Harmonic Oscillator Problems & Solutions

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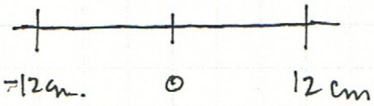
CHAPTER 14 #10, p. 2

QUES: MOTION STARTS AT $x = +12$ cm. WHAT IS THE TIME FOR THE PARTICLE TO MOVE TO $x = -6$ cm?



$|\leftarrow \Delta t = 2.25\,s \rightarrow|$

$T = 2\Delta t = 4.50\,s$

$\omega = \frac{2\pi}{T}$; $A = 12$ cm.

$$x(t) = A\cos\omega t = A\cos\left[\frac{2\pi}{T}t\right]$$

$$x(t) = -6 = 12\cos\left[\frac{2\pi}{4.50}t\right]$$

$$\cos\left[\frac{2\pi}{4.50}t\right] = -\frac{6}{12} = -\frac{1}{2}$$

$$t = \frac{4.50}{2\pi}\cos^{-1}(-1/2)$$

$$\cos^{-1}(-1/2) = 120^\circ$$

$$120^\circ \times \frac{\pi}{180} = \frac{2}{3}\pi$$

$$\boxed{t = \frac{4.50}{2\pi}\cdot\frac{2\pi}{3} = 1.50\,s}$$

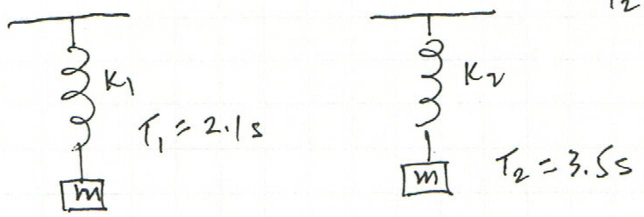
CHAPTER 14, #12, p. 3

SPRING #1 SPRING #2

$k_1 = 0.52\ N/m$ QUES: $k_2 = ?$

$T_1 = 2.1\,s$

$T_2 = 3.5\,s$



$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T_1 = 2\pi\sqrt{\frac{m}{k_1}}\ ; \quad T_2 = 2\pi\sqrt{\frac{m}{k_2}}$$

$$\frac{T_2}{T_1} = \frac{2\pi\sqrt{\frac{m}{k_2}}}{2\pi\sqrt{\frac{m}{k_1}}} = \sqrt{\frac{m}{k_2}}\cdot\sqrt{\frac{k_1}{m}} = \sqrt{\frac{k_1}{k_2}}$$

$$\left(\frac{T_2}{T_1}\right)^2 = \frac{k_1}{k_2}$$

$$k_2 = k_1\left(\frac{T_1}{T_2}\right)^2 = 0.52\left(\frac{2.1}{3.5}\right)^2$$

$$\boxed{k_2 = 0.187\ N/m}$$

Ans: 6.4 s

CHAP 14, #42, p. 9

QUES: $T = ?$

$T = \frac{2\pi}{\omega}$

$v = 6$ cm/s AT $x = 4$ cm

$v = 2$ cm/s AT $x = 7$ cm.

$x = A\cos\omega t$

$v = -\omega A\sin\omega t$

$E_T = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$\frac{k}{m}A^2 = v^2 + \frac{k}{m}x^2$

$\omega^2A^2 = v^2 + \omega^2x^2$

$\omega^2A^2 = 36 + \omega^2 16$

$\omega^2A^2 = 4 + \omega^2 49$

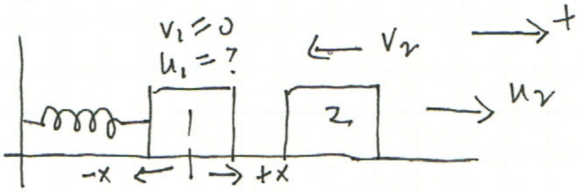
$36 + \omega^2 16 = 4 + \omega^2 \cdot 49$

$32 = 33\,\omega^2$

$\omega^2 = \frac{32}{33}$ $\qquad \omega = \sqrt{\frac{32}{33}}$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{33}{32}} = 6.38\,s}$$

