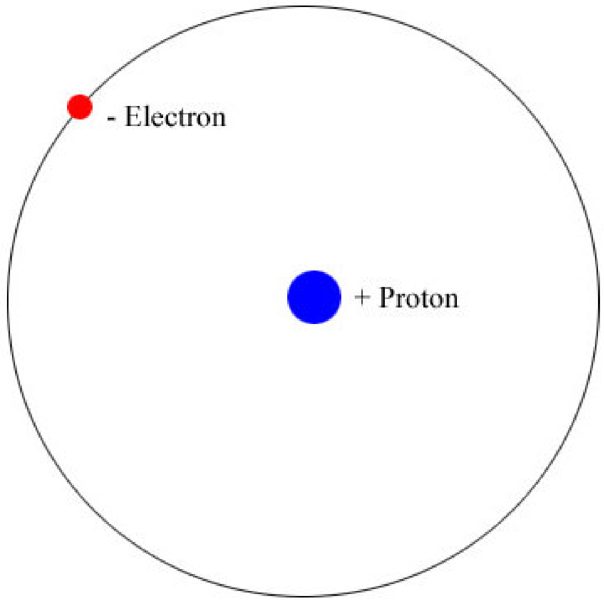
Chapter 21

The Electric Field I

The Electric Field I

- Charge
- Conductors & Insulators
- Coulomb's Law
- The Electric Field
- Electric Field Lines
- Action of the Electric Field on Charges

Hydrogen Atom Dimensions



Diam of p ~ 10^{-15} m

Diam of e - pointlike particle

Radius of H atom ~ 10^{-10} m

Mass of electron: $m_e = 9.109\text{x}10^{-31}$kg = 0.510 Mev

Mass of proton: $m_p = 1.672\text{x}10^{-27}$kg = 938 Mev

$$\frac{m_p}{m_e} = \frac{938}{0.510} = 1839$$

The Electron

It was only a little over a 100 years ago that the electron was discovered.

<u>Discovery of the electron - J.J. Thomson</u>

- The electron was discovered in 1897 by J.J. Thomson.
- Electrons follow well defined paths.
- Well defined charge-to-mass ration: e/m

<u>Atomic Nature</u>

- Rutherford’s nuclear model 1911 - positive charge concentrated in the center of the atom and negatively charged electrons surround the nucleus.

The Electron

Charge of electron: -1.602×10^{-19} Coulombs

The symbol e represents the value 1.602×10^{-19} Coulombs. Therefore the charge of the electron is -e

The charge is quantized - all charged objects contain an integral multiple of e.

Proton charge = +e

The exact balancing of the protonic and electronic charges allows atoms to be effectively neutral.

Electrical Phenomena

Lightning

Static electricity

Triboelectricity - charging by friction - unbalancing the neutral atoms.

Triboelectricity

- Good for creating charged objects.
- Allows the study of basic electrical phenomena.
- More qualitative than quantitative.

The Triboelectric Series

More Positive

* Human Hands (if very dry)
* Leather
* **Rabbit Fur**
* **Glass**
* Human Hair
* Nylon
* Wool
* Fur
* Lead
* **Silk**
* Aluminum
* Paper
* Cotton
* Steel (neutral)
* Wood
* Amber
* **Hard Rubber**
* Nickel, Copper
* Brass, Silver
* Gold, Platinum
* Polyester
* Styrene (Styrofoam)
* Saran Wrap
* Polyurethane
* Polyethylene (scotch tape)
* Polypropylene Vinyl (PVC)
* Silicon
* Teflon

More Negative

Positive

Negative

Positive

Negative

Lose electrons more easily

Triboelectricity

Rubber - Negative $\longleftrightarrow$ Fur - Positive

Glass rod - Positive $\longleftrightarrow$ Silk - Negative

Typical charge transfer is about 10^{-9} C or $N_e \sim 10^{11}$ electrons

In studying electrostatics remember that only the
ELECTRONS CAN MOVE

The positive charges (atomic ions) are stationary.

Conductors and Insulators

<u>Insulators</u>: Glass, rubber, wood - anything that will not conduct electricity.

In an insulator the electrons are <u>not free</u> to move around

<u>Conductors</u>: Metals - Aluminum, iron, copper, steel, brass, silver, gold, etc.

In a conductor electrons are <u>free</u> to move under the influence of other charges and external electric fields.

Good electrical conductors are also good thermal conductors

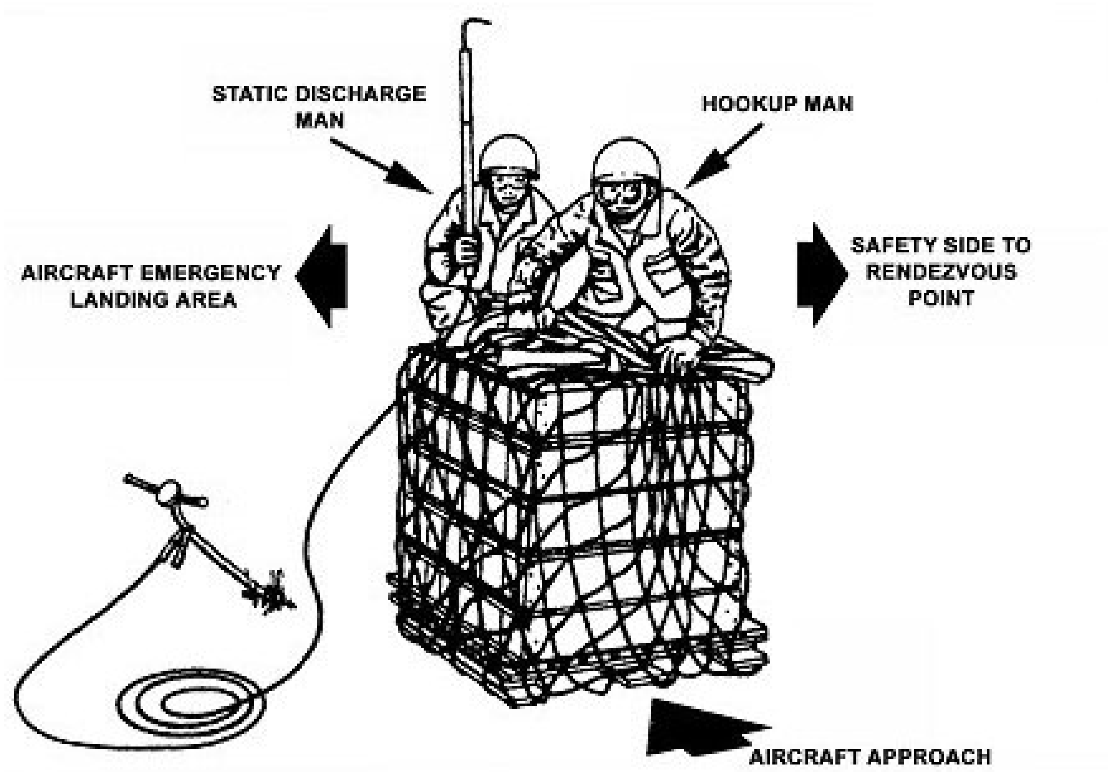
Triboelectricity Examples

The rotation of helicopter blades in the atmosphere will generate a large electrostatic charge.

A grounding hook is used to dissipate the charge on the helicopter before making contact with it.

Static Tales: http://www.pprune.org/archive/index.php/t-309803.html

Static Discharge Operation

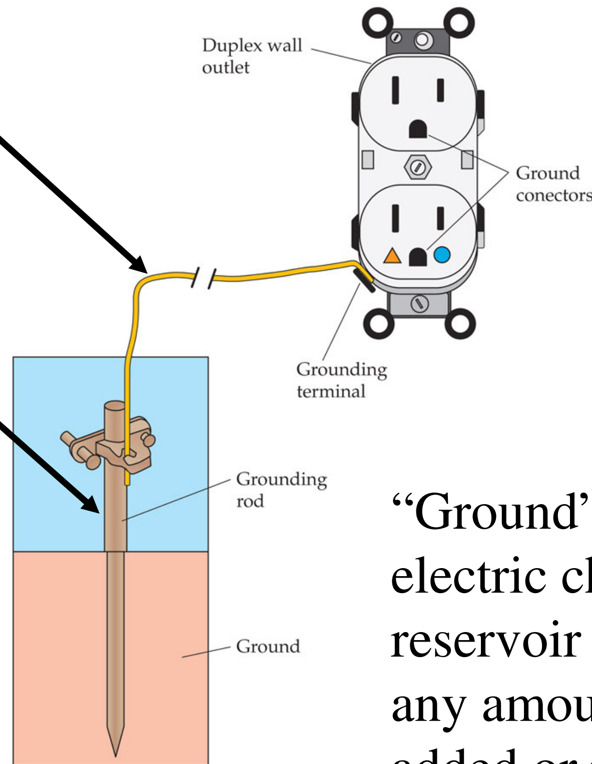


Electrostatic Discharge Example

Walking on a carpet can build up a static charge. Static charges can damage sensitive electronic chips. Conductive wrist straps tied to ground dissipate these dangerous charges.

Home Grounding System

Excellent conductor



"Ground" is a reservoir of electric charge (electrons). A reservoir is large enough that any amount of electrons can be added or removed without changing the ground voltage.

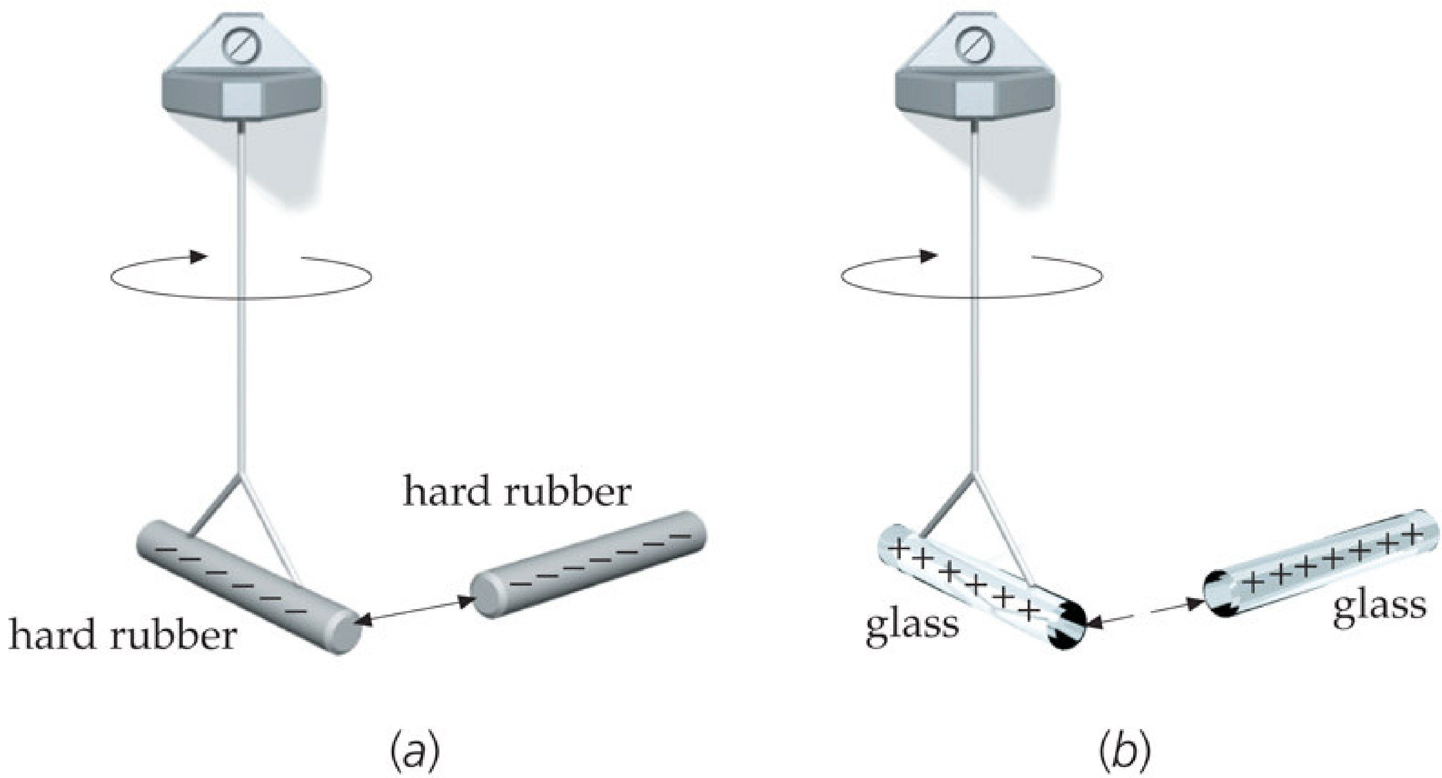
Grounding Systems

Ground is used as a reference voltage in constructing electrical circuits.

In the previous example the ground is providing an alternative pathway for electrons to prevent electrocution in case of a wiring malfunction.

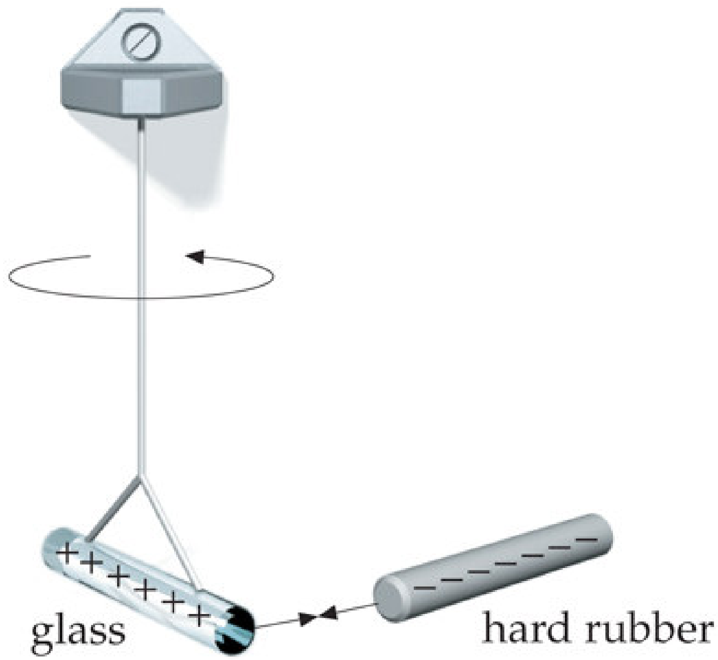
Electrostatic Force

Like charges repel



Electrostatic Force

Unlike charges attract



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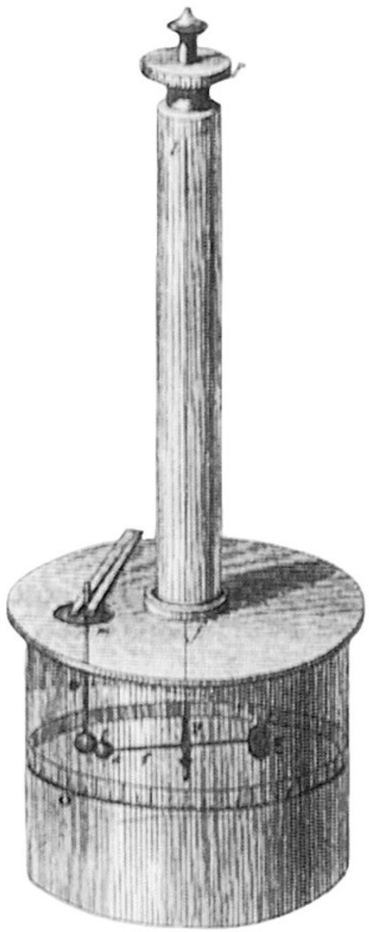
Electrostatic Force Measurements

- Electroscope - semi quantitative measurements
- Torsion meter - quantitative measurements - fairly precise force measurements.
- Electrometer and Charge Sensors

Coulomb's Torsion Balance - Measuring Electrostatic Force



Charles Coulomb - 1736-1806

At one time we tried to duplicate Coulomb's measurements in a lab. It was a very problematic lab since the charge on the spheres would discharge faster than the force measurements could be made.

