Momentum Problems & Solutions
PHY 2425

Michael F. McGraw

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Rev Ques #34, p. 8

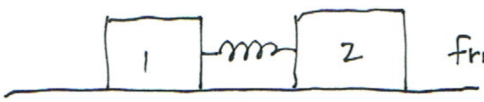
WHEN THE SPRING IS RELEASED AND THE MASSES MOVE APART THEY HAVE A TOTAL KINETIC ENERGY OF $KE_T = 16.8\,J$.

GIVEN: $m_1 = 2.5\,kg$

$m_2 = 4.0\,kg$

$KE_T = 16.8\,J$

$v_1 = ?$



$$P_T^{Before} = P_T^{After} = 0$$

$$0 = m_1 v_1 + m_2 v_2$$

$$v_1 = -\frac{m_2}{m_1} v_2$$

BOTH v_1 AND v_2 ARE UNKNOWN. WE NEED A SECOND EQN.
WE KNOW THE TOTAL KE

$$E_T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\frac{2E_T}{m_1} = v_1^2 + \frac{m_2}{m_1} v_2^2 = \left(-\frac{m_2}{m_1} v_2\right)^2 + \frac{m_2}{m_1} v_2^2$$

$$\frac{2E_T}{m_1} = \left(\frac{m_2}{m_1}\right)^2 v_2^2 + \frac{m_2}{m_1} v_2^2 = \frac{m_2}{m_1}\left(\frac{m_2}{m_1} + 1\right) v_2^2$$

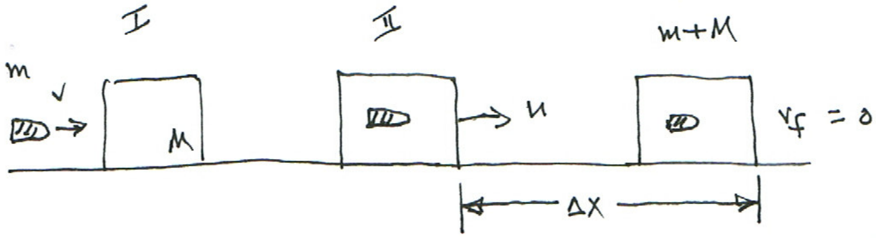
$$v_2^2 = \frac{\frac{2E_T}{m_1}}{\frac{m_2}{m_1}\left(1 + \frac{m_2}{m_1}\right)} = \frac{2E_T}{\frac{m_1 m_2}{m_1}\left(1 + \frac{m_2}{m_1}\right)} = \frac{2E_T}{m_2\left(1 + \frac{m_2}{m_1}\right)}$$

$$v_2^2 = \frac{2(16.8)}{4\left(1 + \frac{4.0}{2.5}\right)} = \frac{8.4}{1 + 1.6} = 3.23$$

$$v_2 = \sqrt{3.23} = \boxed{1.80\,m/s}$$

$$v_1 = -\frac{m_2}{m_1} v_2 = -\frac{4.0}{2.5}(1.80) = \boxed{-2.88\,m/s}$$

Rev. Ques #33. p. 8



GIVEN:

$M = 4.65$ kg

$m = 18$ g $= 0.018$ kg

$v = 725$ m/s

$\mu = 0.35$

QUES: $\Delta x = ?$

PORTION I IS DESCRIBED BY THE CONSERVATION OF MOMENTUM

$$mv = (m+M)u$$

$$u = \frac{m}{m+M} v = \frac{(0.018)(725)}{0.02 + 4.65} = 2.79 \text{ m/s}$$

PORTION II IS DESCRIBED BY THE WORK AGAINST FRICTION

$$\Delta KE = W$$

$$\underbrace{KE_f}_{0} - KE_i = F \cdot d = f \cdot \Delta x \cos(180)$$

$$-\tfrac{1}{2}(m+M)u^2 = -f\,\Delta x \qquad f = \mu N = \mu(m+M)g$$

$$\Delta x = \frac{\tfrac{1}{2}(m+M)u^2}{\mu(m+M)g} = \frac{u^2}{2\mu g}$$

$$\Delta x = \frac{(2.79)^2}{2(0.35)(9.8)} = \frac{7.80}{6.86} = \boxed{1.14 \text{ m.}}$$

