Chapter 25

Electric Potential, Energy, and Capacitance

Units of Chapter 25

Electric Potential Energy and Electric Potential Difference

Equipotential Surfaces and the Electric Field

Capacitors in Series and in Parallel

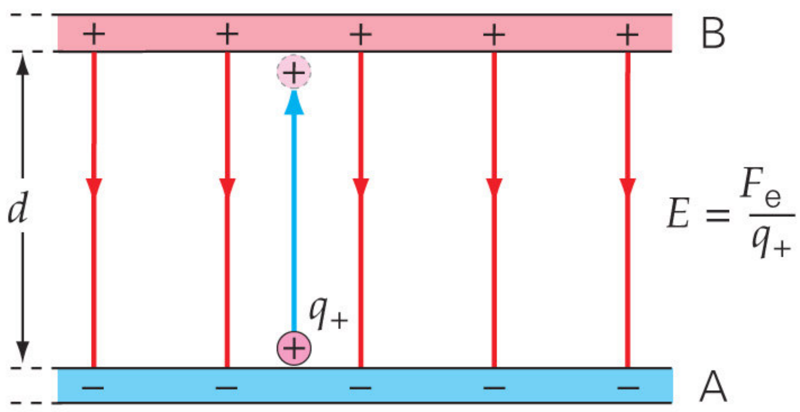
Capacitance

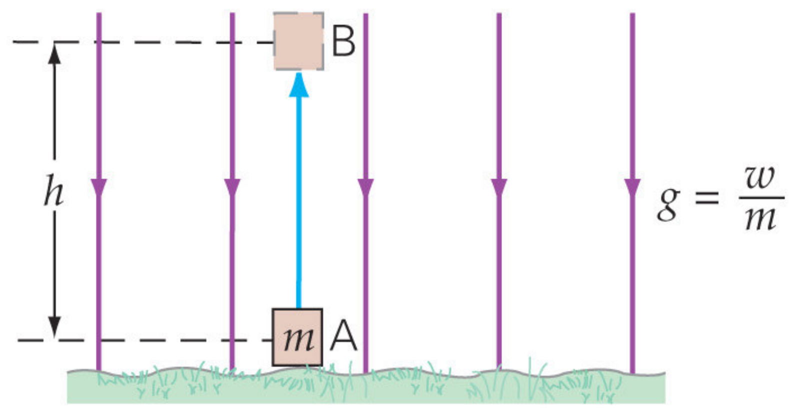
Dielectrics

Electric Potential Energy and Electric Potential Difference

It takes work to move a charge against an electric field. Just as with gravity, this work increases the potential energy of the charge.

$$\Delta U_e = U_B - U_A = q_+ Ed$$





Electric Potential Energy and Electric Potential Difference

Just as with the electric field, it is convenient to define a quantity that is the electric potential energy per unit charge. This is called the electric potential.

$$\Delta V = \frac{\Delta U_e}{q_+}$$

Unit of electric potential: the volt, V.

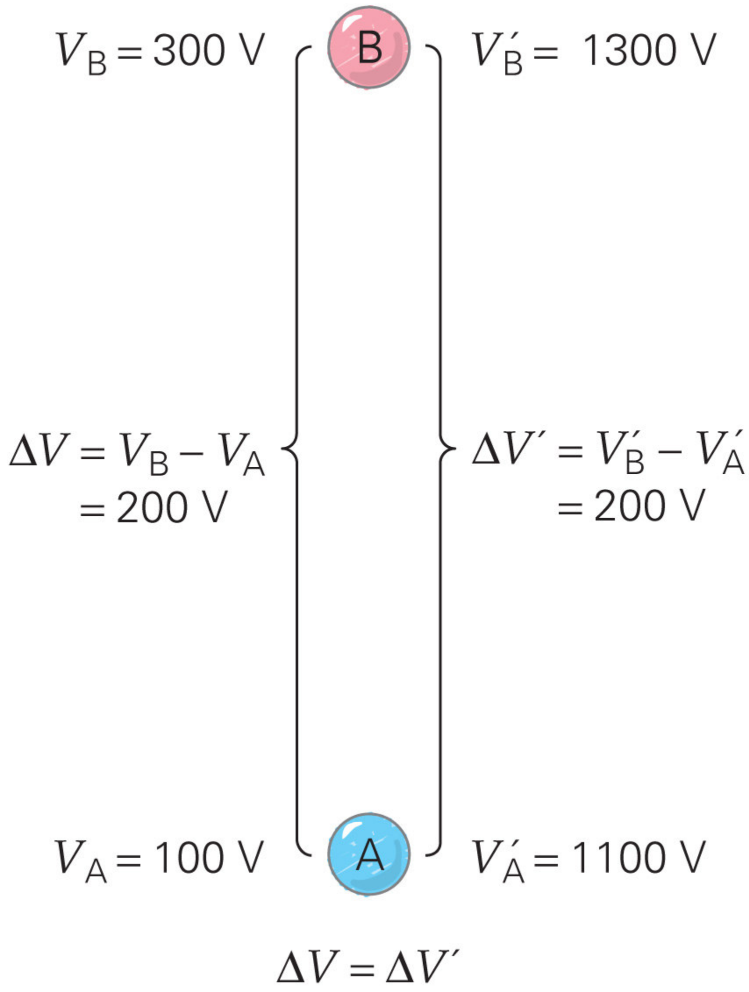
Electric Potential Energy and Electric Potential Difference