Electroscope

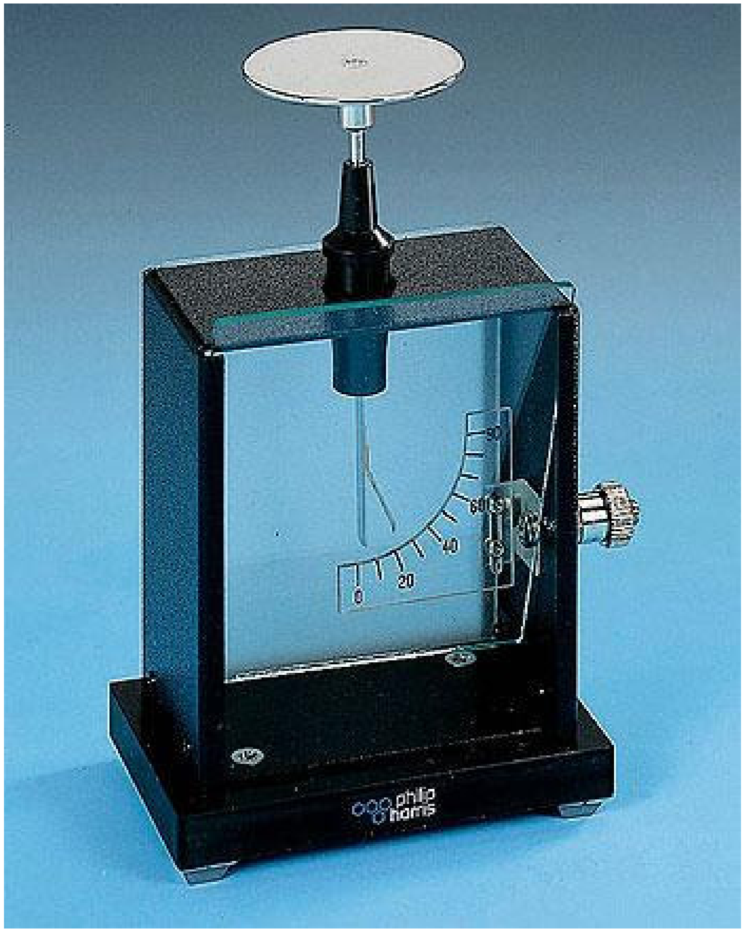
Electroscope fully discharged

Electroscope

Electroscope is
negatively charged.

Thin gold foil leaves

A Quantitative Electroscope



A gold leaf electroscope measures potential difference between the leaf and the base (or earth). The leaf rises because it is repelled by the stem (support). The leaf and its support have the same type of charge.

A typical school electroscope will show a deflection for a charge as small as 0.01 pC.

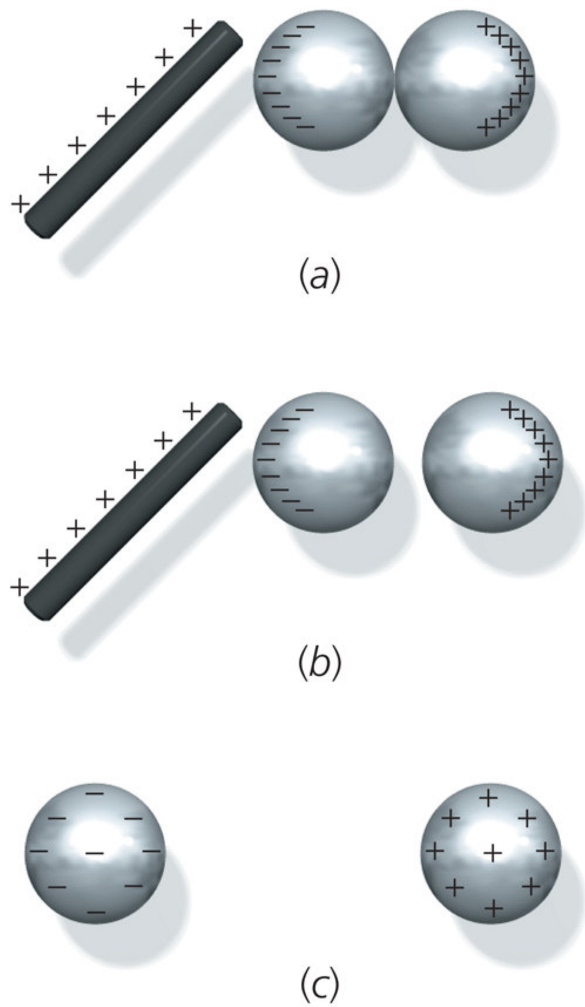
The unit pC is a pico Coulomb,

$1x10^{-12}$ Coulombs, equivalent to the charge on over 6 million electrons.

Electroscope

- Charging by direct contact
- Charging by induction

Charging by Induction - Conductors



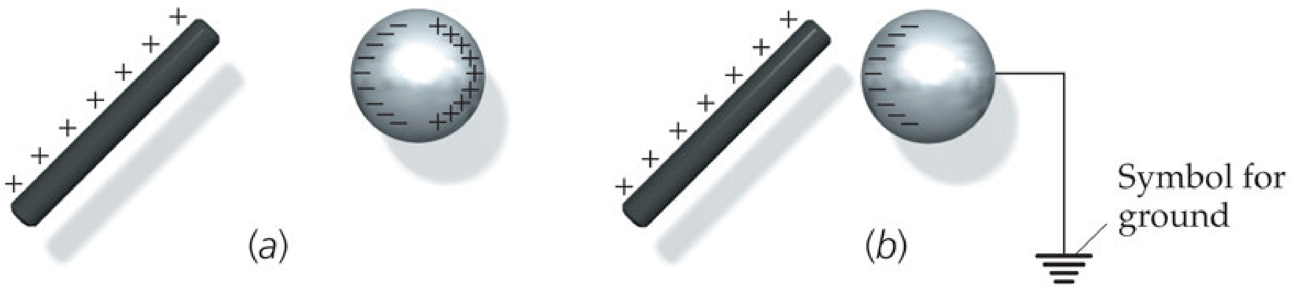
The net charge on the spheres is 0 throughout this demonstration.

Mobile charges in the conductor move in response the the positively charged rod.

With the charged rod still in place but not contacting the spheres the balls are separated. This removes the the conductive connection between the two spheres but the charges are still polarized.

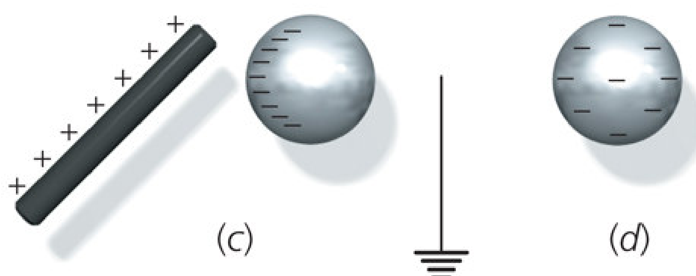
The charged rod is removed and the spheres separated further. The excess charge on each sphere spreads out.

Charging by Induction - Conductors



Ground connection allows electrons to travel to the sphere, neutralizing the positive charge on the far side of the sphere.

Charged rod polarizes the charge on the conducting sphere.



The ground is removed and as the charged rod is removed the excess negative charge spreads out over the sphere

Charge Distributions

In the problems that you will be asked to solve in these first three chapters you will be asked to work with charge distributions and to calculate the resultant electric forces or fields.

This is electrostatics - the charges are not moving when we try to describe them.

However, the charges might move initially until they settle down in an equilibrium position. It is after they achieve this equilbirium position that we will apply the various equations of electrostatics.

Uniform Charge Distributions

These will be found in three forms: linear, surface and volume charge distributions.

- Charge can be positive or negative.
- Charges are fixed in location and don't move.
- Uniform means the charge per unit measure is constant.
- The material containing these charges has only one property - It holds the charges in place and doesn't respond to the Coulomb forces of the other charges.

This is most similar to charges on a dielectric surface.

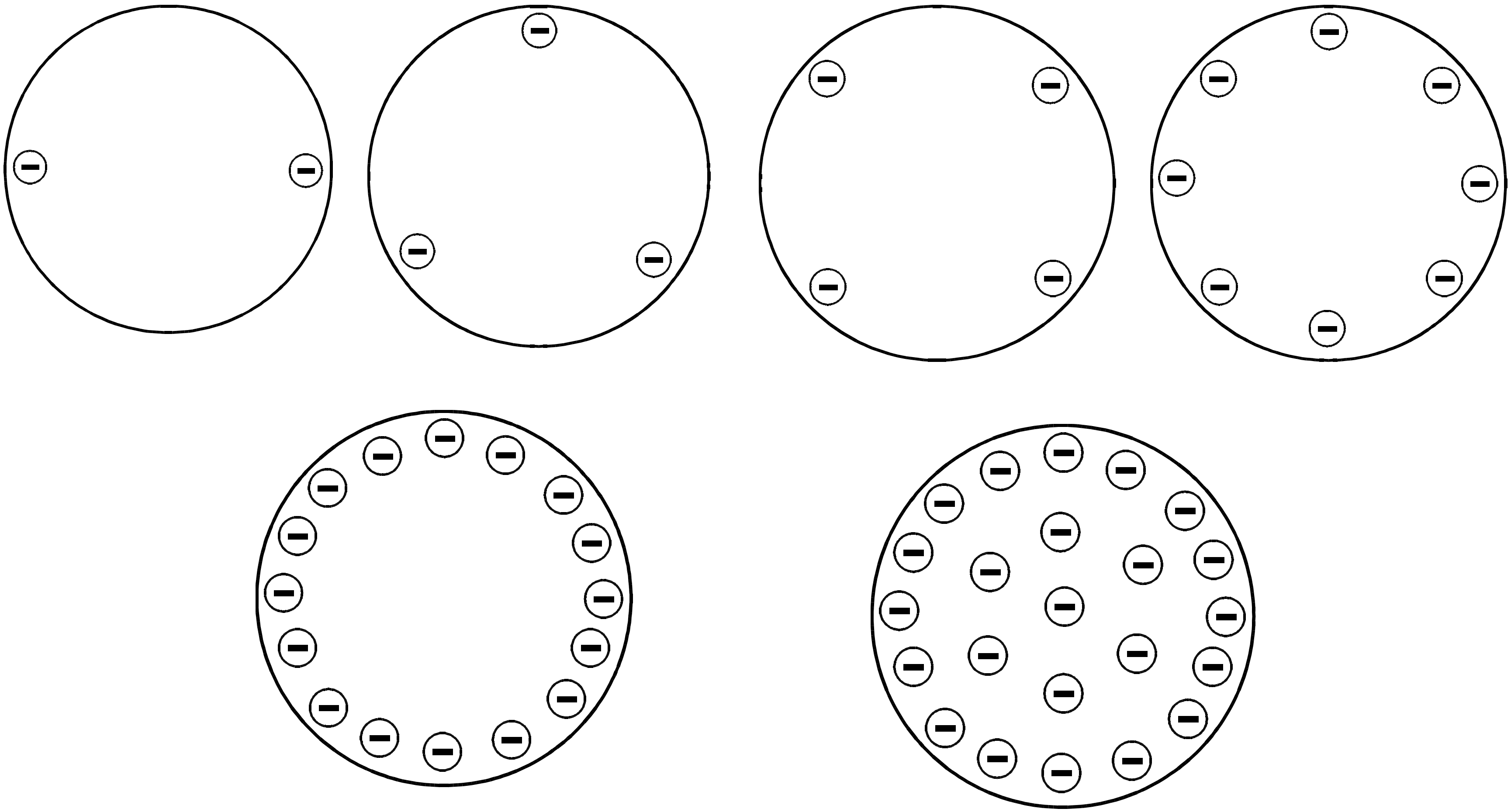
Real Charge Distributions

- In conductors there are free electrons that respond to external charges and fields. Except in high symmetry situations these real charge distributions are difficult to handle mathematically.

- In dielectric materials (insulators) there are bound charges that while not free to move, are able to respond to electric fields in a limited way.

- We will deal with conductors first and later with dielectrics.

Charge Distribution on Metal Disk



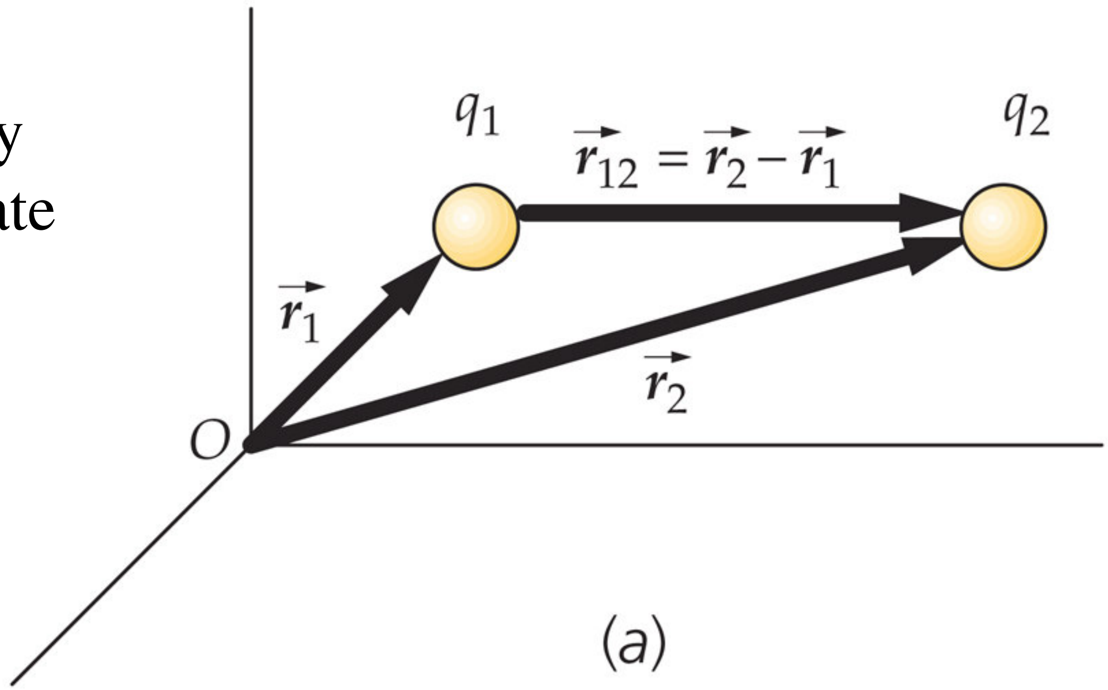
Higher charge density on edges than in the center

Coulomb's Law

- The Coulomb force is a central force directed along a line connecting the two charges.
- The Coulomb force is proportional to the magnitude of the two charges.
- The force is inversely proportional to the square of the distance between the two charges.
- Forces are combined by superposition.

