## Chapter 22 \& 23

Electric Fields II

## Electric Fields II

1. Calculation of E from Coulomb's Law

- E on axis of finite line charge
- E off axis of a finite line charge
- E due to an infinite line charge
- E on the axis of a ring charge
- E on the axis of a uniformly charged disk
- E due to an infinite plane of charge.


## Electric Fields II

2. Gauss's Law
3. Calculation of E using Gauss's Law
4. Discontinuity of $E_{n}$
5. Charge and Field at Conductor Surfaces

## E-Field due to Coulomb's Law



This equation of dE is the beginning of each calculation.

## E on Axis of Finite Line Charge

Origin of coordinate system


The origin is selected at the center of the line charge to provide symmetry in the solution to facilitate our analysis.

## E on Axis of Finite Line Charge



Examine the solution and its limiting cases
$d \vec{E}=\frac{k d q}{r^{2}} \hat{r} \quad d q=\lambda d z$
$d E_{z} \hat{i}=\frac{k d q}{\left(z_{p}-z\right)^{2}} \hat{i}=\frac{k \lambda d z}{\left(z_{p}-z\right)^{2}} \hat{i}$
for $z_{p} \gg \frac{L}{2}$
$E_{z} \approx \frac{k Q}{z_{p}^{2}}$
$E_{z}=k \lambda \int_{\frac{L}{2}}^{\frac{L}{2}} \frac{d z}{\left(z_{p}-z\right)^{2}}=\frac{k \lambda L}{z_{P}^{2}-\left(\frac{L}{2}\right)^{2}}=\frac{k Q}{z_{P}^{2}-\left(\frac{L}{2}\right)^{2}}$

## The Off Axis Line Charge Calculation



## The Off Axis Line Charge Calculation

Changing coordinates
from ( $\mathrm{x}, \mathrm{y}$ ) to ( $\mathrm{r}, \mathrm{z}$ )
The situation is symmetric in r


## The Off Axis Line Charge Calculation



## The Off Axis Line Charge Calculation



Examine the solution and its limiting cases.

$$
\begin{aligned}
& E_{y}=\frac{k Q}{L y}\left(\sin \theta_{2}-\sin \theta_{1}\right) \\
& E_{x}=\frac{k Q}{L y}\left(\cos \theta_{2}-\cos \theta_{1}\right)
\end{aligned}
$$

For a line charge of infinite length

$$
E_{x} \rightarrow 0 \quad E_{y}=\frac{2 k \lambda}{R}
$$ $Q$ and $L$ become infinite.

Physically: Equal charge on each side of the origin

## The Infinite Line Charge Calculation



5th Edition


6th Edition

## The Infinite Line Charge Calculation



Examine the solution and its limiting cases

$$
E_{x} \rightarrow 0 \quad E_{y}=\frac{2 k \lambda}{R}
$$

With this geometry $\mathrm{E}_{\mathrm{x}}$ is zero at every step.

5th Edition

## Solution Comparisons



## Uniform Ring Charge - E on the Axis



## Uniform Ring Charge - E on the Axis

Examine the symmetry first to see which components might cancel out.


## Uniform Ring Charge - E on the Axis



Solution and limiting cases

$$
\begin{aligned}
& d E_{x}=\frac{k d q}{r^{2}} \cos \theta=\frac{k d q}{r^{2}} \frac{x}{r}=\frac{k x d q}{\left(x^{2}+a^{2}\right)^{\frac{3}{2}}} \\
& E_{x}=\frac{k x}{\left(x^{2}+a^{2}\right)^{\frac{3}{2}}} \int d q=\frac{k Q x}{\left(x^{2}+a^{2}\right)^{\frac{3}{2}}}
\end{aligned}
$$

## $\mathrm{E}=0$ in the Plane of the Ring



## Axial E-Field on Both Sides of the Ring

$$
E_{x}=\frac{k Q}{a^{2}} \frac{u}{\left(1+u^{2}\right)^{\frac{3}{2}}}
$$

$$
E_{x}=\frac{k Q x}{\left(x^{2}+a^{2}\right)^{\frac{3}{2}}}=\frac{k Q x}{\left(a^{2}\left(1+\frac{x^{2}}{a^{2}}\right)\right)^{\frac{3}{2}}}=\frac{k Q x}{a^{3}\left(1+\frac{x^{2}}{a^{2}}\right)^{\frac{3}{2}}}
$$

$$
E_{x}=\frac{k Q \frac{x}{a}}{a^{2}\left(1+\frac{x^{2}}{a^{2}}\right)^{\frac{3}{2}}}=\frac{k Q}{a^{2}} \frac{\frac{x}{a}}{\left(1+\frac{x^{2}}{a^{2}}\right)^{\frac{3}{2}}}=\frac{k Q}{a^{2}} \frac{u}{\left(1+u^{2}\right)^{\frac{3}{2}}}
$$