The potential difference between parallel plates can be calculated relatively easily:

$$\Delta V = \frac{\Delta U_e}{q_+} = \frac{q_+ E d}{q_+} = Ed$$

For a pair of oppositely charged parallel plates, the positively charged plate is at a higher electric potential than the negatively charged one by an amount ΔV.

Electric Potential Energy and Electric Potential Difference



As with potential energy, only changes in the electric potential can be defined.

The choice of $V = 0$ is arbitrary.

Only the difference in potential is real.

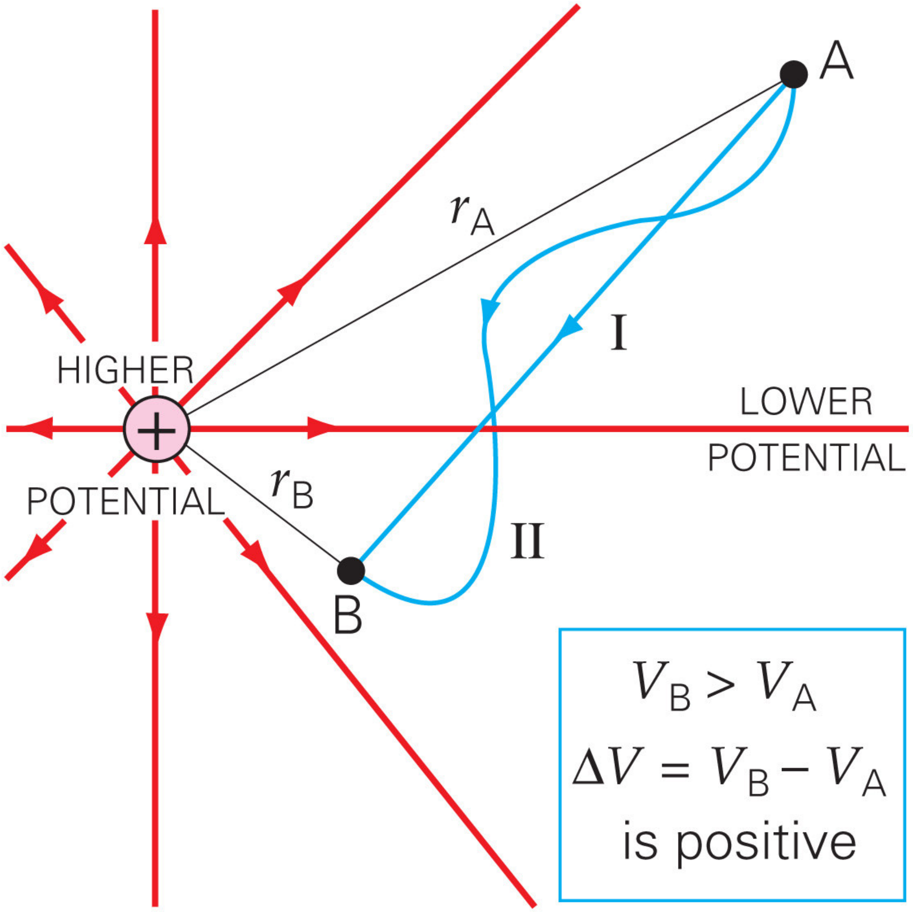
Electric Potential Energy and Electric Potential Difference

Potential differences are defined in terms of positive charges, as is the electric field. Therefore, we must account for the difference between positive and negative charges.

Positive charges, when released, accelerate toward regions of lower electric potential.

Negative charges, when released, accelerate toward regions of higher electric potential.

Electric Potential Energy and Electric Potential Difference



Electric potential difference of a point charge:

$$\Delta V = \frac{kq}{r_B} - \frac{kq}{r_A}$$

Electric Potential Energy and Electric Potential Difference

Whether the electric potential increases or decreases when towards or away from a point charge depends on the sign of the charge.