Relative Distance Vector

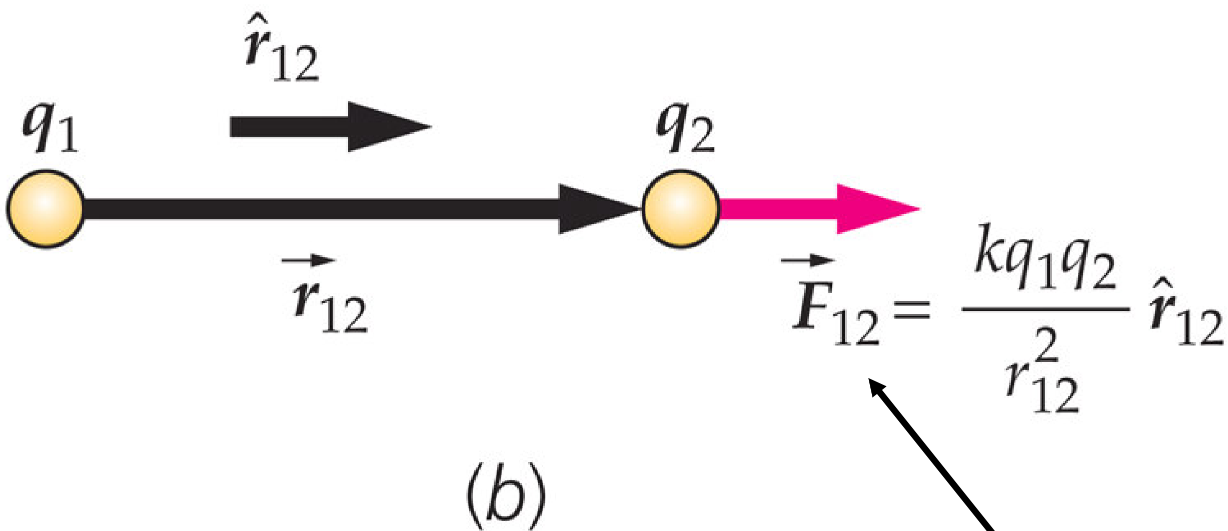
Arbitrary coordinate system



(a)

Electric force problems with point charges will be the most difficult problems that we will solve this semester.

Coulomb Force Vectors



Force on q_2 due to q_1.

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$|\vec{r}_{12}| = r_{12} = \sqrt{(\vec{r}_2 - \vec{r}_1) \bullet (\vec{r}_2 - \vec{r}_1)}$$

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{r_{12}}$$

Coulomb's Law

$$F = k\frac{q_1 q_2}{R^2}$$ Simple scalar magnitude calculation

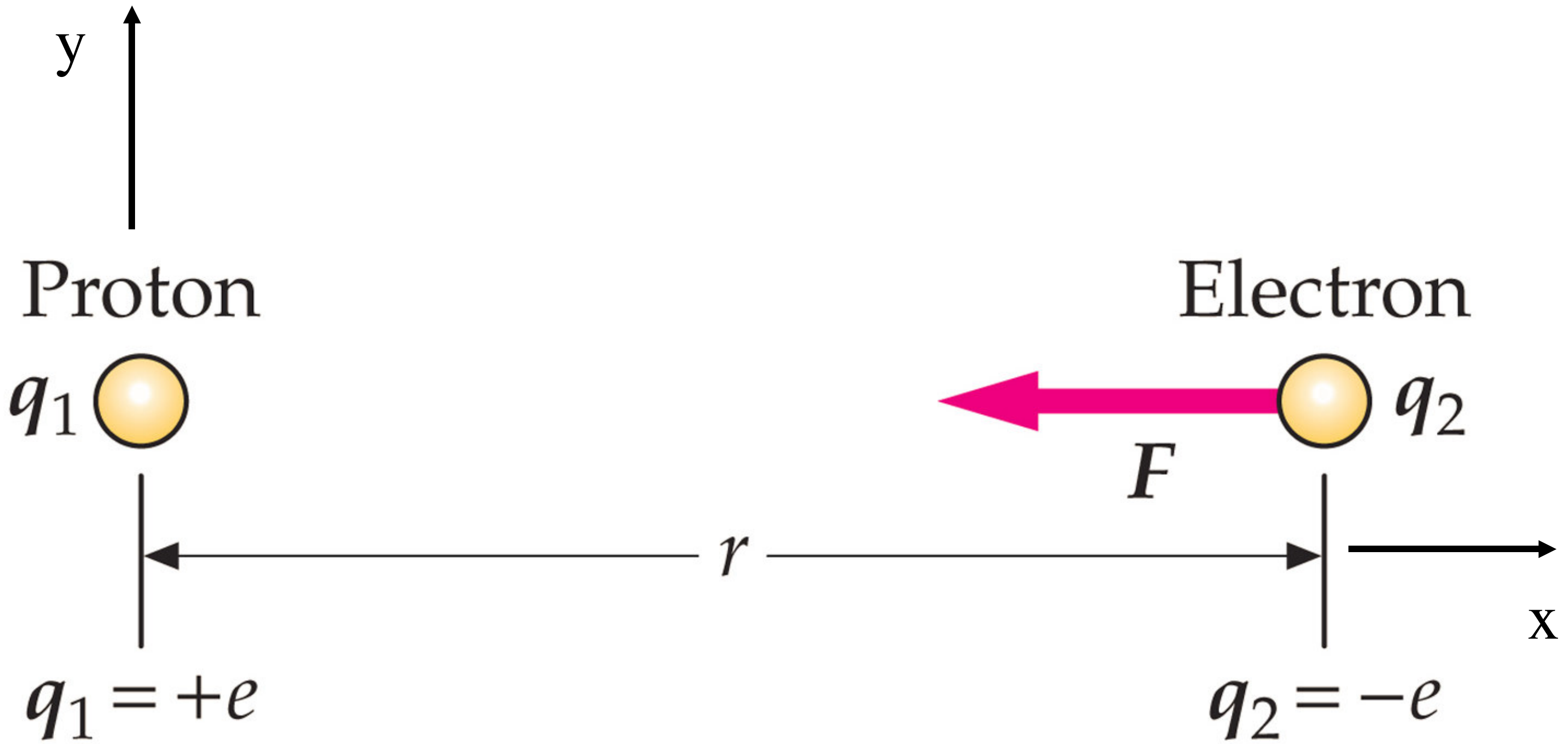
$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$ Full blown vector description

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99x10^9\ Nm^2/C^2$$

ε_0 = Permittivity of free space

$$\varepsilon_0 = 8.85x10^{-12}\ C^2/Nm^2$$

Opposite Charges - Attractive Force



We can pick the coordinate system so that it simplifies the problem.

Opposite Charges - Attractive Force

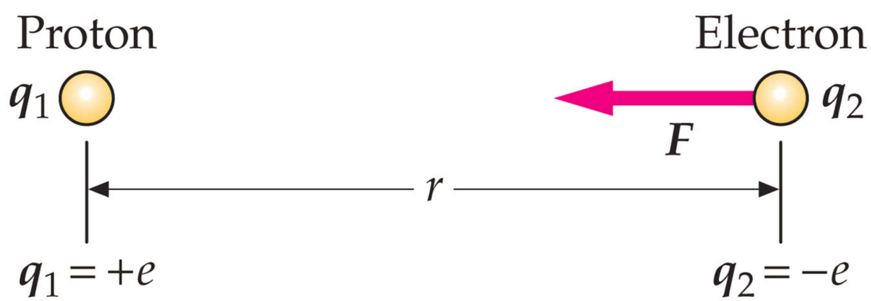
$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$\vec{r}_2 = r\hat{i}$$

$$\vec{r}_1 = 0$$

$$\vec{r}_{12} = r\hat{i} - 0 = r\hat{i}$$

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{\left|\vec{r}_{12}\right|} = \frac{r\hat{i}}{r} = \hat{i}$$



$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} = k\frac{q_1 q_2}{r^2}\hat{i} = k\frac{(+e)(-e)}{r^2}\hat{i}$$

$$\vec{F}_{12} = -k\frac{e^2}{r^2}\hat{i}$$

Opposite Charges - Attractive Force

$$\vec{F}_{12} = -k\frac{e^2}{r^2}\hat{i}$$

$$\vec{F}_{12} = -(8.99x10^9)\frac{(1.602x10^{-19})^2}{(10^{-10})^2}\hat{i}$$

$$\vec{F}_{12} = -23.1x10^{-9}\hat{i}\,(N)$$

Three quantities determine the direction of the force

q_1, q_2, and $\hat{r}_{12}$

Maximize Force

Charges q_1 and q_2 are separated by a distance D. The sum of the charges is held constant. What value of q_2 maximizes the force between them?

$$F = \frac{kq_1q_2}{D^2}; \quad Q = q_1 + q_2$$

$$q_1 = Q - q_2; \quad F = \frac{k\left(Q - q_2\right)q_2}{D^2}$$

$$\frac{dF}{dq_2} = \frac{k}{D^2}\frac{d}{dq_2}\left(\left(Q - q_2\right)q_2\right)$$

Maximize Force

The maximum force is determined by setting the derivative of F with respect to q_2 equal to zero.

$$\frac{dF}{dq_2} = \frac{k}{D^2}\frac{d}{dq_2}\left(\left(Q - q_2\right)q_2\right) = 0$$

$$\frac{dF}{dq_2} = \frac{k}{D^2}\left[\left(Q - q_2\right)\frac{d}{dq_2}\left(q_2\right) + q_2\frac{d}{dq_2}\left(Q - q_2\right)\right] = 0$$

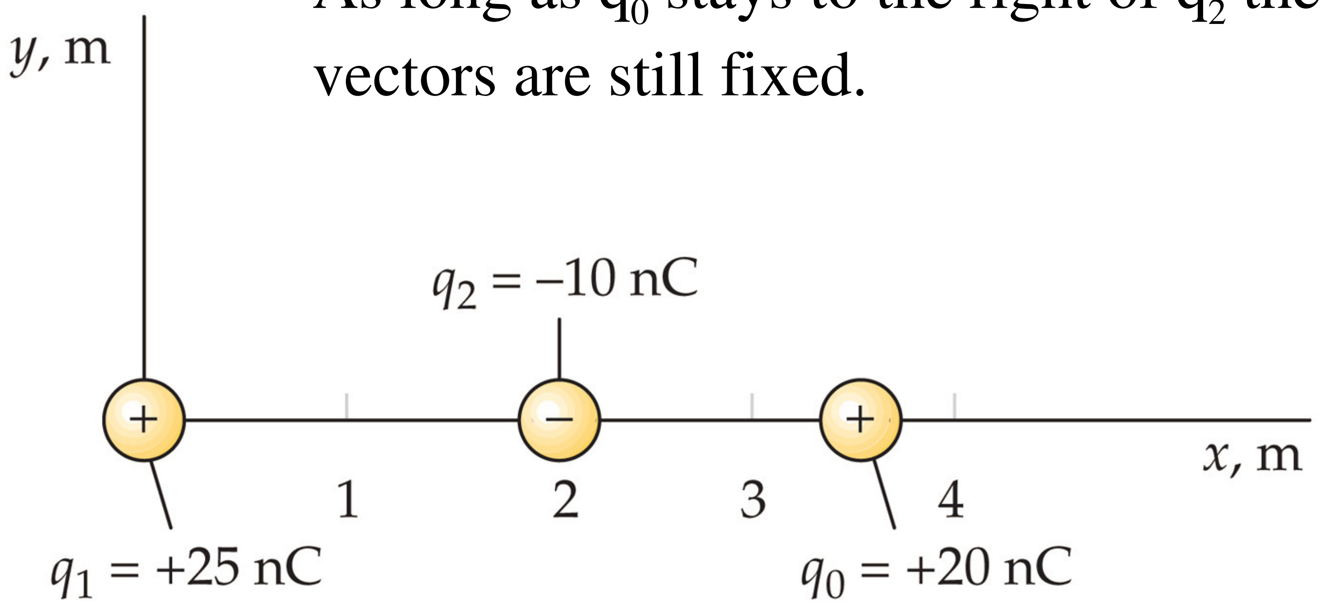
$$\frac{dF}{dq_2} = \frac{k}{D^2}\left[\left(Q - q_2\right) - q_2\right] = 0$$

$$Q - 2q_2 = 0 \qquad q_2 = Q/2$$

Problem Strategy

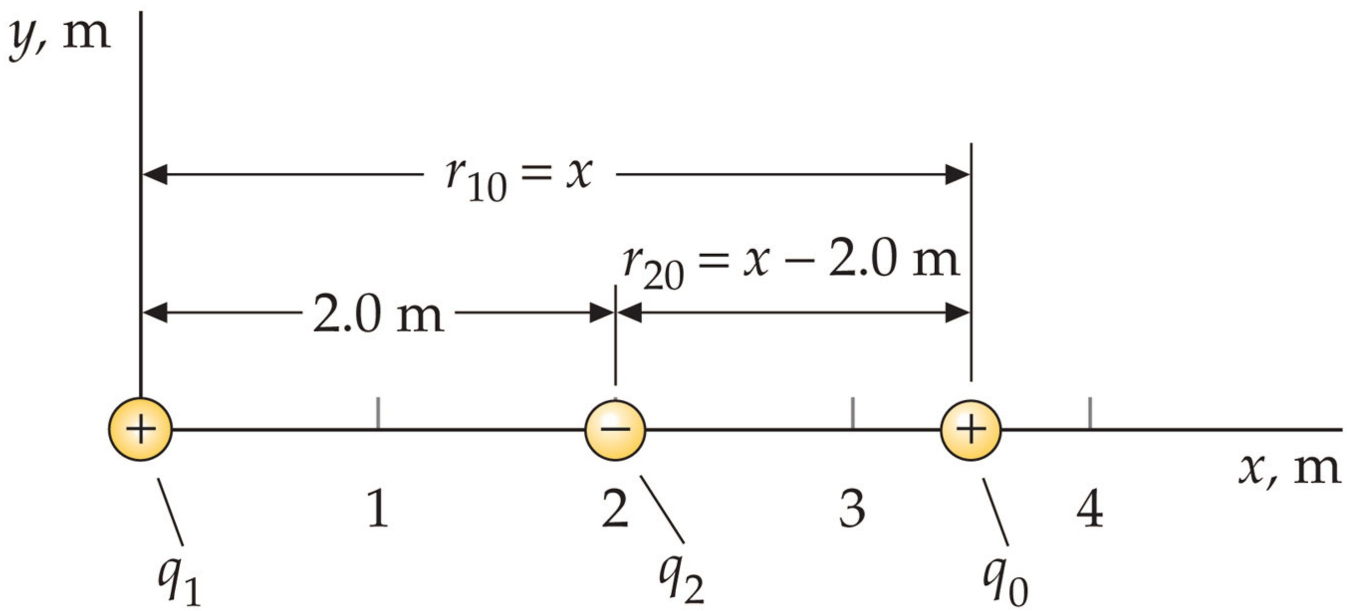
Stationary charges ⟶ Fixed unit vectors

As long as q_0 stays to the right of q_2 the unit vectors are still fixed.