## Uniform Surface Charge - The Disk

Use the ring field results to build up the disk E-field

$$
d E_{x}=\frac{k x d q}{\left(x^{2}+a^{2}\right)^{\frac{3}{2}}}
$$

$$
d q=\sigma d A=\sigma 2 \pi a d a
$$

$$
E_{x}=2 \pi k x \sigma \int_{0}^{R} \frac{a d a}{\left(x^{2}+a^{2}\right)^{\frac{3}{2}}}
$$

$$
E_{x}=-2 \pi k x \sigma\left(\frac{1}{\sqrt{x^{2}+R^{2}}}-\frac{1}{\sqrt{x^{2}}}\right)
$$

## Uniform Surface Charge - The Disk



$$
\begin{aligned}
& E_{x}=-2 \pi k x \sigma\left(\frac{1}{\sqrt{x^{2}+R^{2}}}-\frac{1}{\sqrt{x^{2}}}\right) \\
& \text { for } x \ll R
\end{aligned}
$$

$$
\lim _{x \rightarrow 0} E_{x}=-\lim _{x \rightarrow 0} 2 \pi k x \sigma\left[\frac{1}{R}-\frac{1}{x}\right]=+2 \pi k \sigma=+\frac{\sigma}{2 \varepsilon_{0}}
$$

This is the result we saw earlier for the E-field near the infinite plane. For x sufficiently small the disk will appear to be an infinite plane.

## E-Field Discontinuity Crossing a Surface Charge Distribution



## Solution Comparisons

At small distances the differences are noticeable.

You can't get "far" away from an infinite plane


## Infinite Dielectric Planes with Uniform Fixed Charge

Between the planes


## Electric Flux $-\operatorname{Net} \mathrm{Q}=0$



## Electric Flux - Net $\mathrm{Q} \neq 0$



## Basic Flux Definition

Flux is the product of the electric field vector and the area that it is passing through.


$$
\varphi=\vec{E} \bullet \vec{A}
$$

Passing through means we only count the component of E that is perpendicular to the surface area.

## A More Precise Flux Definition

$$
\varphi=\vec{E} \bullet \hat{n} A_{2}=E A_{2} \cos \theta
$$



This is the electric flux that passes through area $\mathrm{A}_{2}$

In general both the direction and the magnitude of the electric field crossing the surface can be varying.


We will limit our calculations to situations where the electric field is constant in magnitude and direction on the surface.

## Net Flux Calculation

$$
\begin{aligned}
& \varphi_{\text {net }}=\oint_{S} E_{n} d A=E_{n} \oint_{S} d A=\frac{k Q}{R^{2}} 4 \pi R^{2}=4 \pi k Q \\
& \varphi_{\text {net }}=\frac{Q}{\varepsilon_{0}} \\
& \text { To be useful the Gaussian } \\
& \text { surfaces that we pick } \\
& \text { must possess a high } \\
& \text { degree of symmetry. }
\end{aligned}
$$

## Statement of Gauss's Law

The net outward flux through any closed surface equals the net charge inside the surface divided by $\varepsilon_{0}$

$$
\varphi_{n e t}=\oint_{S} E_{n} d A=E_{n} \oint_{S} d A=\frac{Q_{\text {inside }}}{\varepsilon_{0}}
$$

The same as Coulomb's Law for electrostatics, but Gauss's Law is true all the time. Even if we can't find a simple surface to use it to our advantage.

## Given E-field - Find the charge enclosed



The closed surface can consist of several surfaces that together form a closed surface.


From the direction of the field we know that we are dealing with an infinite sheet of positive charge and we want to determine how much charge is contained within the volume formed by our Gaussian surface.

E-values on both sides are the same


$$
\begin{aligned}
& \varphi_{\text {right }}=\vec{E}_{\text {right }} \hat{k} \pi R^{2}=+200 \hat{k}\left(\hat{k} \pi(0.0500)^{2}\right)=1.57 \mathrm{Nm}^{2} / C \\
& \varphi_{\text {left }}=\vec{E}_{\text {left }}(-\hat{k}) \pi R^{2}=-200 \hat{k}\left((-\hat{k}) \pi(0.0500)^{2}\right)=1.57 \mathrm{Nm}^{2} / C \\
& \varphi_{\text {net }}=\varphi_{\text {right }}+\varphi_{\text {left }}+0=1.57+1.57=3.14 \mathrm{Nm}^{2} / \mathrm{C} \\
& Q_{\text {inside }}=\varepsilon \varphi_{\text {net }}=27.8 p \mathrm{C}
\end{aligned}
$$

## Infinite Slab Geometry



Cylindrical surface goes all the way through the slab since it must form a closed surface.

