Rotation Problems

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CHAP 9

A DISK ROTATES WITH A CONSTANT ANGULAR ACCEL
IT TAKES 10 REV TO REACH AN ANGULAR SPEED OF ω.

#6

GIVEN: $\Delta\theta = 10$ rev
α = CONSTANT
$\omega_0 = 0$

QUES: HOW MANY MORE REVS ARE NEEDED TO REACH $\omega' = 2\omega$.

$\omega \rightarrow \omega' = 2\omega$

$$\omega_0 \rightarrow \omega \rightarrow \omega' = 2\omega$$

$$|\leftarrow \Delta\theta \rightarrow|\leftarrow \Delta\theta' \rightarrow|$$

WRITE A RELATIONSHIP FOR ω

$\omega = \omega_0 + \alpha\,\Delta t$ NO

$\omega_{AVG} = \dfrac{\Delta\omega}{2} = \dfrac{\omega - 0}{2}$ NO

YES!

$$\omega^2 = \omega_0^2 + 2\alpha\,\Delta\theta = 2\alpha\,\Delta\theta$$

WRITE AN IDENTICAL RELATIONSHIP FOR ω'

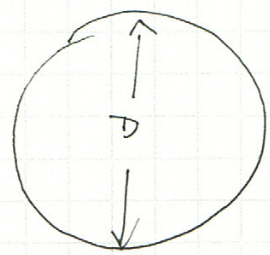
$$(\omega')^2 = \omega^2 + 2\alpha\,\Delta\theta'$$
$$(2\omega)^2 = \omega^2 + 2\alpha\,\Delta\theta'$$
$$4\omega^2 = \omega^2 + 2\alpha\,\Delta\theta'$$
$$3\omega^2 = 2\alpha\,\Delta\theta'$$

SUB RELATIONSHIP FOR ω' IN TERMS OF ω

$$\Delta\theta' = \frac{3\omega^2}{2\alpha} = \frac{3}{2\alpha}\cdot 2\alpha\,\Delta\theta = 3\,\Delta\theta = 3(10) = 30 \text{ rev}.$$

SOLVE FOR QUANTITY OF INTEREST

#7



$D = 135$ m

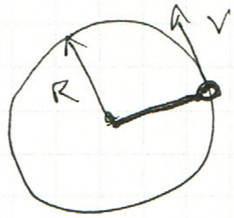
$T = 30$ min $= 30 \text{ min} \dfrac{60 \text{ sec}}{\text{min}} = 1800$ sec

QUES:
V AT PERIMETER = ?

$$V = \omega R = \frac{2\pi}{T}\cdot\frac{D}{2} = \frac{\pi D}{T} = \frac{3.14(135)}{1800} = 0.236 \text{ m/s}$$

CHAP 9

#11.

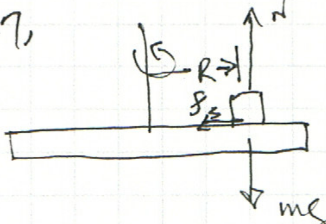


$R = 0.65\,m$
$f = 4.0$ rev/s

QUES: THE STRING BREAKS. AT THAT MOMENT, WHAT IS v?

$v = \omega R = 2\pi f \cdot R = 2\pi(4)(0.65) = 16.3\ m/s$
TANGENT TO CIRCLE.

#27,



$f_s = \mu_s N = \mu_s mg$

$\Sigma F = -f_s = -\dfrac{mv^2}{R}$

$f_s = \dfrac{mv^2}{R}$

$\dfrac{mv^2}{R} \leq f_s = \mu_s mg$

$\dfrac{\cancel{m}v^2}{\mu_s \cancel{m}g} \leq R$

$R \geq \dfrac{v^2}{\mu_s g} = \dfrac{\omega^2 R^2}{\mu_s g}$

$1 \geq \dfrac{\omega^2 R}{\mu_s g}$

$\dfrac{\mu_s g}{\omega^2} \geq R$

QUES: $R_{max} = ?$

$\mu_s = 0.2$
$\omega = 1.4$ rad/s

1. DRAW + LABEL PICTURE

2. ANALYZE WHAT IS GOING ON
STATIC FRICTION IS PROVIDING THE CENTRIPETAL F
$f_s \leq \mu_s mg$

FN FIXED ω; $v = R\omega$
$F_c = \dfrac{mv^2}{R} = mR\omega^2$

AS R INCREASES F_c INCREASES. ABOVE A CERTAIN VALUE OF $R = R_c$
$F_c \geq f_s$.