CHAPTER 14 #48, p.11



$m_1 = 0.50$ kg
$m_2 = 0.20$ kg
$k = 600$ N/m
$A = 0.25$ m

QUES: $u_2 = ?$

UNKNOWNS: v_2, u_1, u_2

ELASTIC HEAD ON COLLISION

$m_2 < m_1 \rightarrow m_2$ WILL BOUNCE BACK

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$\frac{m_2}{m_1} v_2 = u_1 + \frac{m_2}{m_1} u_2$$

$$u_1 = \frac{m_2}{m_1}(v_2 - u_2) \quad (1)$$

MOTION OF m_1 - CONSERVATION OF ENERGY

$$KE_i + PE_i = KE_f + PE_f$$

$$KE_i + 0 = 0 + PE_f$$

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} k A^2$$

$$u_1^2 = \frac{k}{m_1} A^2 \quad (2)$$

SUB (1)

$$\left(\frac{m_2}{m_1}\right)^2 (v_2 - u_2)^2 = \frac{k}{m_1} A^2$$

$$(v_2 - u_2)^2 = \left(\frac{m_1}{m_2}\right)^2 \frac{k}{m_1} A^2$$

$$v_2 - u_2 = \pm \frac{m_1}{m_2} \sqrt{\frac{k}{m_1}} A \quad (3)$$

RELATIVE VELOCITY RELATIONSHIP

$$u_2 - u_1 = -(v_2 - v_1)$$

$$= -v_2$$

$$v_2 + u_2 = u_1 \quad (4)$$

ADD EQN (3) AND (4)

$$2v_2 = \pm \frac{m_1}{m_2} \sqrt{\frac{k}{m_1}} A + u_1 \quad (5)$$

WE CAN GET u_1 FROM (2) BUT WE HAVE TO WATCH THE SIGNS

$$u_1 = \pm \sqrt{\frac{k}{m}} A$$

WE KNOW $u_1 < 0$

$$u_1 = -\sqrt{\frac{600}{0.5}} \left(\frac{1}{4}\right) = -\frac{\sqrt{1200}}{4}$$

$$u_1 = -\frac{10}{4}\sqrt{12} = -\frac{10 \cdot 2\sqrt{3}}{4}$$

$$\boxed{u_1 = -5\sqrt{3} \text{ m/s}}$$

WE CAN GET v_2 FROM (5)

$v_2 < 0$

$$v_2 = -\frac{1}{2}\left(\frac{5}{2}\right)\sqrt{\frac{600}{0.5}}\left(\frac{1}{4}\right) - \frac{5\sqrt{3}}{2}$$

$$v_2 = -\frac{5}{16}\sqrt{1200} - \frac{5\sqrt{3}}{2} = -\frac{50}{8}\sqrt{3} - \frac{5\sqrt{3}}{2}$$

$$\boxed{v_2 = -\frac{70}{8}\sqrt{3} \text{ m/s} = -\frac{35}{4}\sqrt{3} \text{ m/s}}$$

FROM (4) $u_2 = u_1 - v_2 = -5\sqrt{3} - \left(-\frac{35}{4}\sqrt{3}\right)$

$$\boxed{u_2 = -\frac{20}{4}\sqrt{3} + \frac{35}{4}\sqrt{3} = +\frac{15}{4}\sqrt{3} \text{ m/s}}$$

CHAPTER 14 # 48, p. 11 CONTINUED

ELASTIC COLLISION $\rightarrow \Delta KE = 0$

$v_2 = -\frac{35}{4}\sqrt{3} = -15.2$ m/s

$u_1 = -5\sqrt{3} = -8.66$ m/s

$u_2 = \frac{15}{4}\sqrt{3} = +6.50$ m/s

$$\frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

$$v_2^2 = \frac{m_1}{m_2} u_1^2 + u_2^2$$

$$\left(\frac{35}{4}\sqrt{3}\right)^2 = \frac{5}{2}\left(5\sqrt{3}\right)^2 + \left(\frac{15}{4}\sqrt{3}\right)^2$$

$$\left(\frac{35}{4}\right)^2 = \frac{5}{2} \cdot 25 + \left(\frac{15}{4}\right)^2$$

$$76.56 = 62.5 + 14.06$$

$$76.56 \quad 76.56$$

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CHAP 14 #49, p. 11

UNKNOWNS: K, m, v

$E_T = \frac{1}{2}KA^2 = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$

$A = 4.0\text{ cm}$

$x = 2.0\text{ cm}$

$\frac{PE}{E_T} =$

$$\frac{PE}{E_T} = \frac{\frac{1}{2}Kx^2}{\frac{1}{2}KA^2} = \left(\frac{x}{A}\right)^2 = \left(\frac{2}{4}\right)^2 = \left(\frac{1}{2}\right)^2 = \boxed{\frac{1}{4}}$$

