\right)^3$$

$$\left(\frac{r}{R_e}\right)^3 = \frac{9.8 T^2}{4\pi^2 R_e}$$

$$\frac{r}{R_e} = \left(\frac{9.8\, T^2}{4\pi^2 R_e}\right)^{1/3}$$

$$\frac{r}{R_e} = \left[\frac{9.8\,(12 \text{ hr} \times 3600 \text{ sec/hr})^2}{4\pi^2 (6.38 \times 10^6)}\right]^{1/3}$$

$$\frac{r}{R_e} = \left[\frac{1.83 \times 10^{10}}{2.52 \times 10^8}\right]^{1/3}$$

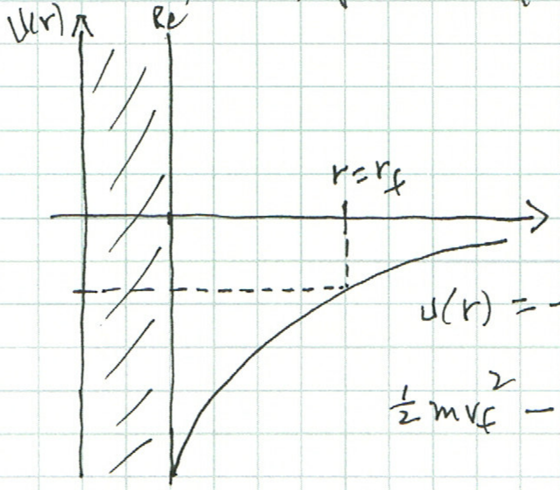
$$\frac{r}{R_e} = \left[\frac{183}{2.52}\right]^{1/3} = [72.6]^{1/3}$$

$$\frac{r}{R_e} = 4.17$$

CHAP 11, #51, p.12

QUES: WHAT IS r IF OBJECT LAUNCH STRAIGHT UP AT $V = V_{esc}/2$ FROM THE SURFACE OF THE EARTH.
($V_{esc} = 11{,}182$ m/s)

Ans. $8.50 \times 10^6$ m




$r_i = R_e$

$v_f = 0$

$v_i = V_{esc}/2 = \frac{11{,}182}{2} = 5591 \frac{m}{s}$

$$u(r) = -\frac{GM_e}{r}$$

$$\frac{1}{2}mv_f^2 - \frac{GM_e m}{r_f} = \frac{1}{2}mv_i^2 - \frac{GM_e m}{r_i}$$

$$0 - \frac{GM_e m}{r_f} = \frac{1}{2}mv_i^2 - \frac{GM_e m}{R_e}$$

MULTIPLY BY $\frac{-1}{GM_e m}$

$$\frac{1}{r_f} = -\frac{v_i^2}{2GM_e} + \frac{1}{R_e} = -\frac{v_i^2}{2(9.8)R_e^2} + \frac{1}{R_e}$$

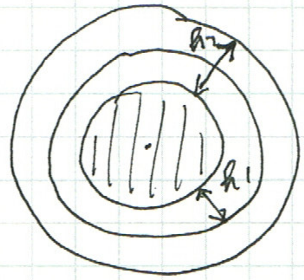
$$\frac{1}{r_f} = \frac{1}{6.37 \times 10^6} - \frac{(5591)^2}{19.6\,(6.37 \times 10^6)^2}$$

$$\frac{1}{r_f} = 1.570 \times 10^{-7} - 3.93 \times 10^{-8} = 1.18 \times 10^{-7}$$

$$r_f = (1.18 \times 10^{-7})^{-1}$$

$$\boxed{r_f = 8.50 \times 10^6 \text{ m}}$$

CHAP 11 #53, p.12




$h_1 = 1000\ km$

$h_2 = 1500\ km$

$m = 1000\ kg$

QUES: HOW MUCH WORK TO GO FROM ORBIT $h_1$ TO ORBIT $h_2$

$$KE_f + U_f = KE_i + U_i + W$$

$$\frac{1}{2}mv_f^2 - \frac{GM_e m}{r_f} = \frac{1}{2}mv_i^2 - \frac{GM_e m}{r_i} + W$$

SINCE $V = \frac{GM_e}{r}$

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{GM_e m}{r}$$

$r_f = R_e + 1500 = 7870\ km$

$r_i = R_e + 1000 = 7370\ km$

$$\frac{1}{2}\frac{GM_e m}{r_f} - \frac{GM_e m}{r_f} = \frac{1}{2}\frac{GM_e m}{r_i} - \frac{GM_e m}{r_i} + W$$

$$-\frac{GM_e m}{2r_f} = -\frac{1}{2}\frac{GM_e m}{r_i} + W$$

$$\frac{1}{2}GM_e m\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = W$$

$$W = \frac{m}{2}\, 9.8\, R_e^2\left(\frac{10^{-3}}{7370} - \frac{10^{-3}}{7870}\right)$$