WE WANT R WHEN
$F_c = f_s$

$R \leq \dfrac{\mu_s g}{\omega^2} = \dfrac{(0.2)(9.8)}{(1.4)^2}$

$R \leq 1.0\,m$

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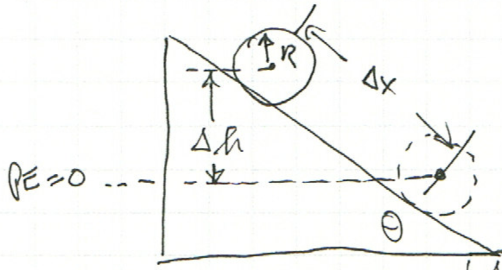

GIVEN: SOLID SPHERE

$I = 0.4MR^2$

$R = 0.06\,m$

$m = 0.50\,kg$

$\boxed{\text{ROLLS WITHOUT SLIPPING.} \quad V_{cm} = R\omega}$

$\Delta x = 14\,m \;;\; \theta = 30^\circ$

QUES: V_{cm} AT BOTTOM OF INCLINE = ?

$\boxed{\text{Ans: } 9.9\ m/s}$

$\boxed{\Delta h = \Delta x \sin\theta}$

STARTING: $E_T = PE_{AT\,TOP} = KE_{cm} + KE_{ROT}$

$$mg\Delta h = \tfrac{1}{2} m v_{cm}^2 + \tfrac{1}{2} I \omega^2$$

NO-SLIPPING

$$= \tfrac{1}{2} m v_{cm}^2 + \tfrac{1}{2}(0.4MR^2)\left(\frac{v_{cm}}{R}\right)^2$$

$$= \tfrac{1}{2} m v_{cm}^2 + \tfrac{1}{2}\left(\tfrac{2}{5}\right) MR^2 \frac{v_{cm}^2}{R^2}$$

$$mg\Delta h = \tfrac{1}{2} m v_{cm}^2 + \tfrac{1}{5} M v_{cm}^2 = \tfrac{7}{10} m v_{cm}^2$$

$$\cancel{m} g\Delta h = \tfrac{7}{10} \cancel{m} v_{cm}^2$$

VERT TO INCLINE DISTANCE

$$v_{cm}^2 = \frac{10}{7} g\Delta h = \frac{10}{7} g \Delta x \sin\theta$$

$$v_{cm}^2 = \frac{10}{\cancel{7}}(9.8)(\cancel{14}) \sin 30 = 98.0$$

$$\boxed{v_{cm} = \sqrt{98.0} = 9.9\ m/s}$$

CHAP 9

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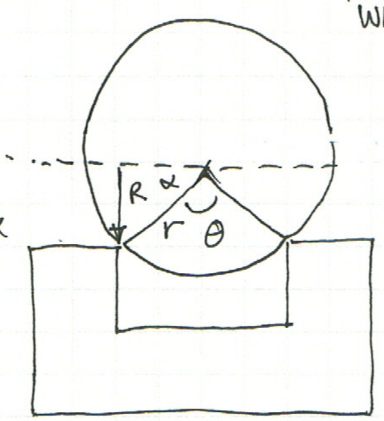
A SOLID SPHERE IS ROLLING IN A GROOVE. WHAT IS ITS ROTATIONAL KE ?