#77, p.19

$p_i = 73\ \frac{kg\cdot m}{s}$ $\quad$ $p_f = 38\ \frac{kg\cdot m}{s}$ $\quad$ QUES: $|\vec{F}| = ?$

$\Delta t = 7.3$ s.

$$\Delta\vec{p} = \vec{F}\,\Delta t$$

$$p_f - p_i = F\Delta t \qquad F = \frac{p_f - p_i}{\Delta t} = \frac{38-73}{7.3} = -\frac{35}{7.3} = \boxed{-4.79\ N}$$

#81, p. 20 $\qquad$ DON'T ASSUME AN ELASTIC COLLISION

BEFORE: $v_1 = 0.5\ m/s$, $v_2 = 0.3\ m/s$ $\qquad m_1 = 2.5 m_2$

AFTER: $u_1 = ?$, $u_2 = +0.55\ m/s$

$$p^{BEFORE} = m_1 v_1 + m_2 v_2 = 2.5 m_2 (0.5) + m_2 (0.3)$$

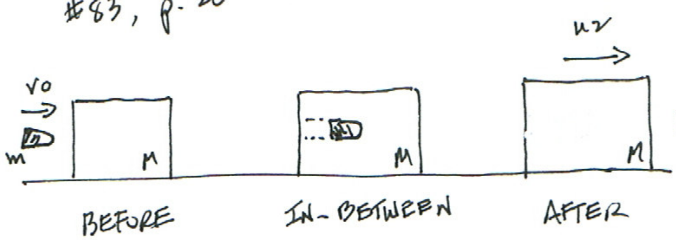
$$p^{AFTER} = m_1 u_1 + m_2 u_2 = 2.5 m_2 u_1 + m_2 (0.55)$$

m_2 CANCELS OUT

$$\frac{5}{4} + 0.3 = \frac{5}{2} u_1 + 0.55$$

$$u_1 = \frac{2}{5}(1.25 + 0.3 - 0.55) = \boxed{0.40\ m/s}$$

#83, p. 20



$u_1 = \frac{v_0}{3}$

QUES: SHOW $u_2 = \frac{2}{3}\frac{m}{M} v_0$

$$m v_0 + M(0) = M u_2 + m u_1 = M u_2 + m\left(\frac{v_0}{3}\right)$$

$$m v_0 = M u_2 + \frac{m v_0}{3}$$

$$M u_2 = m v_0 - \frac{m v_0}{3} = \frac{2}{3} m v_0$$

$$\boxed{u_2 = \frac{2}{3}\frac{m}{M} v_0}$$

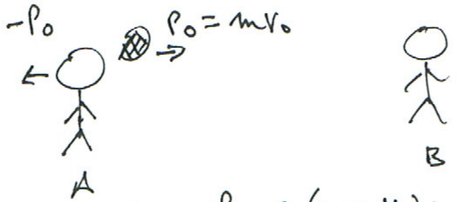
Rev. Ques. #6, p. 2

THIS PROBLEM CONTAINS ELASTIC AND INELASTIC COLLISIONS. THE DIRECTIONS OF THE MOMENTUM VECTORS ARE HANDLED IN THE PICTURES. THE VARIABLES REPRESENT THE MAGNITUDES.

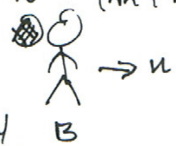
TWO EQUAL MASS PERSONS TOSS A BALL BACK AND FORTH ON AN FRICTIONLESS ICE POND.