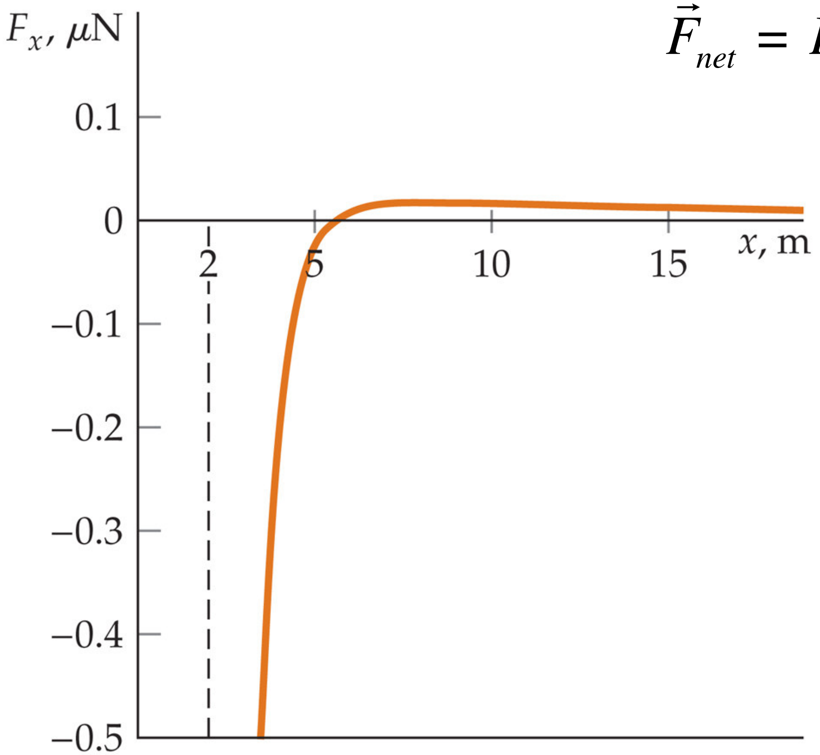


Problem Strategy

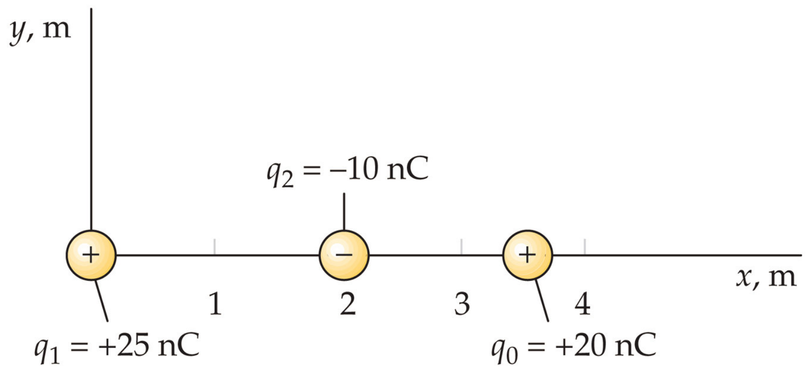
$$\vec{F}_{net} = \vec{F}_{10} + \vec{F}_{20} = \left(\frac{k\left|q_1 q_0\right|}{x^2} - \frac{k\left|q_2 q_0\right|}{(x - 2.0)^2} \right)\hat{i}$$



Problem Strategy

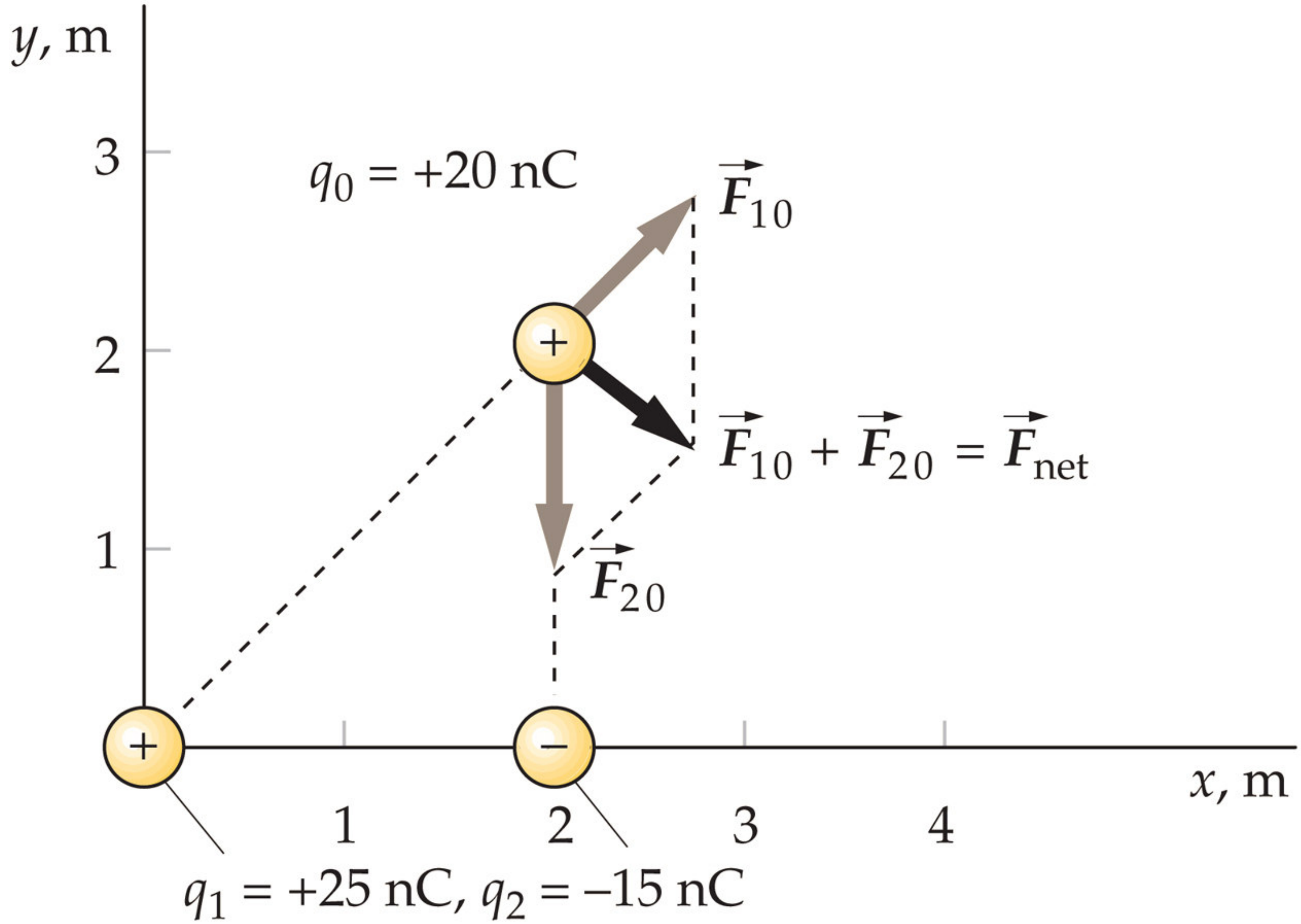


$$\vec{F}_{net} = \vec{F}_{10} + \vec{F}_{20} = \left(\frac{k|q_1 q_0|}{x^2} - \frac{k|q_2 q_0|}{(x - 2.0)^2} \right) \hat{i}$$



The solution only describes the region x > 2.0m

Electric Forces in 2-D



Electric Forces in 2-D

$$\vec{F}_{10} = \frac{kq_1q_0}{\left(2\sqrt{2}\right)^2}\cos(45^0)\hat{i} + \frac{kq_1q_0}{\left(2\sqrt{2}\right)^2}\sin(45^0)\,\hat{j}$$

$$\vec{F}_{20} = +\frac{kq_2q_0}{2^2}\left(-\hat{j}\right)$$

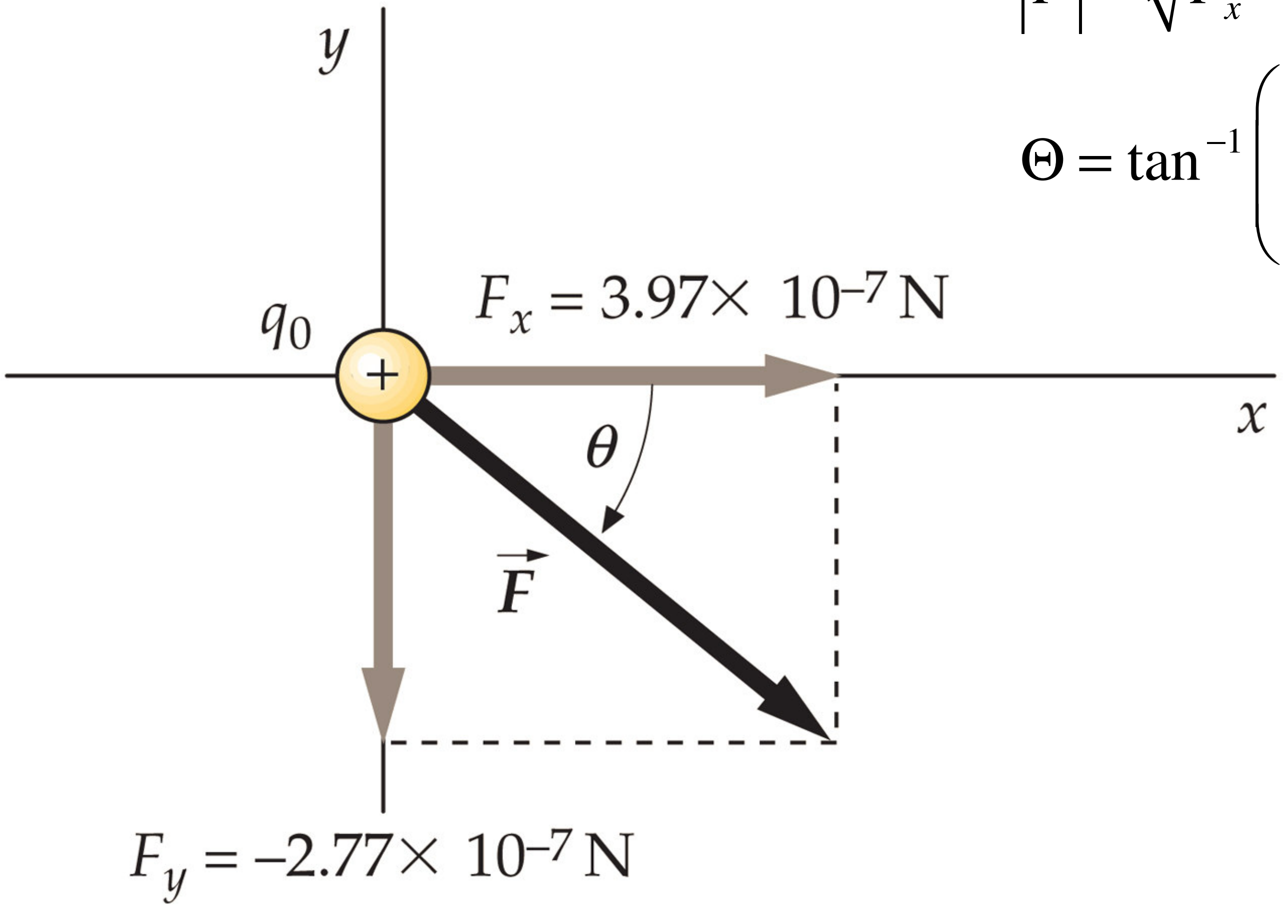
$$\vec{F}_x = \frac{kq_1q_0}{\left(2\sqrt{2}\right)^2}\cos(45^0)\hat{i}$$

$$\vec{F}_y = +\frac{kq_1q_0}{\left(2\sqrt{2}\right)^2}\sin(45^0)\,\hat{j} - \frac{k\left|q_2\right|q_0}{2^2}\,\hat{j}$$

Electric Forces in 2-D

$$\left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2}$$

$$\Theta = \tan^{-1}\left(\frac{\left|F_y\right|}{\left|F_x\right|}\right)$$



The Electric Field

The concept of an electric field surrounding the electric charge eliminates the problems of “action at a distance.”

The field exists even in the absence of a “positive test charge” to sample the force generated by the field.

$$\vec{E} = \lim_{q \to 0} \frac{\vec{F}}{q} \qquad \vec{F}_{i0} = k\,\frac{q_i q_0}{r_{ip}^2}\,\hat{r}_{ip}$$

$$\vec{E}_{ip} = \frac{\vec{F}_{i0}}{q_0} = k\,\frac{q_i}{r_{ip}^2}\,\hat{r}_{ip}$$

Electric Field Vectors

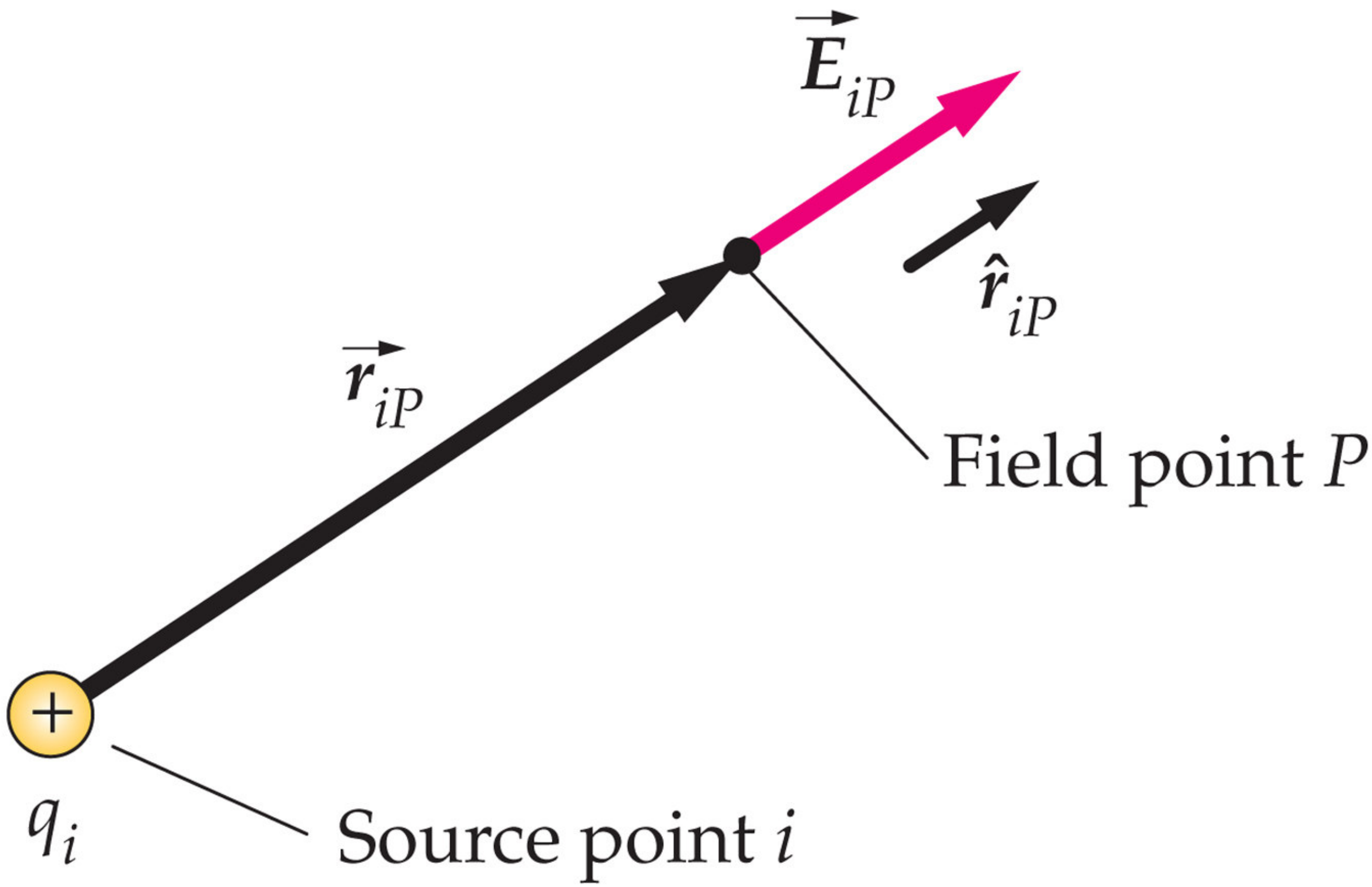
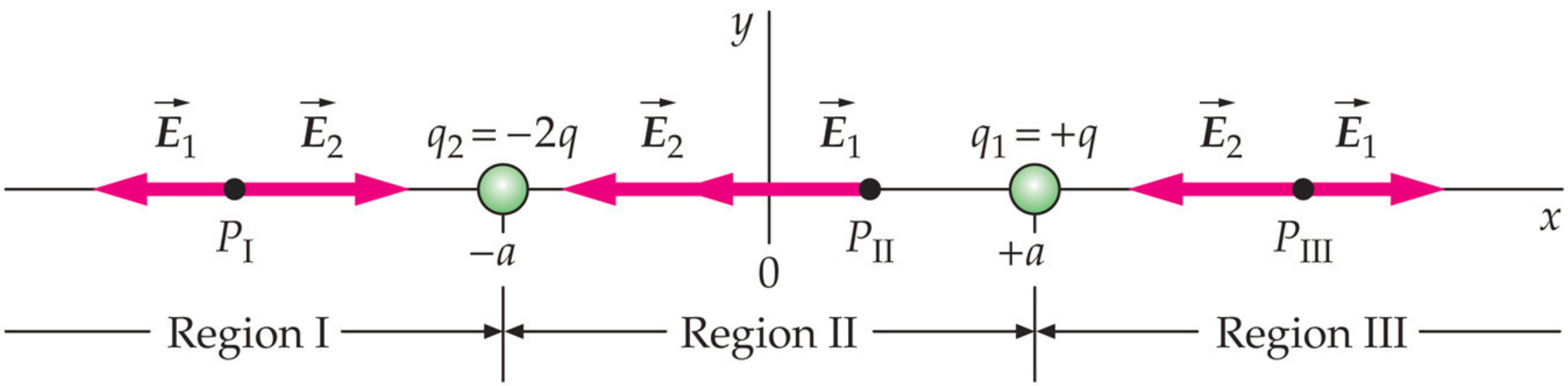