$R = r\sin\alpha$

SOLID SPHERE

$r = 5\,cm = 0.05\,m$

$m = 0.2\,kg$

$\theta = 120°$

$V_{cm} = 10\,mm/s = 10^{-2}\,m/s$

QUES: $KE_R = ?$

STARTING POINT: $KE_R = \frac{1}{2} I \omega^2$

$I = \frac{2}{5} m r^2$

INDEPENDENT OF HOW IT ROLLS BECAUSE IT IS A SPHERE

$$KE_R = \frac{1}{2}\left(\frac{2}{5} m r^2\right)\left(\frac{V_{cm}}{R}\right)^2$$

$V_{cm} = R\omega$

NON-SLIP CONDITION

$$KE_R = \frac{1}{5} m r^2 \frac{V_{cm}^2}{R^2} = \frac{1}{5} m \left(\frac{r}{R}\right)^2 V_{cm}^2$$

$$KE_R = \frac{1}{5} m \left(\frac{r}{r\sin\alpha}\right)^2 V_{cm}^2$$

$2\alpha + \theta = 180$

$2\alpha = 180 - 120 = 60$

$\alpha = 30°$

$$KE_R = \frac{1}{5} \frac{m V_{cm}^2}{\sin^2 30} = \frac{1}{5} \frac{(0.20)(10^{-2})^2}{(1/2)^2}$$

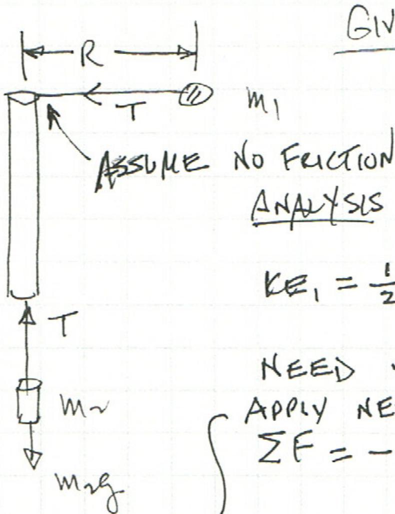
$$KE_R = \frac{1}{5} \cdot 4 \left(\frac{2}{10}\right) 10^{-4} = \frac{8}{5} \times 10^{-5}$$

$$KE_R = 1.6 \times 10^{-5} = 16 \times 10^{-6}$$

$$\boxed{KE_R = 16\,\mu J}$$

CHAP 9

\#48



GIVEN: $R = 0.800\,m$

$m_2 = 5.00\,kg$

CONS. VELOCITY

QUES: $KE_1 = ?$

Ans: 19.6 J

ANALYSIS: m_1 IN CIRCULAR MOTION AT $v = $ CONST AND RADIUS $= R$

$$KE_1 = \frac{1}{2} m_1 v^2 = \frac{1}{2} I \omega^2 \qquad v = R\omega.$$

NEED v OR ω

APPLY NEWTON'S LAW TO BOTH MASSES

$$\left\{ \begin{array}{l} \Sigma F = -T = -\dfrac{m_1 v^2}{R} \\ \\ \Sigma T - m_2 g = 0 \quad T = m_2 g \end{array} \right.$$

SOLVE FOR v: $\quad T = m_2 g = \dfrac{m_1 v^2}{R}$

$$v^2 = \frac{m_2}{m_1} g R$$

SUB INTO KE_1

$$KE_1 = \frac{1}{2} m_1 v^2 = \frac{1}{2} m_1 \left(\frac{m_2}{m_1} \right) g R$$

$$KE_1 = \frac{1}{2} m_2 g R = \frac{1}{2}(5)(9.8)(0.8) = 2(9.8) = 19.6\,J$$

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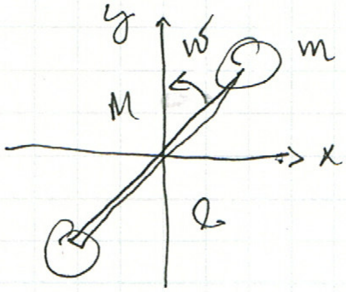
GIVEN:
2 SMALL MASSES m
BAR OF MASS M; LENGTH ℓ
ANGULAR SPEED $= \omega$

ROTATION IS IN X-Y PLANE



$\vec{L} = 0$ direction is $\hat{k}$

QUES: FIND $\vec{L}$

$$I_T = I_M + I_m$$
$$= \frac{1}{12}M\ell^2 + 2\left(m\left(\frac{\ell}{2}\right)^2\right)$$
$$= \frac{1}{12}M\ell^2 + \frac{m\ell^2}{2}$$

$$\vec{L} = \left(\frac{1}{12}M + \frac{m}{2}\right)\omega\ell^2$$
$$= \frac{1}{2}\left(m + \frac{1}{6}M\right)\omega\ell^2\,\hat{k}$$