GIVEN: $m_A = m_B = M = 90.5\text{ kg}$
$v_0 = 21.5\text{ m/s}$
$m = 0.200\text{ kg}$

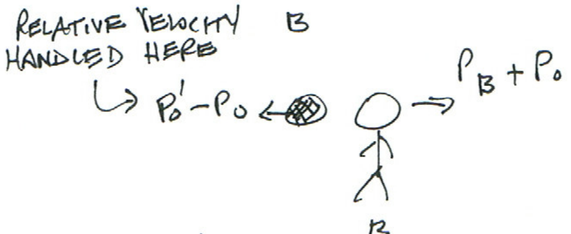
QUES: $v_A = ?$ AFTER CATCHING THE BALL.



$$P_0 = (m+M)u = P_0' + P_B$$
$$ \;\; mu + Mu$$

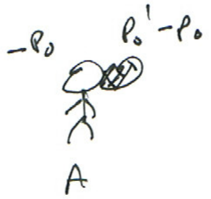


$$P_0 = (m+M)u$$
$$mv_0 = (m+M)u$$
$$u = \frac{m}{m+M} v_0$$



HANDLING THE RELATIVE VELOCITY

PERSON B IS THROWING AN OBJECT FROM A MOVING PLATFORM. THE MOMENTUM EQN NEEDS THE VELOCITY OF THE OBJECT RELATIVE TO THE GROUND.



$$(m+M)v_f = -p_0 + (p_0' - p_0)$$
$$= -2p_0 + p_0'$$
$$= -2(mv_0) + mu$$
$$= -2mv_0 + m\left(\frac{m}{m+M}\right)v_0 = \left[-2 + \frac{m}{m+M}\right]mv_0$$

$$(m+M)v_f = \left[-2 + \frac{0.200}{90.7}\right](0.20)(21.5) = -8.60\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$v_f = \frac{-8.60}{m+M} = \frac{-8.60}{90.7} = \boxed{-9.48 \times 10^{-2}\text{ m/s}}$$

Rev Ques #44, p. 11

THE SLED JUMPER

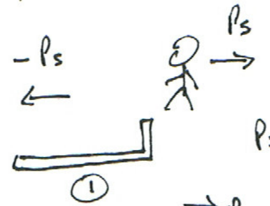
TWO SLEDS ARE ALIGNED HEAD TO TAIL. A PERSON JUMPS FROM THE FIRST SLED TO THE SECOND AND THEN IMMEDIATELY JUMPS BACK TO THE FIRST SLED.
INITIALLY ALL OBJECTS ARE AT REST.

GIVEN: $m = 40\ kg$
$m_s = 20\ kg$

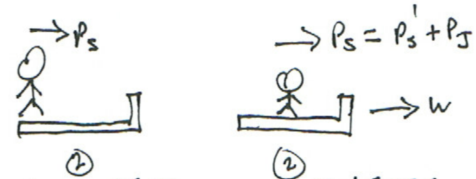
QUES: WHAT IS THE VELOCITY OF THE 2ND SLED AFTER THE SECOND JUMP?

JUMPERS HORIZONTAL SPEED RELATIVE TO THE GROUND IS GIVEN AS 2m/s

$v_0 = 2\ m/s$



$P_s = m_s v_0 = 20(2) = 40\ \frac{kg\cdot m}{s}$



$$P_s + 0 = (m_s + m)u = \underbrace{m_s u}_{P_s'} + \underbrace{m u}_{P_J}$$

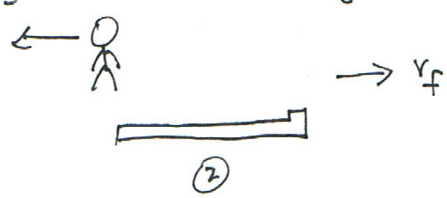
$$u = \frac{P_s}{m_s + m}$$

$$u = \frac{m_s v_0}{m_s + m}$$

$$u = \frac{v_0}{1 + \frac{m}{m_s}} = \frac{2}{1 + \frac{40}{20}} = \frac{2}{3} = 0.67\ m/s$$