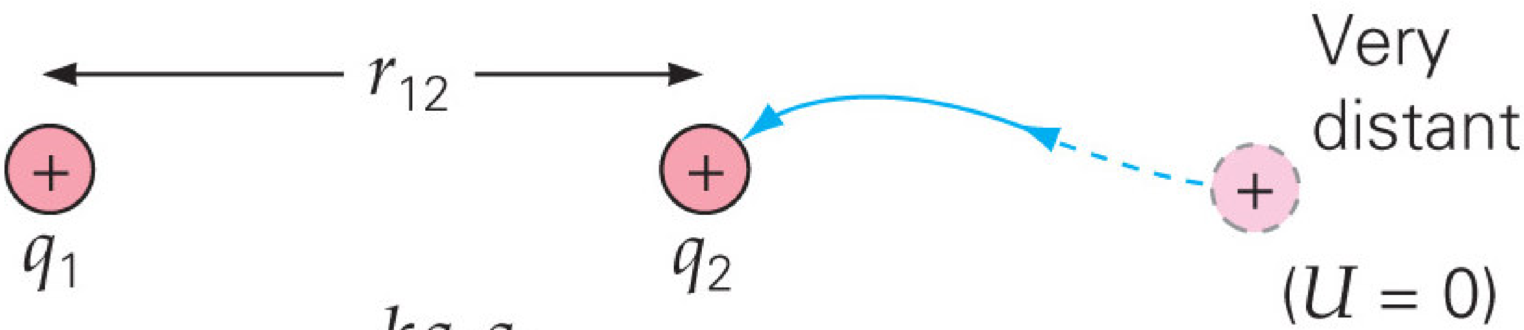
Electric potential increases when moving nearer to positive charges or farther from negative charges.

Electric potential decreases when moving farther from positive charges or nearer to negative charges.

Electric Potential Energy and Electric Potential Difference

The electric potential energy of a system of two charges is the change in electric potential multiplied by the charge.

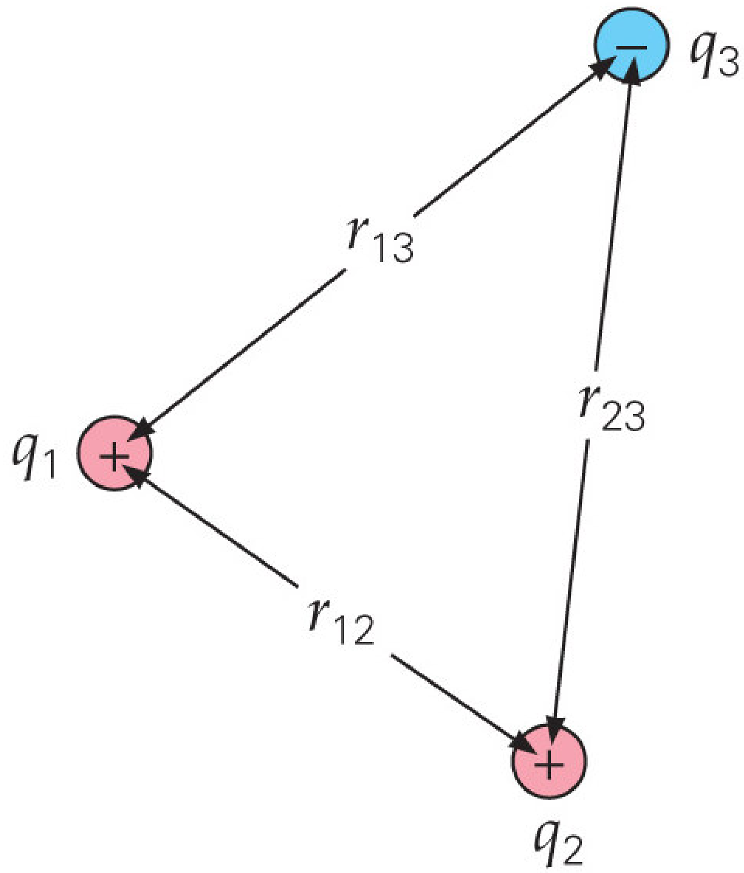
$$\Delta U_e = q_2 \Delta V = q_2 (V_1 - V_\infty) = q_2 \left(\frac{kq_1}{r_{12}} - 0 \right) = \frac{kq_1 q_2}{r_{12}}$$



$$U_{12} = \frac{kq_1 q_2}{r_{12}}$$

(by convention)