## Infinite Slab Geometry



## This is the same surface we used in the previous problem.

In this case we know the charge distribution and we want to calculate the E-field everywhere along the z -axis.

The material is a dielectric. The charge is distributed uniformly throughout the volume of the dielectric slab.

Since the slab is infinite the electric field is everywhere perpendicular to the surface of the slab.

## E-Field Outside the Infinite Slab


$\sigma=\rho 2 a \quad \begin{array}{ll}\text { Integrating the volume density over one dimension creates a } \\ \text { surface density }\end{array}$

## E-Field Inside the Infinite Slab



## E-Field Discontinuity - A More Realistic View

$$
\begin{aligned}
& E_{z}= \pm \frac{\sigma}{2 \varepsilon_{0}} \\
& k=1 / 4 \pi \varepsilon_{0} ; \quad \sigma=2 a \rho \\
& E_{z}= \pm \frac{\sigma}{2 \varepsilon_{0}}
\end{aligned}
$$

## E-Field Due to a Thin Spherical Shell of Charge



## Conductive Sphere



## Dielectric Sphere



## Dielectric Sphere



$$
\begin{aligned}
& Q^{\prime} \text { within } r \quad Q^{\prime}=\frac{4}{3} \pi \rho r^{3} \\
& Q \text { within } R \quad Q=\frac{4}{3} \pi \rho R^{3} \\
& Q^{\prime}=\frac{r^{3}}{R^{3}} Q
\end{aligned}
$$

$$
\begin{aligned}
& \oint_{S} E d A=\frac{Q^{\prime}}{\varepsilon_{0}} \\
& E \cdot 4 \pi r^{2}=\frac{Q^{\prime}}{\varepsilon_{0}}=\frac{r^{3}}{R^{3}} Q \frac{l}{\varepsilon_{0}} \\
& E=\frac{1}{4 \pi \varepsilon_{0}} \frac{l}{r^{2}} \frac{r^{3}}{R^{3}} Q=\frac{Q}{4 \pi \varepsilon_{0} R^{3}} r
\end{aligned}
$$

## E-Field Due to a Uniform Line Charge



$$
\begin{aligned}
& \varphi_{\text {curved }}=E_{R} 2 \pi R L \\
& \varphi_{\text {net }}=\varphi_{\text {curved }}=E_{R} 2 \pi R L=\frac{Q_{\text {inside }}}{\varepsilon_{0}}
\end{aligned}
$$

$$
E_{R} 2 \pi R L=\frac{Q_{\text {inside }}}{\varepsilon_{0}}=\frac{\lambda L}{\varepsilon_{0}}
$$

$$
E_{R}=\frac{\lambda}{2 \pi \varepsilon_{0} R}
$$

## Zero E-Field Inside a Conductor

The net positive charge on the conductor is achieved by removing some of the electrons from the formerly neutral conductor. The remaining electrons then spread out and find new equilibrium positions that results in a net positive charge on the surface of the conductor.


## Charges and Fields at Conductor Surfaces


(a)

(b)

(c)

## Imaging Electric Field Lines



Thin pieces of thread suspended in oil line up along the electric field lines.

A potential difference is set up between the two conductors. Hence there is a net charge on each surface.

All field lines contact the conductor surfaces at right angles.

Inside the closed circular electrode there is no electric field.

## Electric Fields, Charges and Conductors



Therefore the total charge inside $S$ is zero. Hence the induced negative charge is equal and opposite to the positive charge.

## Electric Fields, Charges and Conductors


"Inside the conductor" means inside the actual conducting material itself and not inside the hollow volume inside the spherical shell.

The E-field outside the conductor is the same as if only the point charge was present and the conductive shell was not there.

## Electric Fields, Charges and Conductors



> What changes in the unsymmetrical case, where the positive charge is moved off the center position?

The number of electric field lines eminating from the positive charge and ending on the negative charges on the inner surface of the hollow sphere are equal in number.

As the positive charge moves closer to the inner surface the concentration of negative charge increases there - the number of negative charges remains the same.

## Electric Fields, Charges and Conductors



The drawing is a little misleading. The total induced negative charge is equal to the induced positive charge. When the center positive charge moves off center both induced charge densities change but the size of the induced charges remain the same.

Up close the field outside the conductor no longer resembles that of an isolated positive charge.
As the distance from the sphere becomes large compared to the radius of the sphere the external field approaches that of a point charge once again.

## Homework Problems