RELATIVE VELOCITY HANDLED HERE
$\hookrightarrow P_J - P_J'$

$$P_J' + P_s' = m_s v_f$$



RELATIVE VELOCITY LINE

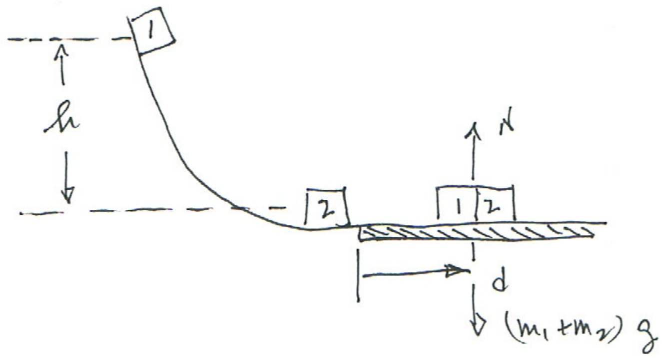
$$P_J - P_J' = P_J' + P_s' = m_s v_f$$

$$m u - m(u - v_0) = m_s v_f$$

$$m v_0 = m_s v_f$$

$$v_f = \frac{m}{m_s} v_0 = \frac{40}{20}(2) = 4\ m/s$$

Rev Ques. # 89, p.22



GIVEN:

$h = 1.5\,m$
$m_1 = 5\,kg$
$m_2 = 8\,kg$
$\mu = 0.65$

QUES:

WHAT IS d WHEN THE BLOCKS STOP MOVING.

$\Sigma F_y = N - (m_1 + m_2)g = 0$

$N = (m_1 + m_2)g$

$f = \mu N = \mu (m_1 + m_2) g$

A.) Drop: Cons of Energy

B.) HITTING + STICKING TO #2: CONS. OF MOMENTUM

C.) SLOW DOWN: WORK AGAINST FRICTION

A.) $mgh = \frac{1}{2} m_1 v_0^2 \longrightarrow v_0^2 = 2gh$

B.) $m_1 v_0 = (m_1 + m_2) u \longrightarrow u = \frac{m_1}{m_1 + m_2} v_0$

C.) $\Delta KE = WORK = f d \cos(180)$

$KE_f - KE_i = -fd$

$-KE_i = -fd$

$d = \frac{KE_i}{f}$

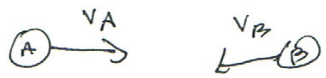
$$d = \frac{\frac{1}{2}(m_1 + m_2) u^2}{\mu (m_1 + m_2) g} = \frac{u^2}{2\mu g} = \left(\frac{m_1}{m_1 + m_2}\right)^2 \frac{v_0^2}{2\mu g}$$

$$d = \left(\frac{m_1}{m_1 + m_2}\right)^2 \frac{2gh}{2\mu g} = \left(\frac{m_1}{m_1 + m_2}\right)^2 \frac{h}{\mu}$$

$$d = \left(\frac{5}{13}\right)^2 \frac{1.5}{0.65} = 0.34\,m$$

#28 p.7 ONE DIMENSIONAL ELASTIC COLLISION

QUES: FIND u_A AND u_B AFTER THE COLLISION



GIVEN:

$m_A = 1\ 0\,g$ $\qquad m_B = 60\,g$ $\qquad \frac{m_A}{m_B} = \frac{120}{60} = 2 = M$