Electric Potential Energy and Electric Potential Difference

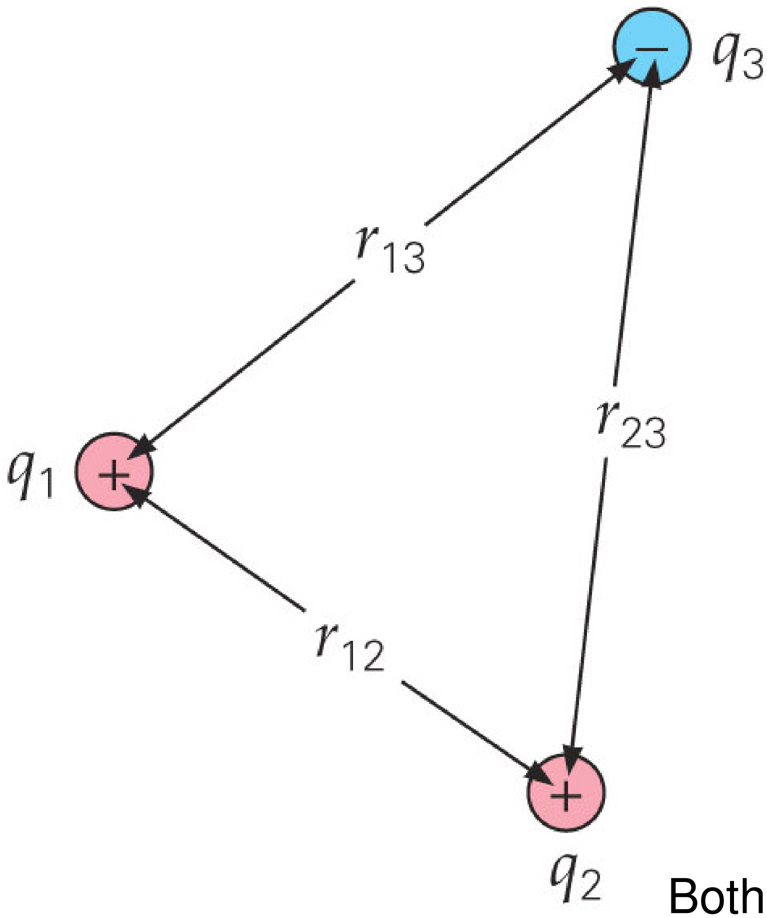


$$U = U_{12} + U_{23} + U_{13}$$

The additional potential energy due to a third charge is the sum of its potential energies relative to the first two. Further charges extend the sum.

$$U = U_{12} + U_{23} + U_{13} + U_{14} \cdots$$

Electric Potential Energy and Electric Potential Difference

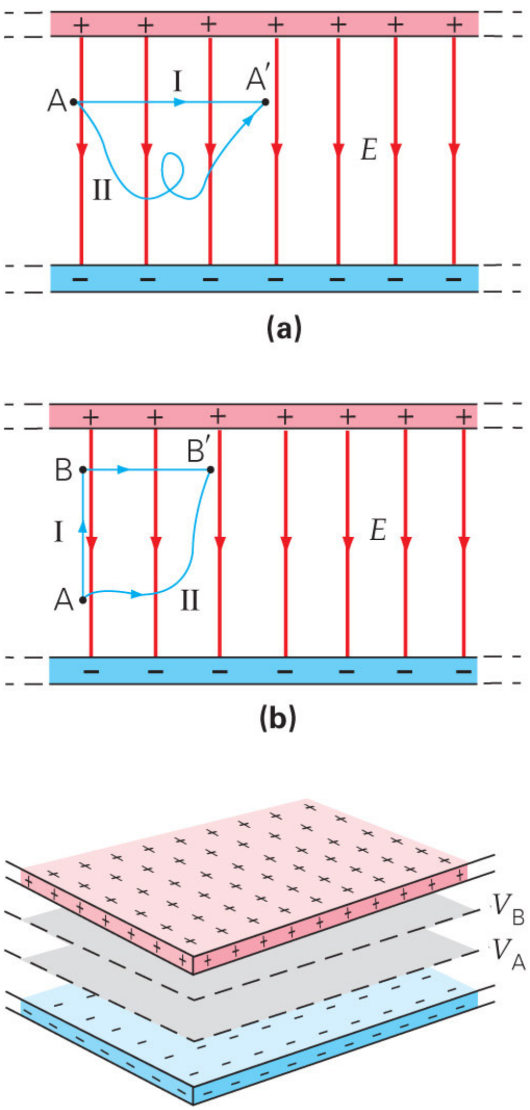


The factor of 1/2 removes the double counting brought about by making the summation over both indices

$$U = \frac{1}{2}\sum_{i,j} q_i V_{ij}$$

Both V_{ij} and V_{ji} will be included – double counting

Equipotential Surfaces and the Electric Field



(a)