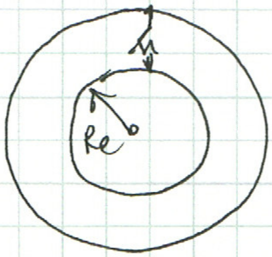
SINCE $9.8 = \frac{GM_e}{R_e^2}$

$$W = \frac{1000}{2}(9.8)(6370 \times 10^3)^2(8.62 \times 10^{-6})\, 10^{-3}$$

$$W = 500\,(3.98 \times 10^{14})(8.62 \times 10^{-9})$$

$$W = 1.72 \times 10^9$$

CHAP 11, #35, p. 8




$R_e = 6.436 \times 10^6$ m

$h = 3.20 \times 10^6$ m

QUES: WHAT IS ACCELERATION OF SATELLITE i.e. $g(h)$

$$g(r) = \frac{GM_e}{r^2}$$

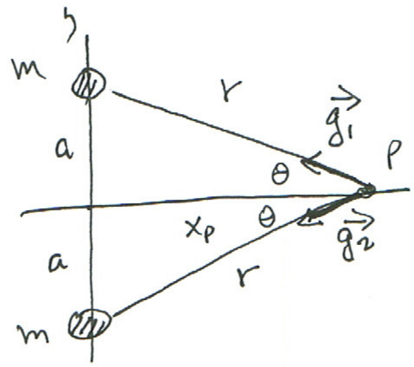
$$g(h) = \frac{GM}{(R_e+h)^2} = \frac{9.8\,R_e^2}{(R_e+h)^2}$$

$$g(r) = \frac{9.8\,R_e^2}{(R_e+h)^2} = \frac{9.8}{\left(1+\frac{h}{R_e}\right)^2}$$

$$g(r) = \frac{9.8}{\left(1+\frac{3.20}{6.44}\right)^2} = \frac{9.8}{(1.497)^2}$$

$$\boxed{g(r) = 4.37\ m/s^2}$$

# GRAVITATIONAL FIELD OF TWO POINT PARTICLES

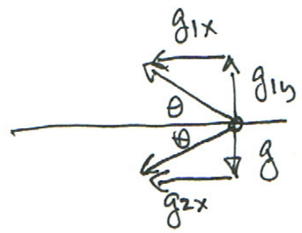



$r^2 = a^2 + x_p^2$

FIND $g(x)$ FOR ALL POINTS ON THE X-AXIS

THE MAGNITUDE OF $\vec{g}_1$ AND $\vec{g}_2$ ARE IDENTICAL

$$|\vec{g}_1| = |\vec{g}_2| \equiv g = \frac{GM}{r^2}$$




THE y-COMPONENTS CANCEL BY SYMMETRY

$$\vec{g} = \vec{g}_1 + \vec{g}_2$$

$\vec{g}$ ONLY HAS AN X- MPONENT

$$\vec{g} = g_{1x}(-\hat{\imath}) + g_{2x}(-\hat{\imath})$$

$$g_{1x} = g\cos\theta$$

$$g_{2x} = g\cos\theta$$

$$\vec{g} = -2g\cos\theta\,\hat{\imath}$$

$$\cos\theta = \frac{x_p}{r}$$

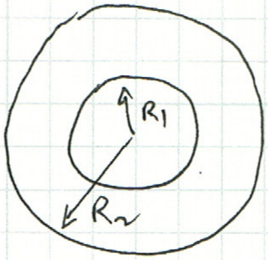
$$\cos\theta = \frac{x_p}{\sqrt{a^2 + x_p^2}}$$

$$\vec{g} = -\frac{2g\,x_p}{\sqrt{a^2 + x_p^2}}\,\hat{\imath}$$

$$\vec{g} = -\frac{2GM}{a^2 + x_p^2}\cdot\frac{x_p\,\hat{\imath}}{\sqrt{a^2 + x_p^2}} = \frac{-2GMx_p\,\hat{\imath}}{(a^2 + x_p^2)^{3/2}}$$

CHAP 11, #30, p.7

Ans: $4.38\ m/s^2$




QUES:
FIND $g(r)$ AT $r = R = 5000\,km$
THIS IS THE FIELD OUTSIDE BOTH MASSES, AT THE SURFACE OF THE LARGEST MASS

$\rho_1 = 4000\ kg/m^3$
$\rho_2 = 3000\ kg/m^3$

$R_1 = \frac{R}{2}$ $\quad R_2 = R$

$M_1 = \rho_1 \frac{4}{3}\pi R_1^3$ ; $M_2 = \rho_2 \cdot \frac{4}{3}\pi (R_2^3 - R_1^3)$

$M_1 = 4000\left(\frac{4}{3}\pi\right)(2500 \times 10^3)^3$ ; $M_2 = 3000\left(\frac{4}{3}\pi\right)(5000^3 - 2500^3)10^9$

$M_1 = 4 \times 10^3\left(\frac{4}{3}\pi\right)(2.5 \times 10^3)^3 10^9$ ; $M_2 = 3 \times 10^3\left(\frac{4}{3}\pi\right)(5^3 \times 10^9 - 2.5^3\, 10^9)10^9$

$M_1 = 4\left(\frac{4}{3}\pi\right)(2.5)^3\, 10^3 \cdot 10^9 \cdot 10^9$ ; $M_2 = 3\left(\frac{4}{3}\pi\right)(5^3 - 2.5^3)\, 10^{18} \cdot 10^3$

$M_1 = 261.8 \times 10^{21}$ $\qquad M_2 = 1374 \times 10^{21}$

#1 $g_1 = \frac{GM_1}{R^2}$ ; #2 $g_2 = \frac{GM_2}{R^2}$

$g(R) = g_1(R) + g_2(R)$

$g(R) = \frac{G(M_1 + M_2)}{R^2} = \frac{G(1636 \times 10^{21})}{R^2}$

$g(R) = \frac{6.67 \times 10^{-11}(1.636 \times 10^{24})}{(5.0 \times 10^6)^2} = \frac{6.67(1.636)}{5^2}\ \frac{10^{-11}\ 10^{24}}{10^{12}}$

$\boxed{g(R) = 0.436 \times 10 = 4.36\ m/s^2}$