Table 21-2 Some Electric Fields in Nature

	E, N/C
In household wires	10^{-2}
In radio waves	10^{-1}
In the atmosphere	10^{2}
In sunlight	10^{3}
Under a thundercloud	10^{4}
In a lightning bolt	10^{4}
In an X-ray tube	10^{6}
At the electron in a hydrogen atom	6×10^{11}
At the surface of a uranium nucleus	2×10^{21}

Electric Field Problems

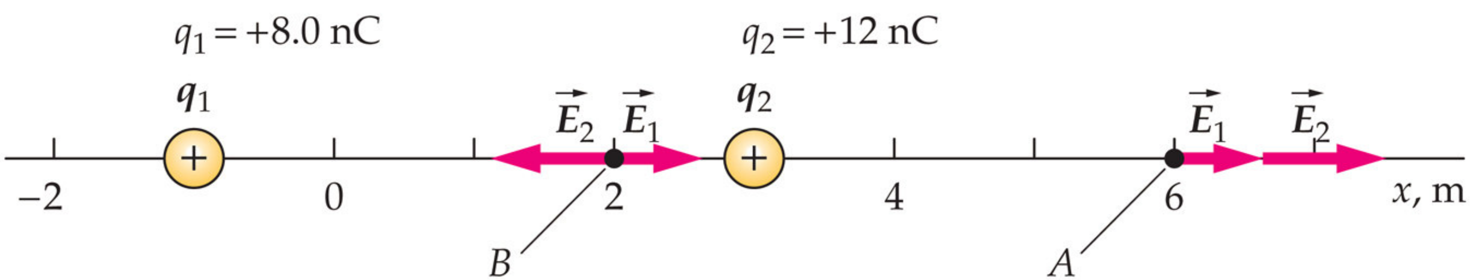
Looking for zero E-field



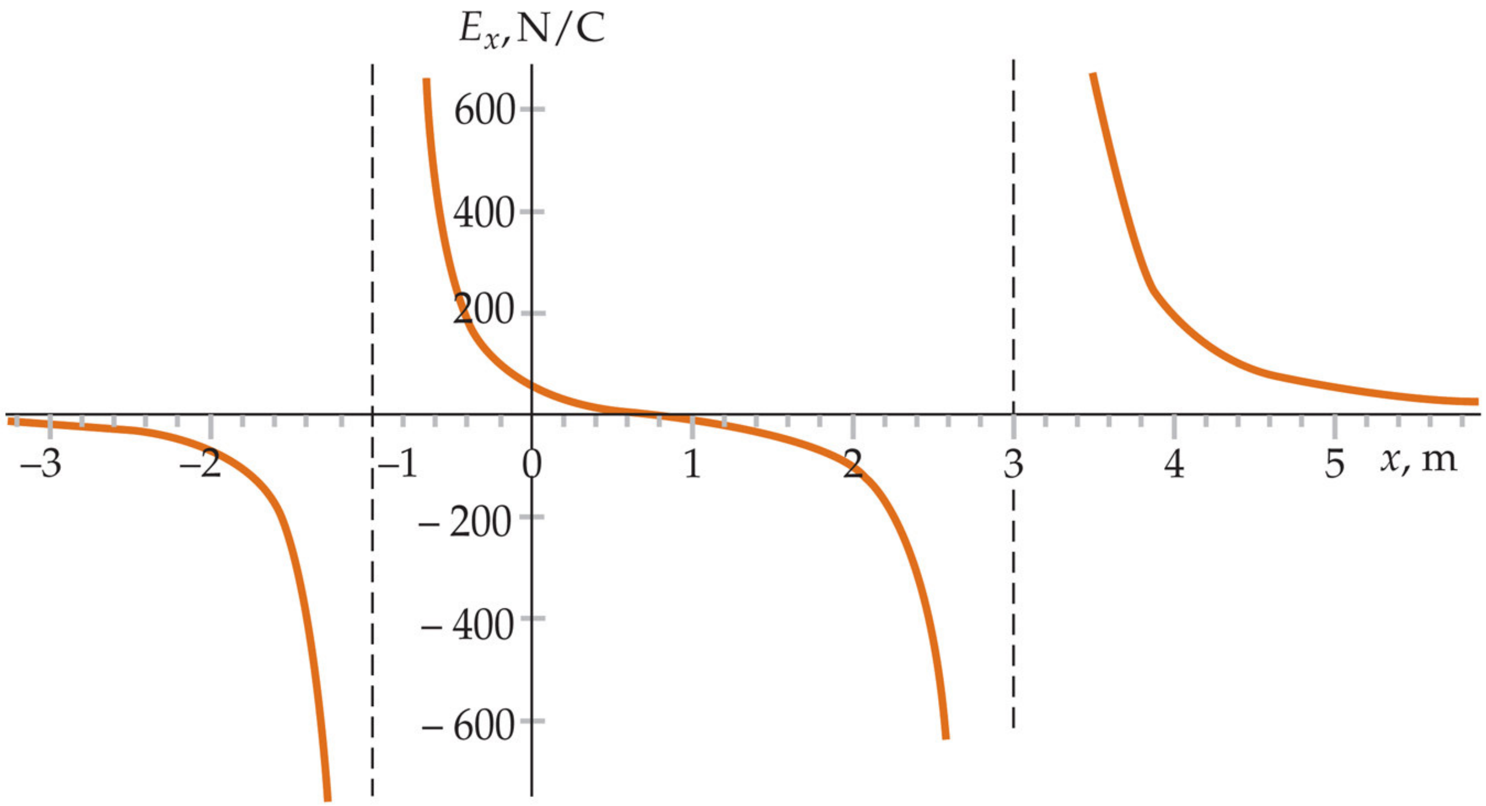
No zero | No zero | Zero possible

Dividing the problem into three regions avoids the need to develop a set of unit vectors that will work in all three regions.

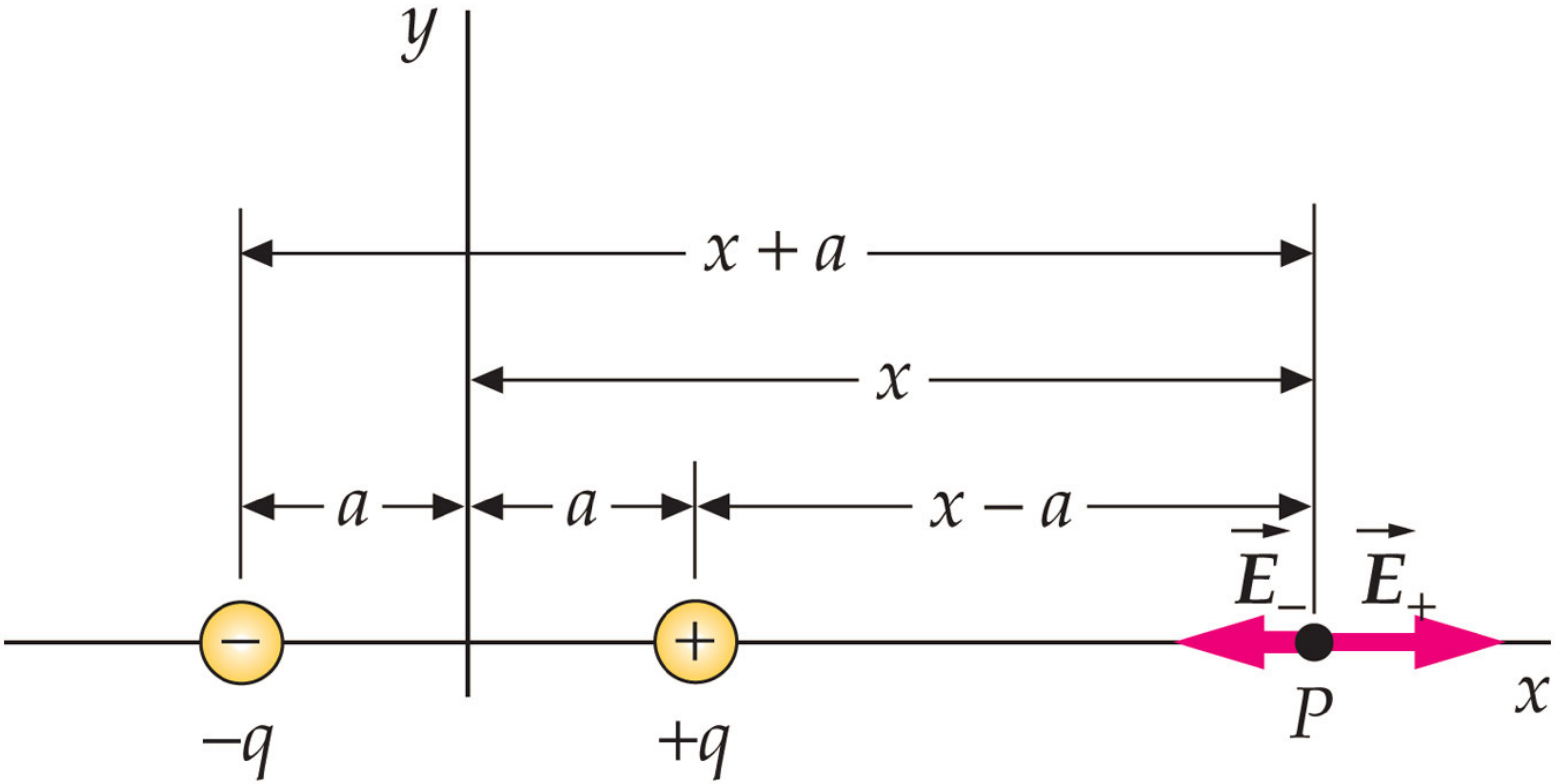
Electric Field Problems



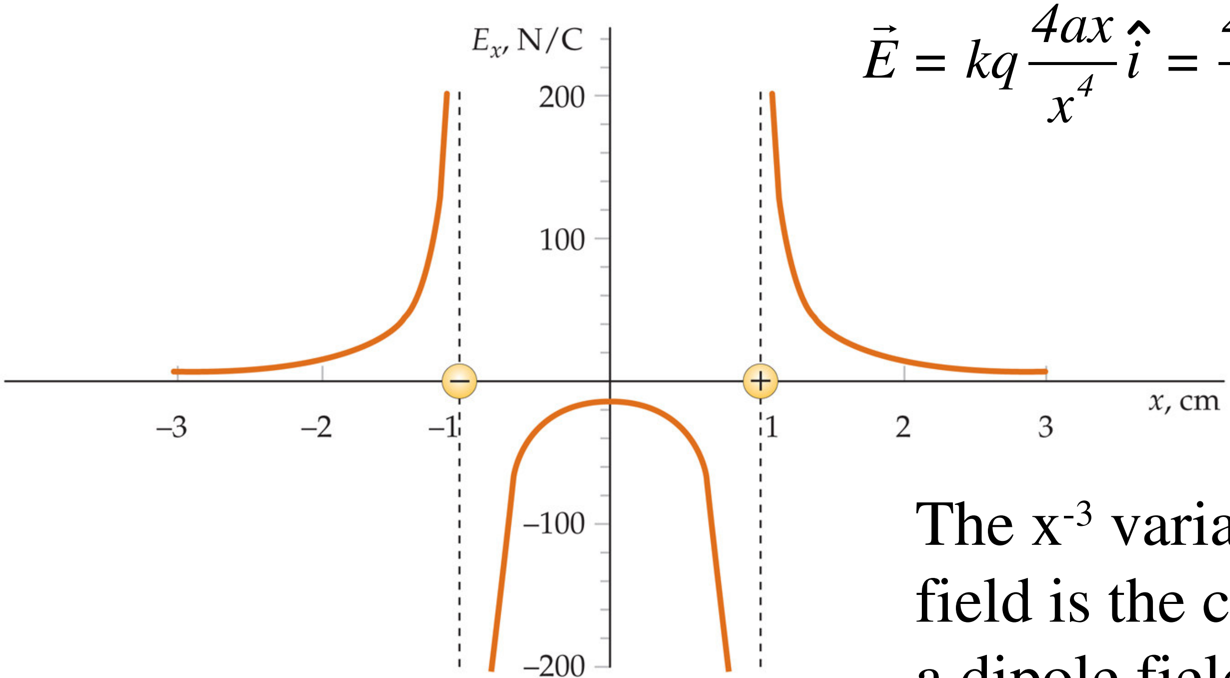
Electric Field Problems



Electric Dipole Geometry



Electric Dipole Field

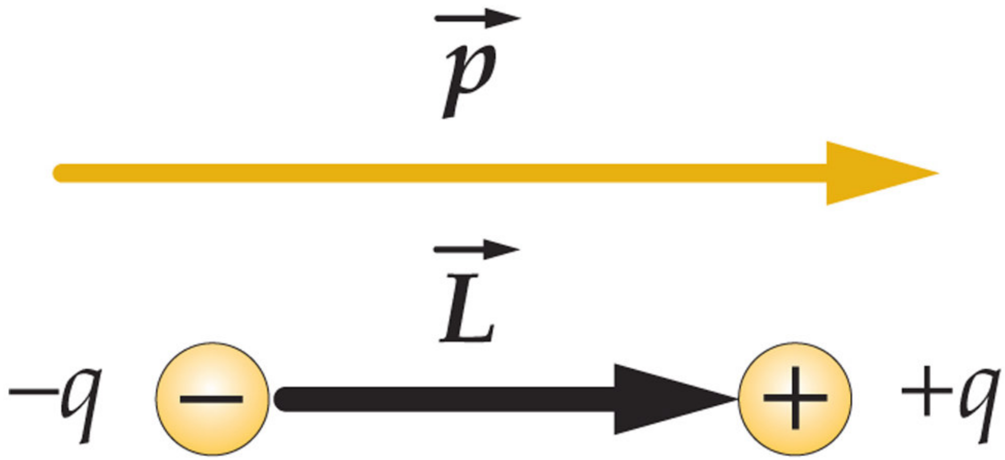


$$\vec{E} = kq \frac{4ax}{\left(x^2 - a^2\right)^2} \hat{i} \quad for\ x > a$$

$$\vec{E} = kq \frac{4ax}{x^4} \hat{i} = \frac{4kqa}{x^3} \hat{i} \quad for\ x >> a$$

The x^{-3} variation of the E-field is the characteristic of a dipole field - it is short ranged.