$v_A = 80\ m/s$ $\qquad v_B = -60\ m/s$

$$\Sigma \vec{p}_i = \Sigma \vec{p}_f$$

$$140(80) + 60(-60) = m_A u_A + m_B u_B$$

$$11,700 - 3600 = 7600 = m_A u_A + m_B u_B =$$

ELASTIC $\rightarrow \Delta KE = 0$

$$m_A v_A^2 + m_B v_B^2 = m_A u_A^2 + m_B u_B^2$$

$$\frac{m_A}{m_B} v_A^2 + v_B^2 = \frac{m_A}{m_B} u_A^2 + u_B^2$$

$$\boxed{M v_A^2 + v_B^2 = M u^2 + u_B^2}$$

REDOING MOMENTUM

$$\frac{m_A}{m_B} 80 + (-60) = \frac{m_A}{m_B} u_A + u_B$$

$$\boxed{80M - 60 = M u_A + u_B}$$

$$u_B = 80M - u_A M - 60 = (80 - u_A)M - 60$$

$$M(80)^2 + (-60)^2 = M u_A^2 + u_B^2$$

$$6400M + 3600 = M u_A^2 + [(80 - u_A)M - 60]^2 \checkmark$$

$$6400M + 3600 = M u_A^2 + (80 - u_A)^2 M^2 - 120(80 - u_A)M + 3600$$

$$6400M = M u_A^2 + (6400 - 160u_A + u_A^2)M^2 - 9600M + 120 M u_A$$

#28 p.7 (CONTINUED)

$$6400M = Mu_A^2 + 6400M^2 - 160M^2u_A + M^2u_A^2 - 9600M + 120M\,u_A$$

$$6400M = M(M+1)\,u_A^2 + (120M - 160M^2)u_A + 6400M^2 - 9600M$$

$$0 = \underbrace{[M(M+1)]}_{A}u_A^2 + \underbrace{[(120-160M)M]}_{B}u_A + \underbrace{[6400M^2 - 16000M]}_{C}$$

$$0 = Au_A^2 + Bu_A + C$$

$$u_A = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

WITH $M = 2$

$$A = M(M+1) = 2 \cdot 3 = \underline{\underline{6}}$$

$$B = [120 - 160(2)]2 = (120 - 320)2 = -400$$

$$C = 6400(2)^2 - 16{,}000(2)$$

$$C = 25{,}600 - 32{,}000$$

$$C = \underline{\underline{-6400}}$$

$$u_A = \frac{+400 \pm \sqrt{400^2 - 4(6)(-6400)}}{12}$$

$$u_A = \frac{+400 \pm \sqrt{160000 + 153{,}600}}{12}$$

$$u_A = \frac{+400 \pm \sqrt{313{,}600}}{12} = \frac{400 \pm 560}{12}$$

$$u_A^+ = \frac{960}{12} = 80 \text{ m/s}$$

$$u_A^- = -\frac{160}{12} = \boxed{-13.3 \text{ m/s}}$$

$$u_B^+ = (80 - u_A)M - 60$$

$$u_B^+ = (80 - u_A)2 - 60$$

$$u_B^+ = (80 - 80)2 - 60$$

$$= -60 \text{ m/s}$$

$$u_B^- = (80 - (-13.3))2 - 60$$

$$= 93.3(2) - 60$$

$$= 186.6 - 60$$

$$u_B^- = \boxed{126.6 \text{ m/s}}$$

#28 p.7. METHOD USING THE RELATIVE VELOCITY RELATIONSHIP

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

$$v_1 - v_2 = u_2 - u_1 \qquad \text{ELASTIC COLLISION}$$

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 \qquad \text{MOMENTUM CONSERVATION}$$

$$\left.\begin{aligned} \frac{m_1}{m_2} v_1 + v_2 &= \frac{m_1}{m_2} u_1 + u_2 \\ v_1 - v_2 + u_1 &= u_2 \end{aligned}\right\}$$

$$\frac{m_1}{m_2} v_1 + v_2 = \frac{m_1}{m_2} u_1 + \left(v_1 - v_2 + u_1\right) \qquad \text{SUB FROM REL VELOCITY}$$

$$\left(\frac{m_1}{m_2} - 1\right) v_1 + 2v_2 = \left(\frac{m_1}{m_2} + 1\right) u_1$$

$$u_1 = \frac{\left(\frac{m_1}{m_2} - 1\right) v_1 + 2v_2}{1 + \frac{m_1}{m_2}} = \frac{(2-1) v_1 + 2v_2}{3} = \frac{80 + 2(-60)}{3}$$

$$u_1 = -\frac{40}{3} = -13.9 \text{ m/s}$$

$$u_2 = v_1 - v_2 + u_1 = 80 - (-60) + (-13.9)$$

$$u_2 = 140 - 13.9 = 126.1 \text{ m/s}$$