$w^2 = \frac{K}{m}$

$PE_{max} = KE_{max}$

$\frac{1}{2}KA^2 = \frac{1}{2}mv_{max}^2$

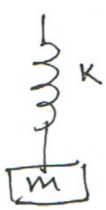
$\frac{K}{m}A^2 = v_{max}^2$

$\sqrt{\frac{K}{m}}A = v_{max}$

$wA = v_{max}$

$w = \frac{v_{max}}{A}$

CHAP #52, p. 11



$A = 10.0\text{ cm}$

$m = 2.5\text{ kg}$

$K = 4500\text{ N/m}$

QUES: KE WHEN $x = 5\text{ cm}$?

$E_T = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2 = \frac{1}{2}KA^2$

$KE = \frac{1}{2}K(A^2 - x^2)$

$KE = \frac{4500}{2}(0.10^2 - 0.05^2)$

$\boxed{KE = 16.9\text{ J}}$

CHAP 14 #56, p.12

QUES: WHAT IS THE POSITION OF THE MASS WHEN KE = PE ?

$A = 5.0\ cm$

$$E_T = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

WHEN KE AND PE ARE EQUAL THEY EACH ARE $E_T/2 = \frac{1}{4}kA^2$

$$PE = \frac{1}{2}kx^2 = \frac{E_T}{2} = \frac{1}{4}kA^2$$

$$x^2 = \frac{1}{2}A^2$$

$$\boxed{x = \pm\frac{A}{\sqrt{2}} = \pm\frac{5.0}{\sqrt{2}} = 3.53\ cm.}$$

CHAP 14 #74, p.18

QUES: IF THIS MASS IS SET INTO

A 0.10 kg MASS STRETCHES A MASSLESS SPRING 0.20m FROM ITS EQUILIBRIUM POSITION

$F = -kx$

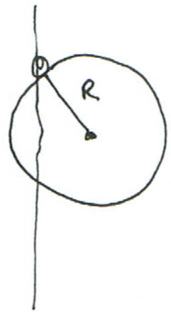
$mg = kx \qquad k = \frac{mg}{x} = \frac{0.10(9.8)}{0.20} = \frac{9.8}{2} = 4.90\ \frac{N}{m}$

$$w = \sqrt{\frac{k}{m}} \qquad f = \frac{w}{2\pi}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.90}{0.10}}$$

$$\boxed{f = 1.11\ Hz}$$

CHAPTER 14 #92, p. 22



QUES: $T = ?$

$$I_{cm} = \frac{1}{2}MR^2$$

$$T = 2\pi\sqrt{\frac{I}{MgD}}$$

$$I = I_{cm} + MR^2 = \frac{1}{2}MR^2 + MR^2$$

$$I = \frac{3}{2}MR^2$$

$$T = 2\pi\sqrt{\frac{\frac{3}{2}MR^2}{MgR}} = 2\pi\sqrt{\frac{3}{2}\frac{R}{g}}$$

$$\boxed{T = 2\pi\sqrt{\frac{3}{2}\frac{R}{g}}}$$

Ans: $\ell = 6.10$ cm.

CHAPTER 14 #102, p. 25

$f = 2.0$ Hz

$g = 9.81\ m/s^2$

QUES: WHAT IS THE LENGTH OF A SIMPLE PENDULUM THAT WOULD OSCILLATE AT THE SAME FREQ.

$f_c = f_s \rightarrow T_c = T_s$

$f_c = 2\text{Hz} \quad T_c = \frac{1}{2}$

COMPOUND

$$T_c = 2\pi\sqrt{\frac{I}{MgD}}$$

SIMPLE

$$T_s = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T_S = T_C$$

$$\sqrt{\frac{\ell}{g}} = \sqrt{\frac{I}{MgD}}$$

$$\frac{\ell}{g} = \frac{I}{MgD}$$

$$\ell = \frac{I}{MD}$$

$$T_c = \frac{1}{2} = 2\pi\sqrt{\frac{I}{MgD}}$$

$$\frac{1}{4\pi} = \sqrt{\frac{I}{MgD}}$$

$$\left(\frac{1}{4\pi}\right)^2 g = \frac{I}{MD}$$

$$\ell = \left(\frac{1}{4\pi}\right)^2 g = \frac{g}{16\pi^2} = 0.0621\ m$$

$$\boxed{\ell = 6.21\ \text{cm}}$$