Electric Dipole Vectors



$$\vec{p} = q\vec{L}$$

$$\vec{E} = kq\frac{4ax}{x^4}\hat{i} = \frac{2k(2qa)}{x^3}\hat{i} = \frac{2k\vec{p}}{x^3} \; for\ x >> a$$

Electric Field Lines

1. Electric field lines begin on positive charges (or at infinity) and end on negative charges (or at infinity).

2. The lines are drawn uniformly spaced entering or leaving an isolated point charge.

3. The number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude fo the charge.

4. The density of the lines (the number of lines per unit area perpendicular to the lines) at any point is proportional to the magnitude of the field at that point.

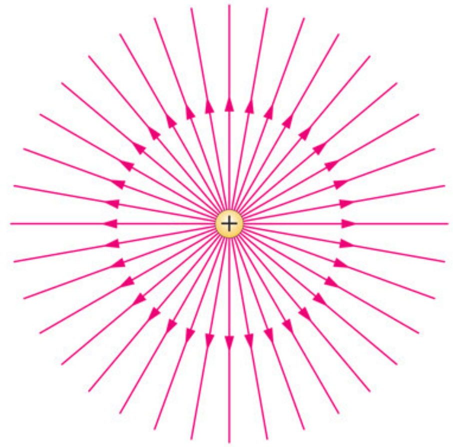
Electric Field Lines

5. At large distances from the system of charges with a net charge, the field lines are equally spaced and radial, as if they came from a single point charge equal to the net charge of the system.

6. Field lines do not cross. If two field lines crossed, that would indicate two directions for E at the point of the intersection.

The electric field lines of Michael Faraday were a brilliant conceptual device that allowed the scientists of his day to visualize the field. These field concepts are still much in use today.

Electric Field Distribution



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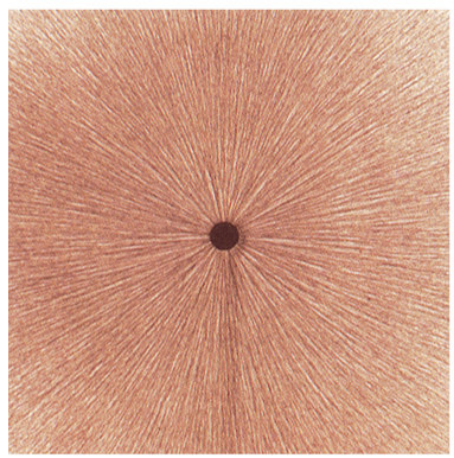

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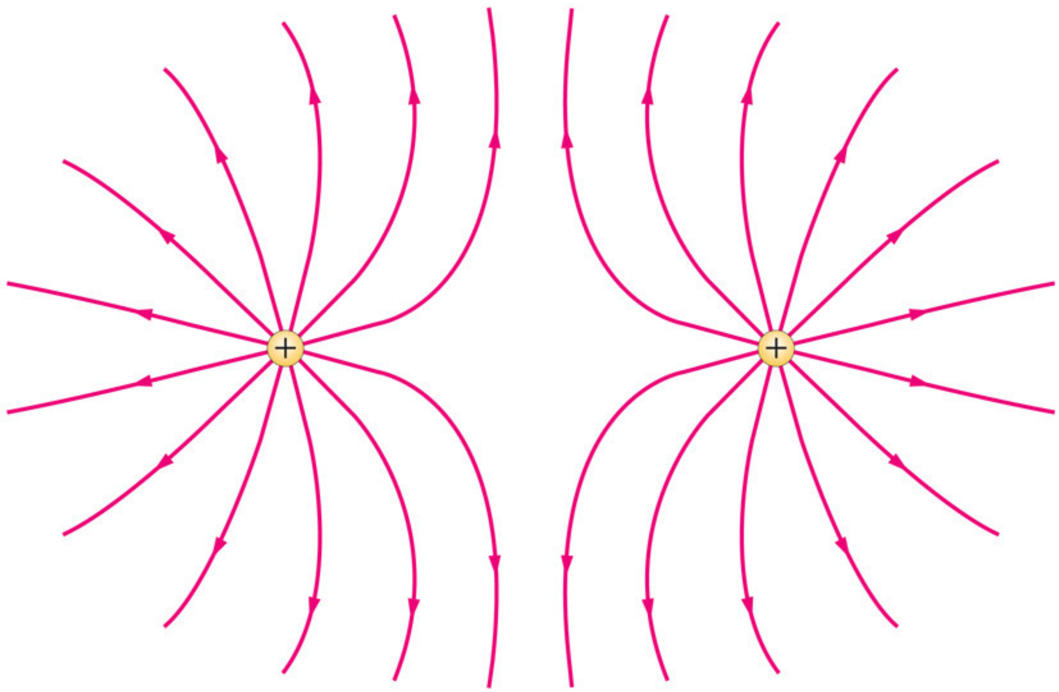
Electric Field Distribution



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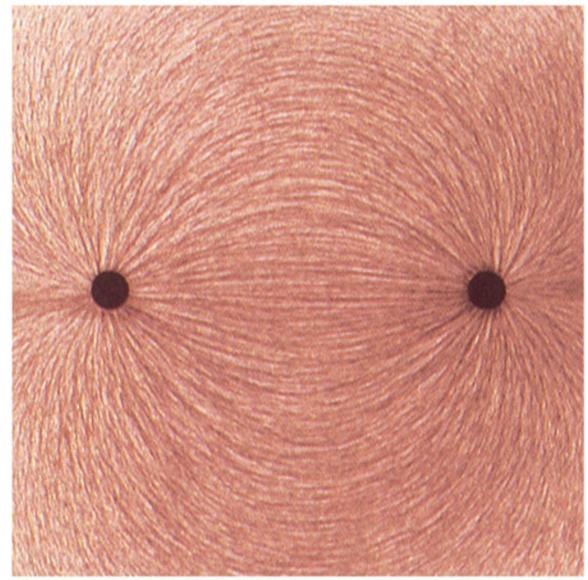

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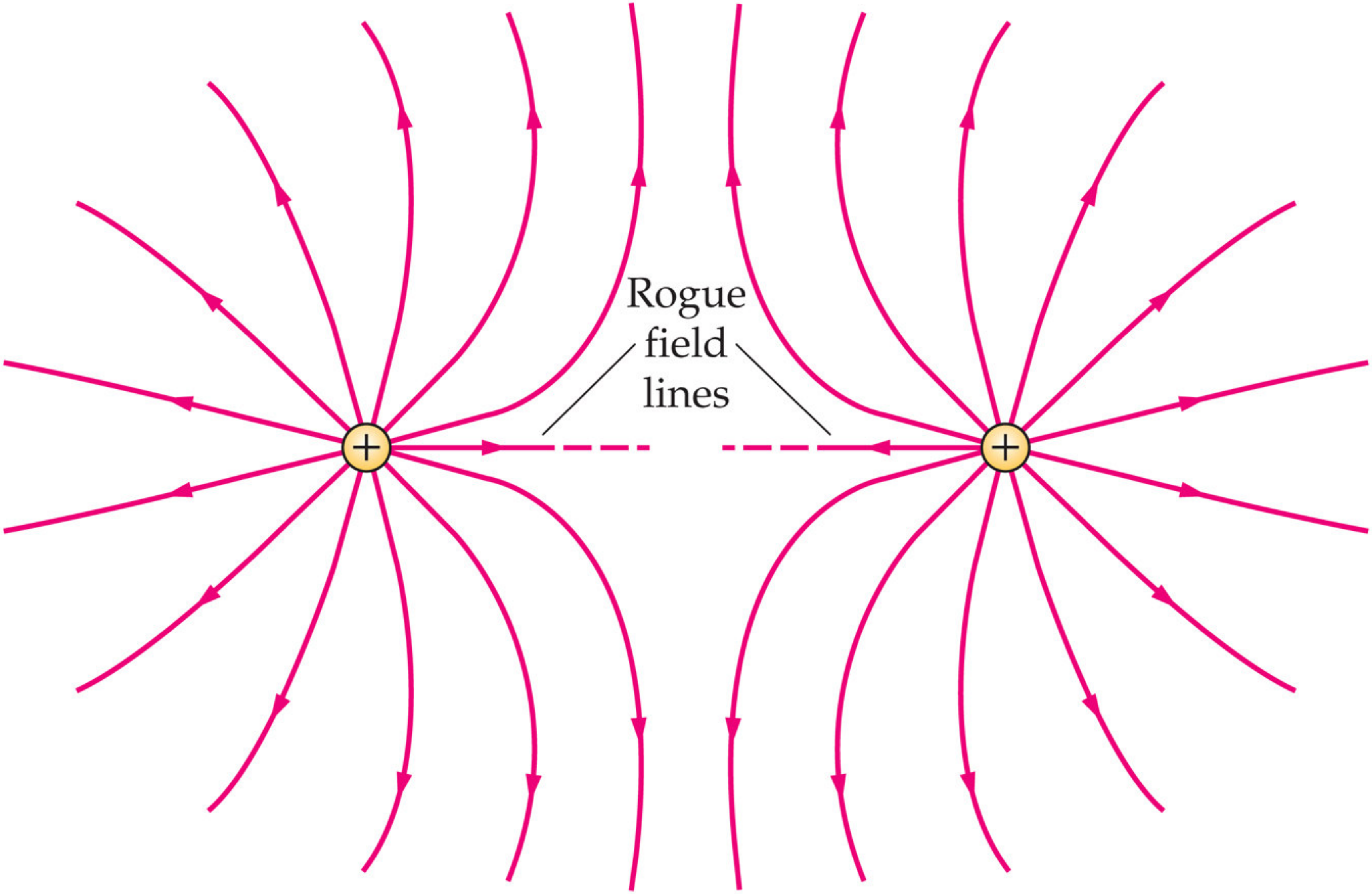
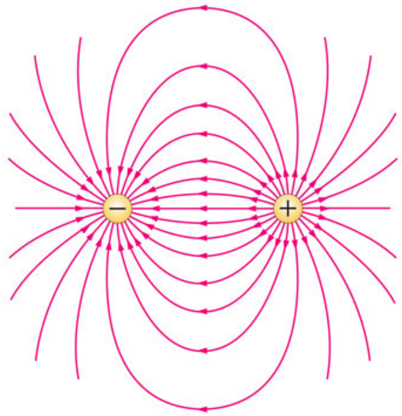
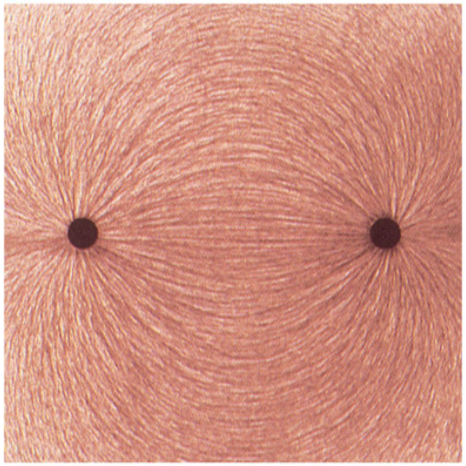
Electric Field Distribution



Electric Field Distribution

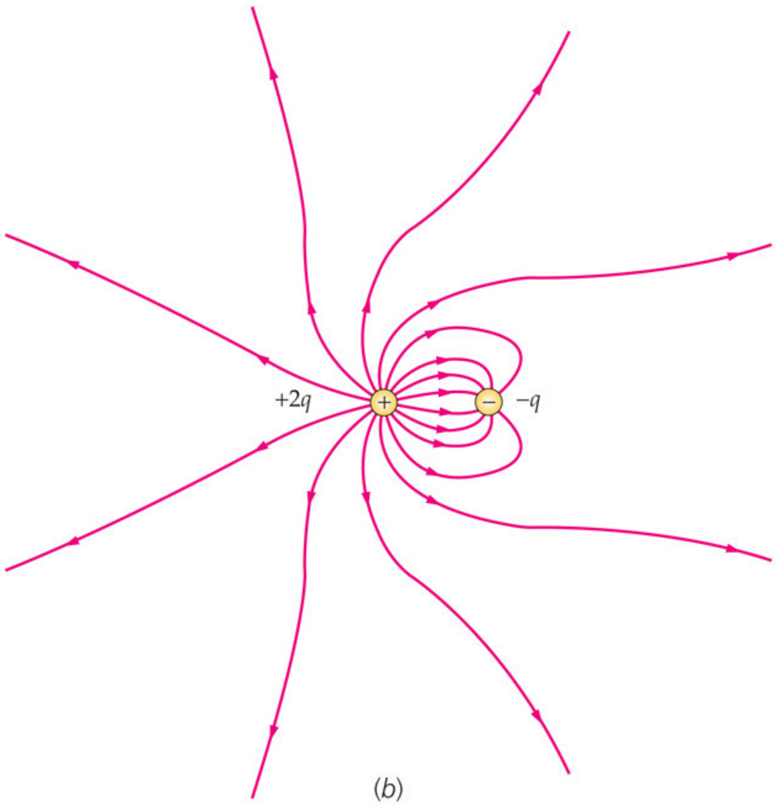


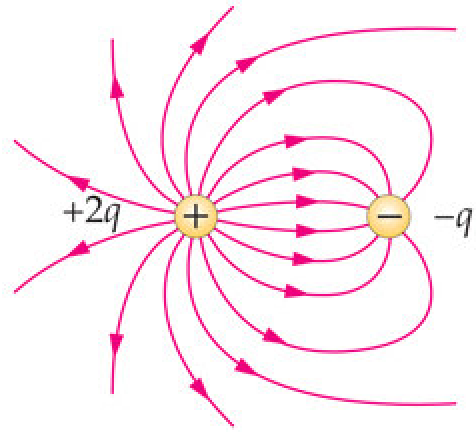
(a)



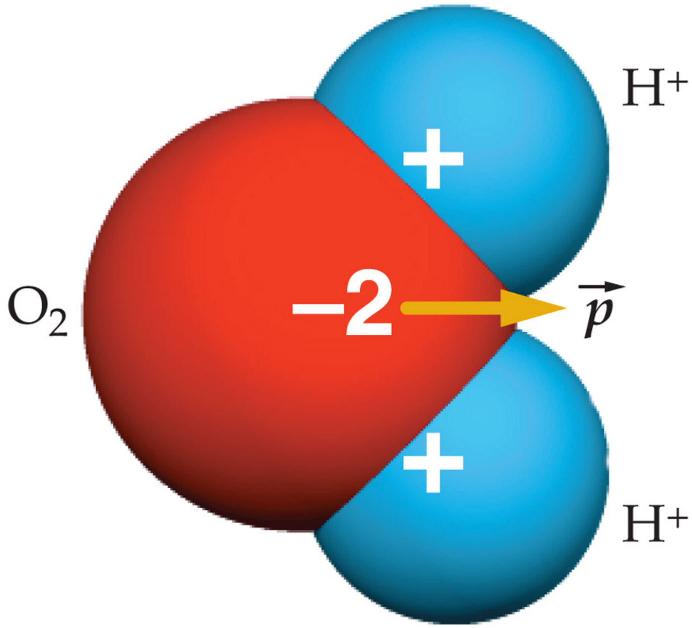
(b)