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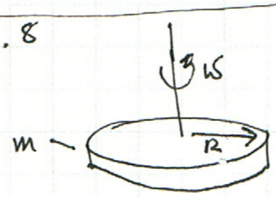


$R = 8.0$ cm
$m = 3.0$ kg
$f = 600 \frac{\text{rev}}{\text{min}}$

Run 10 s after power off

QUES: KE_i WHEN POWER OFF

$$KE = \frac{1}{2}I\omega^2 \qquad I = \frac{1}{2}mR^2$$

$$\omega = 2\pi f = 2\pi \frac{600 \text{ rev}}{\text{min}} \frac{1 \text{ min}}{60 \text{ s}}$$
$$= 2\pi(10) = 20\pi \frac{\text{rad}}{\text{s}}$$

$$KE = \frac{1}{2}\left(\frac{1}{2}MR^2\right) = \frac{1}{4}MR^2\omega^2$$

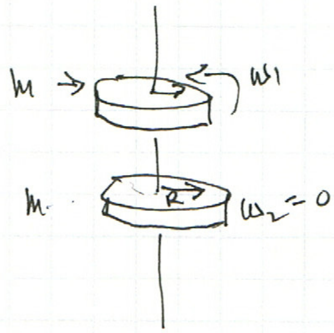
$$KE = \frac{1}{4}MR^2(2\pi 10)^2 = \frac{400\pi^2}{4}MR^2 = 100\pi^2 MR^2$$

$$KE = 100\pi^2(3)(0.08)^2$$

$$KE = 18.9 \text{ J}$$

NOTE: THE 10 s RUN TIME AFTER THE POWER IS TURNED OFF IS NOT NEEDED.

Ch 10 #36



DISK 1 IS INITIALLY SPINNING WHILE DISK 2 IS AT REST. THE TWO ARE BROUGHT INTO CONTACT AND STICK TOGETHER. WHAT HAPPENS?

ANGULAR MOMENTUM WILL BE CONSERVED SINCE $\sum \tau_{EXT} = 0$

$L_i = I_1 \omega_1$ $\qquad$ $L_f = (I_1 + I_2)\omega_f$ $\qquad$ $I_1 = I_2 = \frac{1}{2}MR^2 = I$

$$L_i = L_f$$
$$I_1\omega_1 = (I_1 + I_2)\omega_f$$
$$I_1\omega_1 = 2I_1\omega_f$$

$$\omega_f = \frac{\omega_1}{2}$$

$KE_i = \frac{1}{2}I\omega_1^2$ $\qquad$ $KE_f = \frac{1}{2}I\omega_f^2 + \frac{1}{2}I\omega_f^2$

$$= I\omega_f^2 = I\left(\frac{\omega_1}{2}\right)^2 = \frac{I\omega_1^2}{4} = \frac{1}{2}\left(\frac{1}{2}I\omega_1^2\right)$$

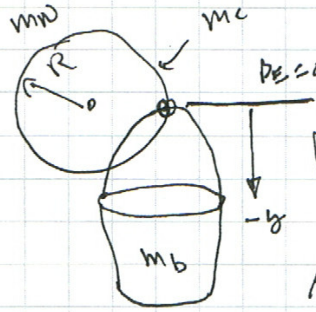
$$KE_f = \frac{1}{2}KE_i$$

IN GENERAL

$\omega_f = \frac{I_1}{I_1 + I_2}\omega_1$ $\qquad$ $\frac{KE_f}{KE_i} = \frac{\frac{1}{2}(I_1 + I_2)\omega_f^2}{\frac{1}{2}I_1\omega_1^2} = \left(\frac{I_1 + I_2}{I_1}\right)\frac{1}{\omega_1^2}\left(\frac{I_1}{I_1 + I_2}\cdot\omega_1\right)^2$

$\frac{KE_f}{KE_i} = \frac{I_1}{I_1 + I_2}$ $\qquad$ APPLIED TO ABOVE EXAMPLE $I_1 = I_2 = I$

$$\frac{KE_f}{KE_i} = \frac{1}{2}$$



BUCKET DROPS A DISTANCE d
UNWINDING ROPE OFF THE WINCH

PULLEY IS MASSIVE – IT HAS INERTIA
AND IT WILL SLOW THINGS DOWN

A LENGTH OF MASSIVE CABLE WILL ALSO
DROP A DISTANCE d.