Electric Field Distribution





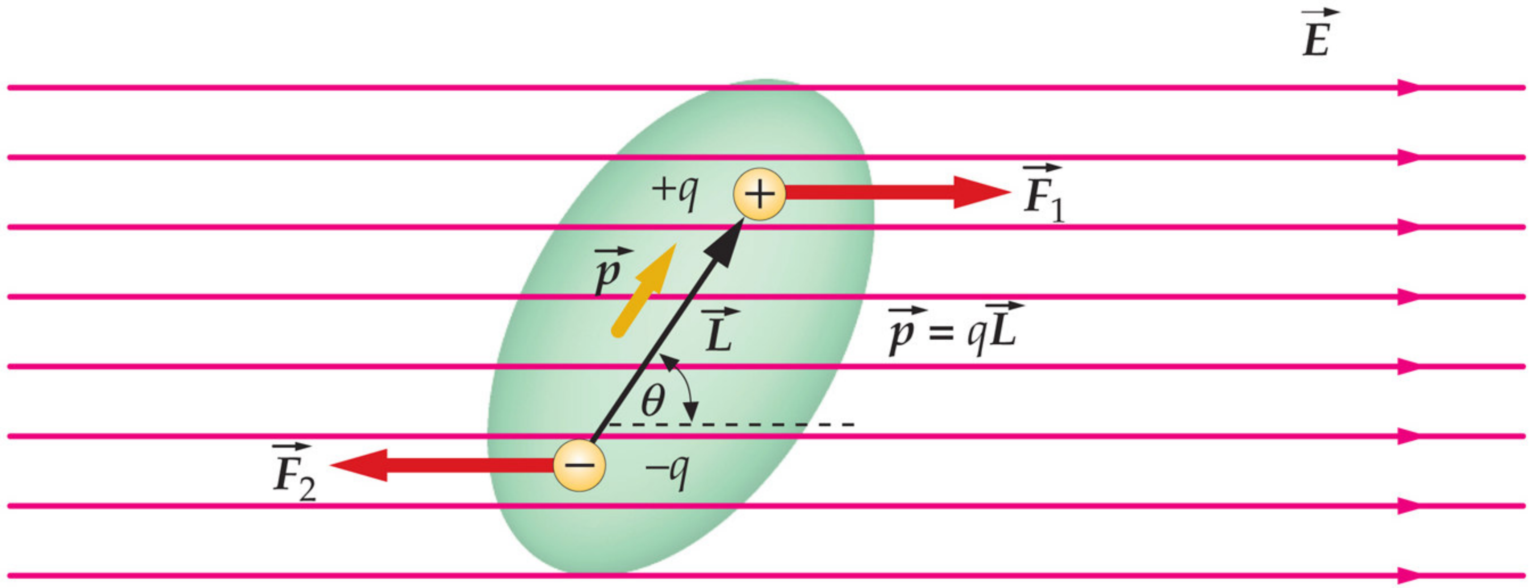
H_20 Molecule



The charge distribution of the water molecule gives rise to a permanent dipole moment.

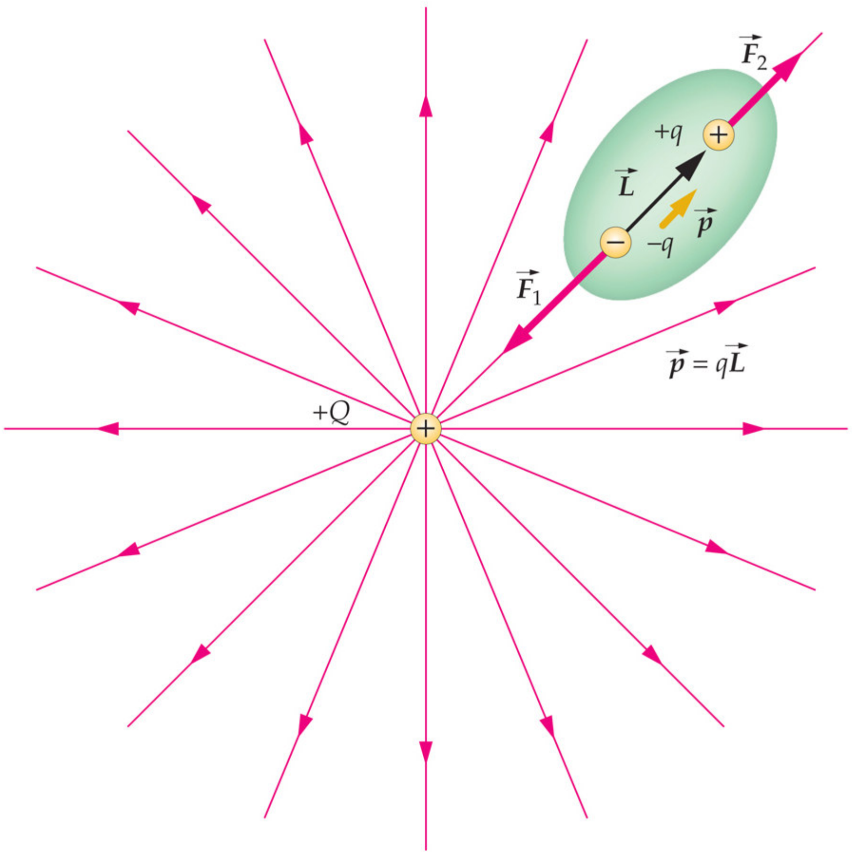
It's the dipole moment that causes the water molecule to oscillate back and forth in the presence of a microwave field.

Polarized Object in an External Electric Field



The microwave field alternates direction at a frequency ~ 10^9 Hz

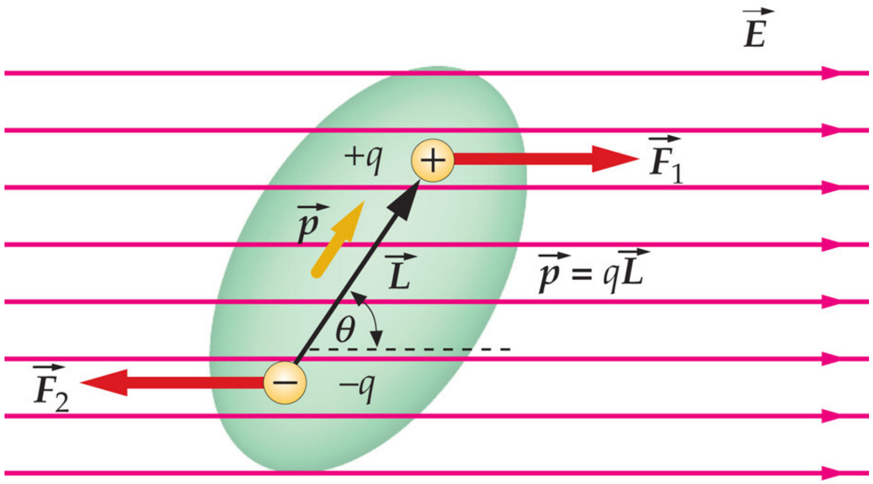
Polarized Object in an External Electric Field



An electric field can polarize objects that don't have a permanent dipole moment

A non-uniform electric field can both polarize an object and attract it.

Polarized Object in an External Electric Field



A uniform electric field causes a torque on a dipole but there is no net force.

$$\vec{\tau} = \vec{p} \times \vec{E}$$

By rotating the dipole through an angle dθ the electric field does work.

$$dW = -\tau d\theta = -pE\sin\theta d\theta$$

Set -dW equal to the change in PE (dU).

$$dU = -dW = pE\sin\theta d\theta$$

Integrating

$$U = -pE\cos\theta + U_0$$

$$Choose\ U_0 = 0\ when\ \theta = 90^0$$

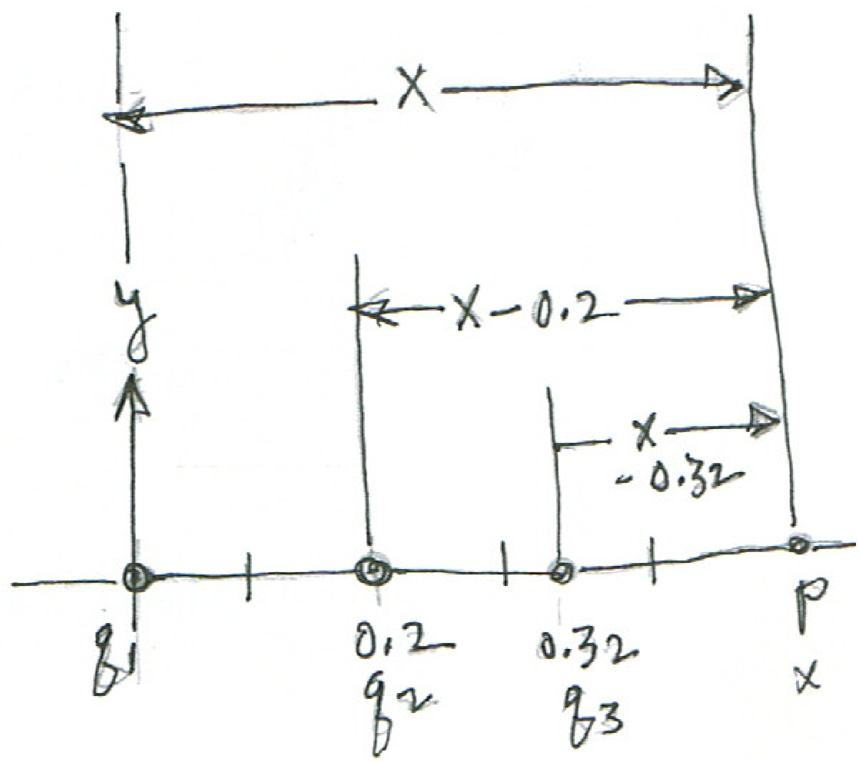
$$U = -pE\cos\theta = -\vec{p} \bullet \vec{E}$$

Homework Problems

Hmwk Problem #73

Three charges on a line. q_1 at x=0; q_2 at x = 0.2 m; Q at x = 0.32m.

$\vec{F}_2 = 240N\,\hat{i}$ Question: a.) Determine Q; b.) Find x so that E(x)=0



$$q_1 = -3.0\mu C$$

$$q_2 = +4.0\mu C$$

$$q_3 = Q = -97.1\,\mu C$$

Hmwk Problem #73

$$\vec{F}_2 = \vec{F}_{1,2} + \vec{F}_{Q,2}$$

$$240\hat{i} = \frac{kq_1q_2}{r_{1,2}^2}\hat{i} + \frac{kQq_2}{r_{Q,2}^2}\left(-\hat{i}\right) \qquad \hat{r}_{Q,2} = \frac{\vec{r}_{Q,2}}{\left|\vec{r}_{Q,2}\right|} = -\hat{i}$$

$$240\hat{i} = \frac{kq_1q_2}{r_{1,2}^2}\hat{i} - \frac{kQq_2}{r_{Q,2}^2}\hat{i}$$

$$Q = \frac{-r_{Q,2}^2}{kq_2}\left[240 - \frac{kq_1q_2}{r_{1,2}^2}\right]$$

Hmwk Problem #73

$$Q = \frac{-r_{Q,2}^2}{kq_2}\left[240 - \frac{kq_1q_2}{r_{1,2}^2}\right]$$

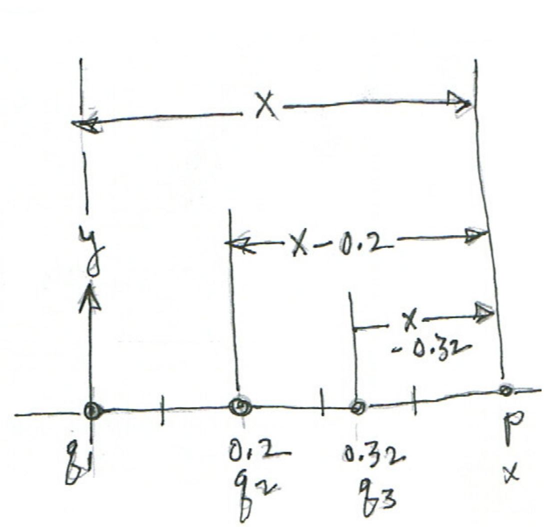
This is when the sign of the charge goes in.

$$Q = \frac{-(0.12)^2}{(8.99E+09)(4E-06)}\left[240 - \frac{(8.99E+09)(-3E-06)(4E-06)}{(0.2)^2}\right]$$

Q = -97.1 μC

Hmwk Problem #73

Determine x so that E(x)=0 For x > 0.32m



$$\vec{E}(x) = \vec{E}_{1,p} + \vec{E}_{2,p} + \vec{E}_{Q,p} = 0$$

$$\vec{E}(x) = E_{1,p}\hat{r}_{1,p} + E_{2,p}\hat{r}_{2,p} + E_{Q,p}\hat{r}_{Q,p} = 0$$

$$\vec{E}(x) = E_{1,p}\hat{i} + E_{2,p}\hat{i} + E_{Q,p}\hat{i} = 0$$

$$E(x) = E_{1,p} + E_{2,p} + E_{Q,p} = 0$$

$$E(x) = \frac{kq_1}{r_{1,p}^2} + \frac{kq_2}{r_{2,p}^2} + \frac{kQ}{r_{Q,p}^2} = 0$$

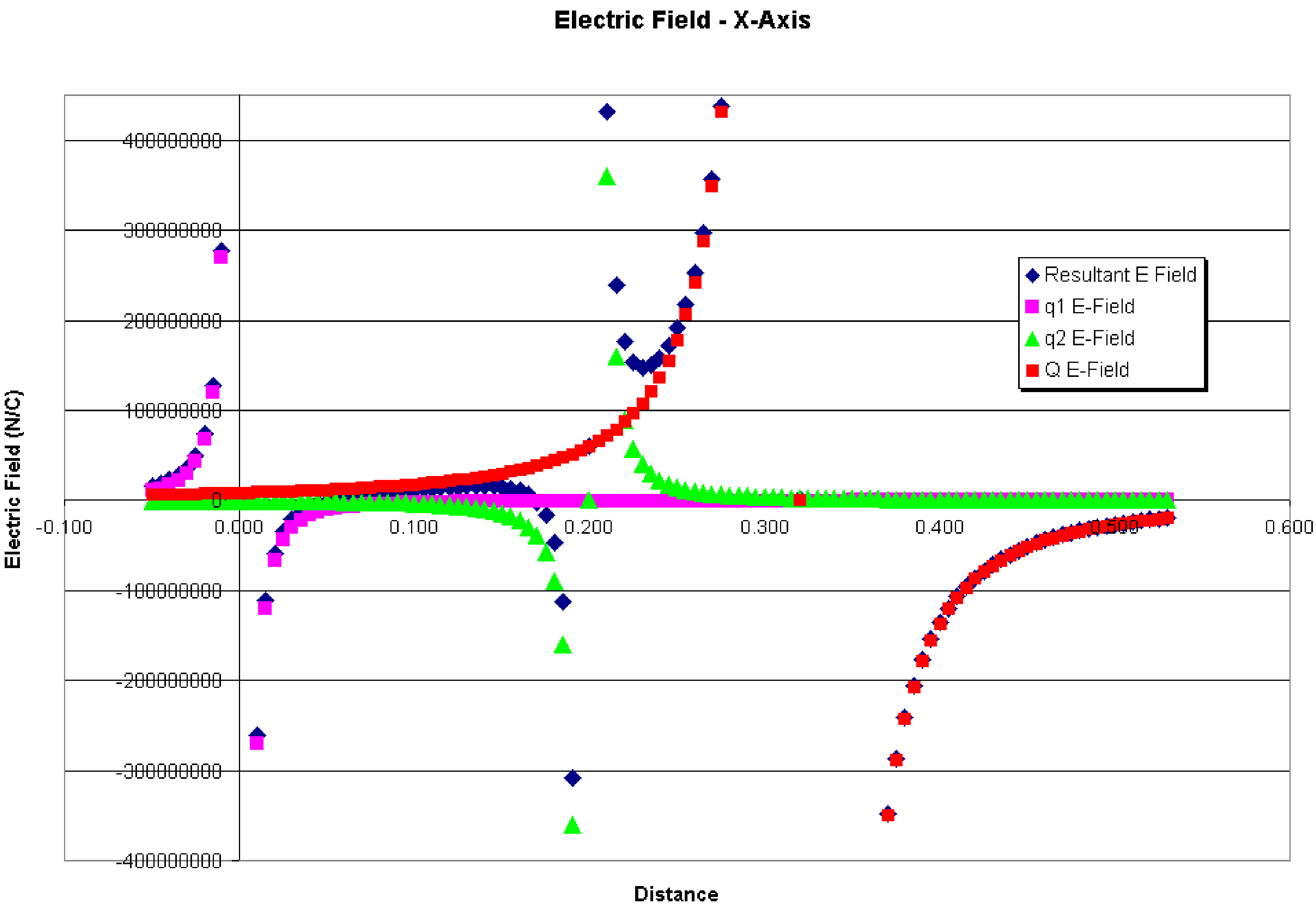
$$\frac{-3}{x^2} + \frac{4}{(x - 0.20)^2} - \frac{97.1}{(x - 0.32)^2} = 0$$

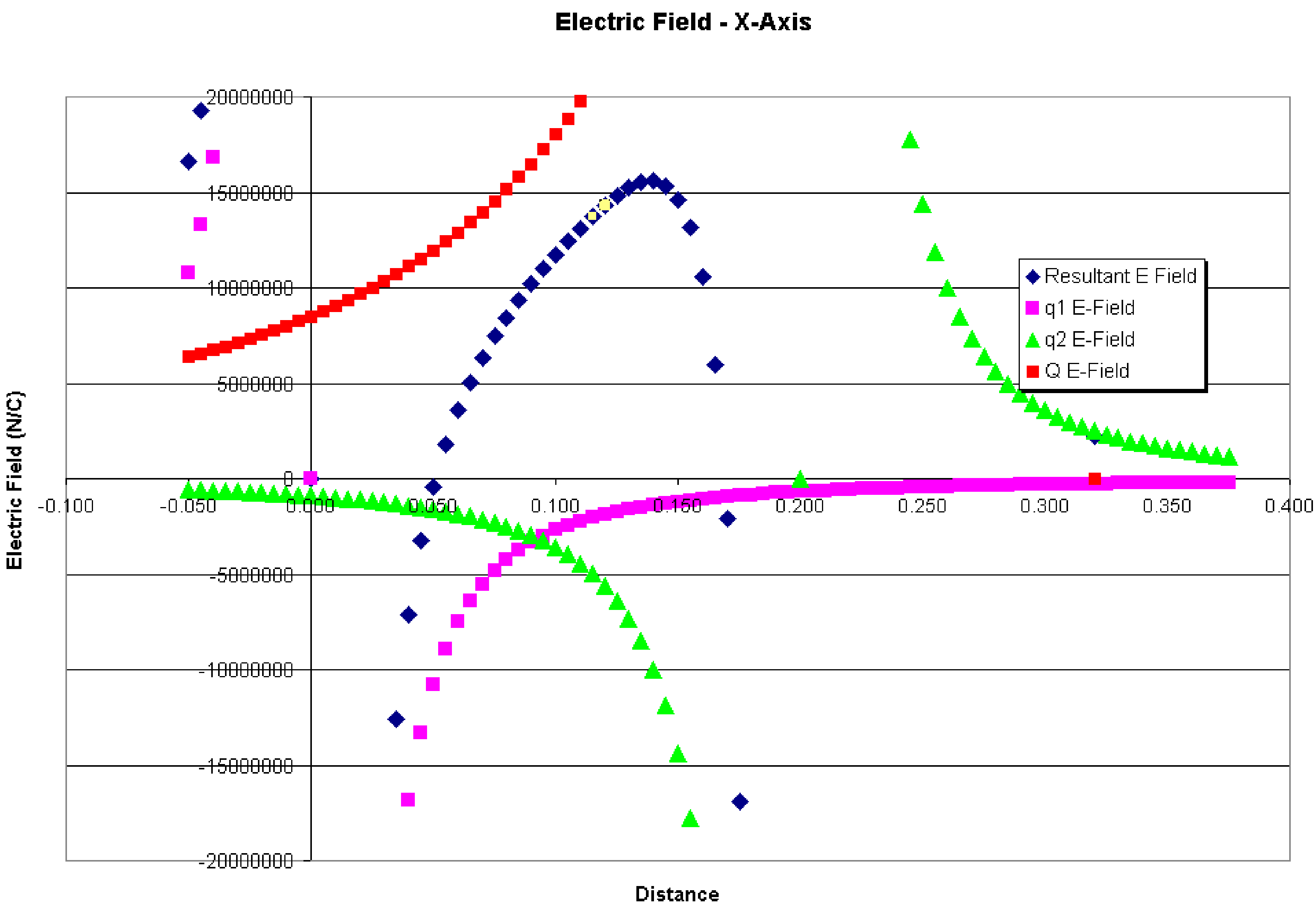
No solution for a real x value.

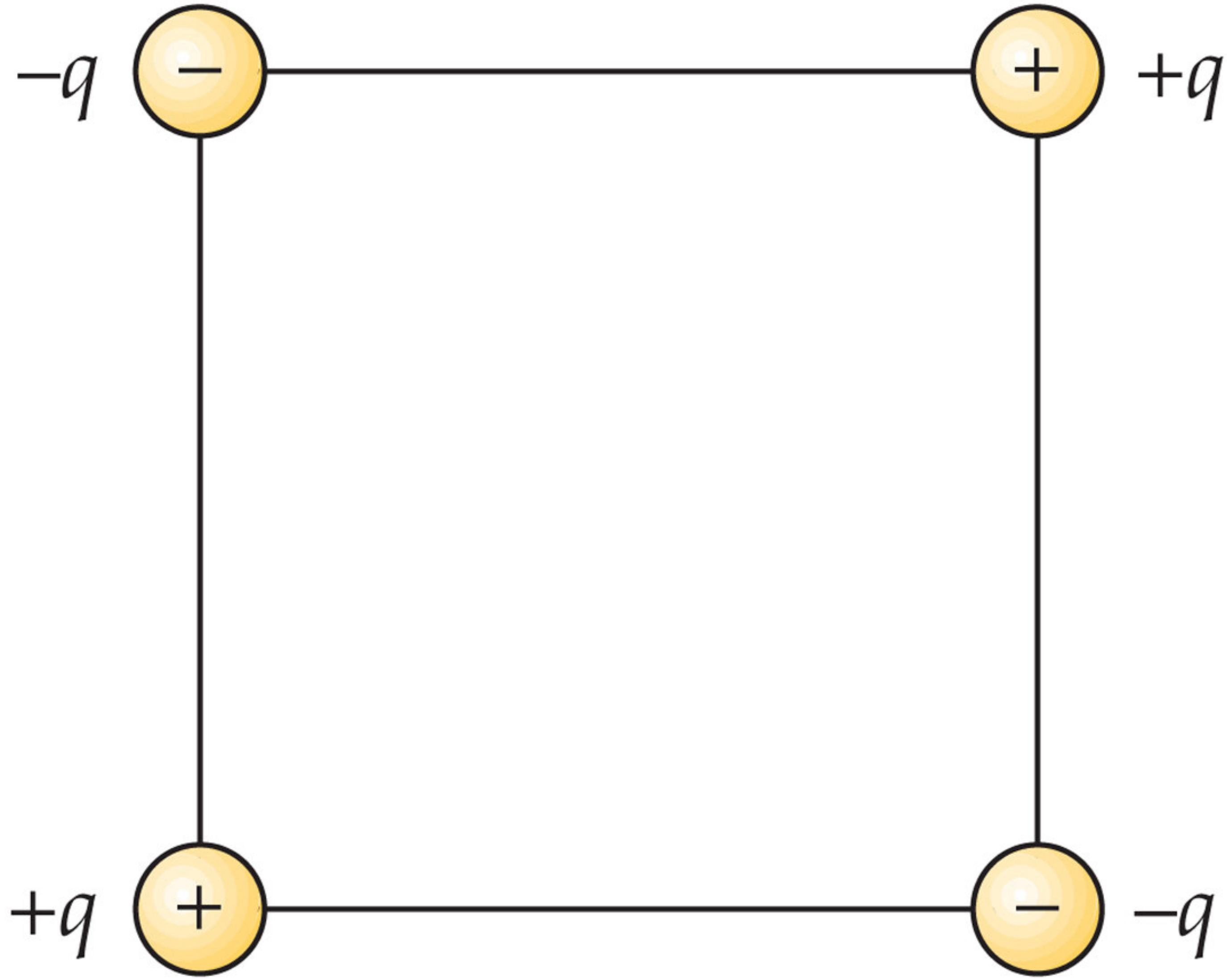
Hmwk Problem #73

A solution for all possible x

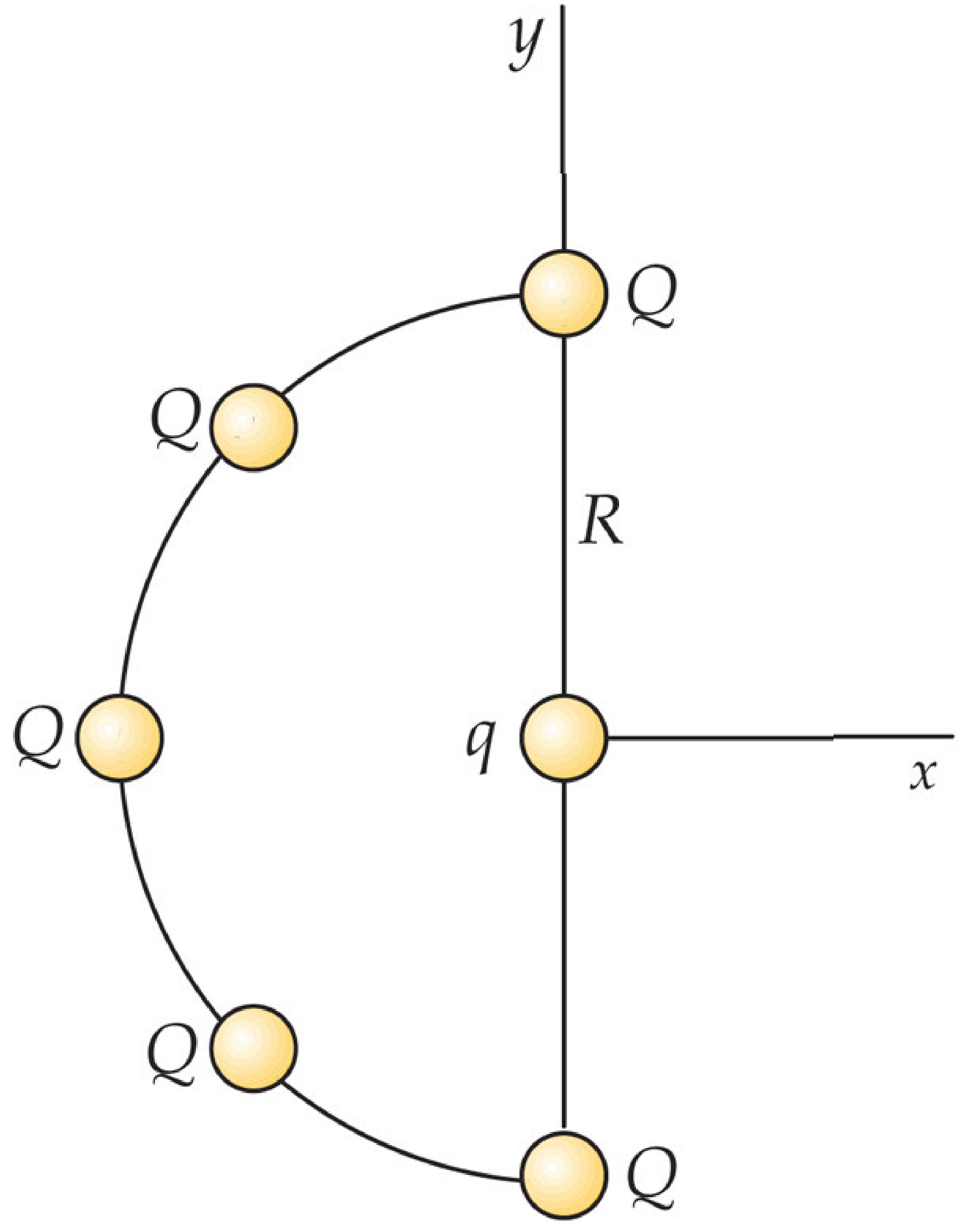
$$\frac{-3x}{x^3}\hat{i} + \frac{4\left(x - 0.20\right)}{\left(x - 0.20\right)^3}\hat{i} - \frac{97.1\left(x - 0.32\right)}{\left(x - 0.32\right)^3}\hat{i} = 0$$

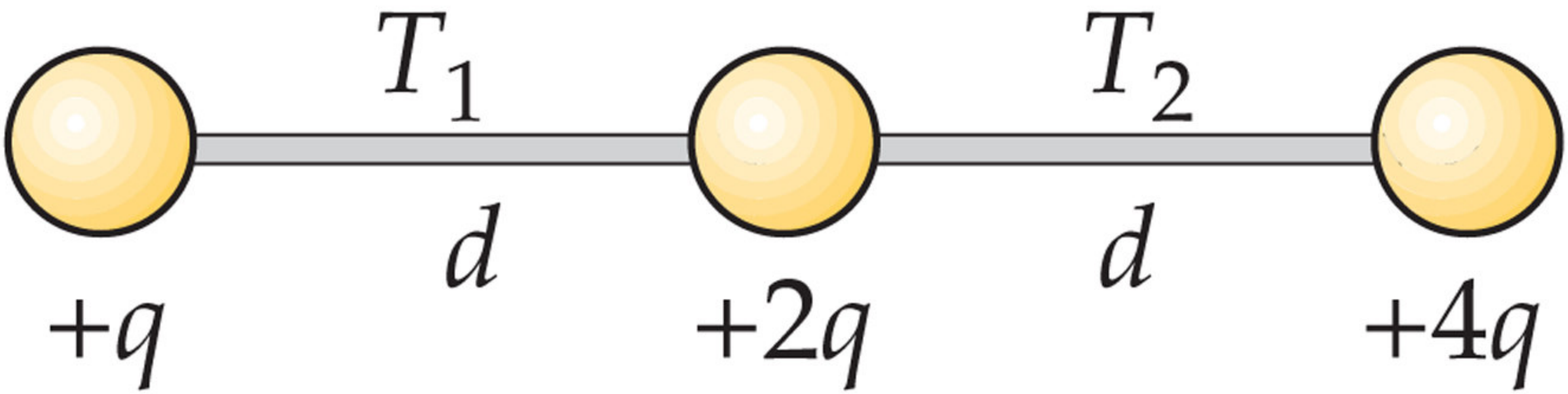

Electric Field - X-Axis
400000000
300000000
200000000
100000000
0
-100000000
-200000000
-300000000
-400000000
-0.100
0.000
0.100
0.200
0.300
0.400
0.500
0.600
Electric Field (N/C)
Distance
Resultant E Field
q1 E-Field
q2 E-Field
Q E-Field


Electric Field - X-Axis
Electric Field (N/C)
Distance
20000000
15000000
10000000
5000000
0
-5000000
-10000000
-15000000
-20000000
-0.100
-0.050
0.000
0.050
0.100
0.150
0.200
0.250
0.300
0.350
0.400
Resultant E Field
q1 E-Field
q2 E-Field
Q E-Field



−q
+q
+q
−q


y
Q
Q
R
Q
q
x
Q
Q


T_1
T_2
d
d
$+q$
$+2q$
$+4q$