USE CONSERVATION OF MECH ENERGY

LOOKING FOR v_b (THE ROPE IS TRAVELING AT THE SAME SPEED)

MEASURE DISTANCE FROM THE TOP ~~##~~ OF THE HANDLE ON THE BUCKET
THIS IS ZERO LEVEL OF PE (GOES THRU CM OF WINCH)

∴ WINCH — ONLY ROTATIONAL KE m_w (TREAT AS LOOP); R
CABLE TRANS KE ; GPE m_L; LENGTH L
BUCKET TRANS KE ; GPE m_b

U = GPE
WINCH CM DOESN'T MOVE $\Delta U_w = 0$

WINCH $KE_R = \frac{1}{2} I \omega^2$

$= \frac{1}{2} m_w R^2 \left(\frac{v}{R}\right)^2$

$= \frac{1}{2} m_w v^2$

CABLE EXTENDS FROM THE CONTACT POINT ON THE WINCH TO THE BUCKET HANDLE OVER A LENGTH d

TWO WAYS TO LOOK AT IT CABLE CM FALLS d

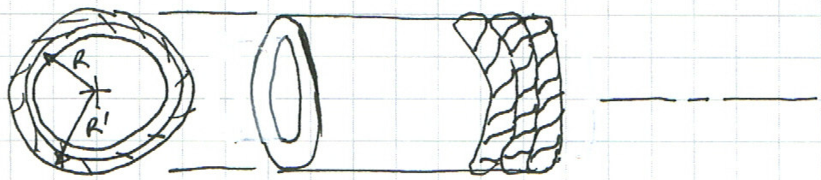
OR THE LENGTH OF CABLE d FALLS AN AVG DIS. $\frac{d}{2}$

CABLE ON WINCH HAS KE

BUCKET

BUCKET HANDLE (AND THE BUCKET CM) FALL A DISTANCE d

WHAT ABOUT COIL OF CABLE STILL ON WINCH?



DIAMETER OF CABLE $= D_c$

RADIUS OF WINCH $= R$

MOMENT INERTIA OF CABLE $= I_c = \frac{1}{2} M_c R'^2 \qquad R' = R + \frac{D_c}{2}$

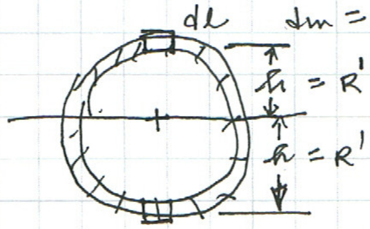
THIS ADDS TO I_W

$$I_T = I_W + I_c = \frac{1}{2} m_W R^2 + \frac{1}{2} m_c R'^2$$

FOR A LOOP GEOMETRY $KE_{ROT} = KE_T$

$U_G = GPE$ FOR THE CABLE

$dl \quad dm = \rho\, dl$



AVG GPE FOR CABLE $= 0$
(FOR A FULL LOOP)

ONLY WHEN THE CABLE UNWINDS FROM THE WINCH IS ITS MASS ABLE TO APPLY A TORQUE TO THE WINCH.

BY PLACING THE GPE $= 0$ LEVEL AT THE LEVEL OF THE AXIS OF ROTATION MAKE AVG GPE $= 0$

$$E_{T_1} = 0 = U_f + KE_f$$

$$+ m_b g(-d) + m_c' g\left(-\frac{d}{2}\right) + \frac{1}{2} m_c v^2 + \frac{1}{2} m_b v^2 + \frac{1}{2} m_w v^2$$

$$-\left(m_b + \frac{1}{2} m_c'\right) g d + \frac{1}{2}\left(m_c + m_b + m_w\right) v^2$$

$$\frac{m_c'}{d} = \frac{m_c}{L}$$

$$v = \sqrt{\frac{(2 m_b L + m_c d) g d}{(m_c + m_b + m_w) L}}$$

$$m_c' = m_c \frac{d}{L}$$