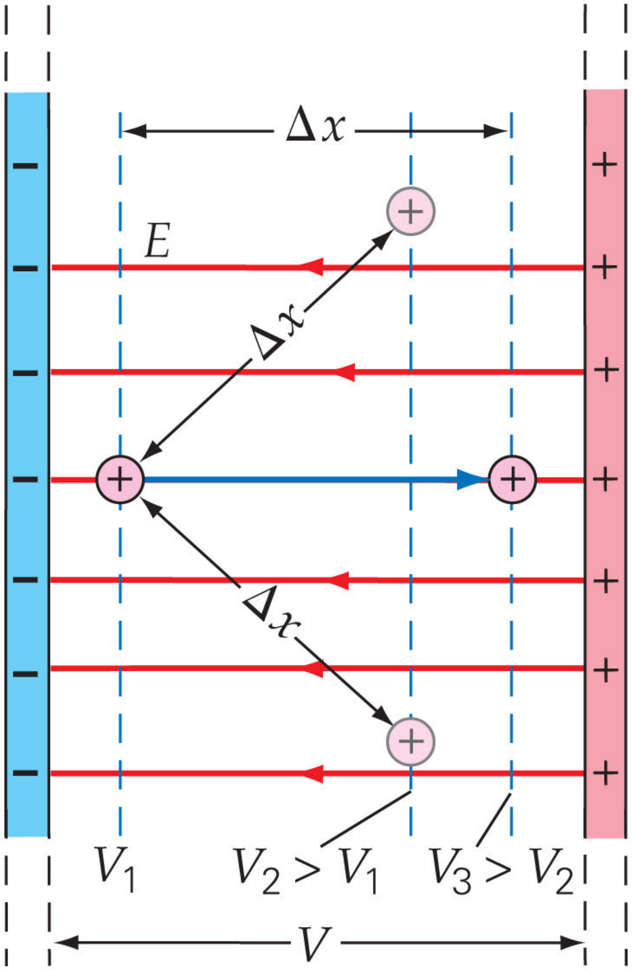
(b)

(c)

An equipotential surface is one on which the electric potential does not vary; it takes no work to move a charge along an equipotential surface.

Equipotential surfaces follow the shape of the nearest conductor.

Equipotential Surfaces and the Electric Field



The direction of the electric field E is that in which the electric potential decreases the most rapidly.

Its magnitude is given by:

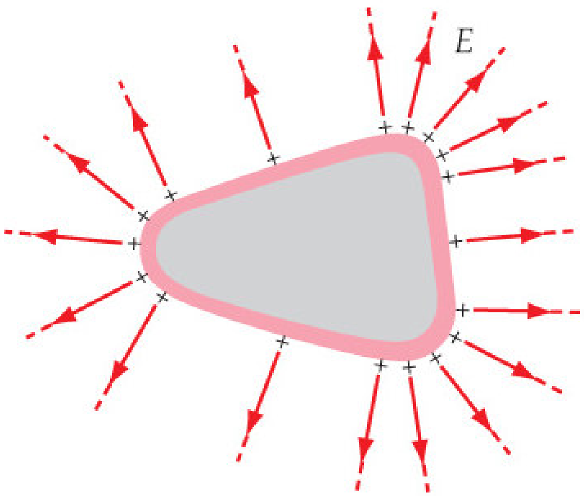
$$E = \left|\frac{\Delta V}{\Delta x}\right|_{max}$$

Equipotential Surfaces and the Electric Field

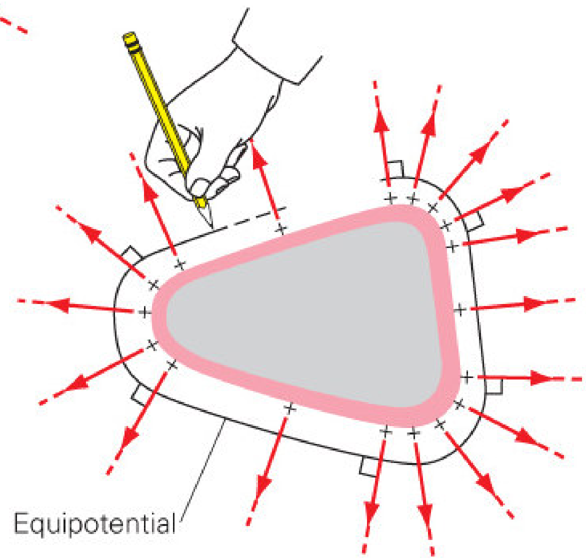
TABLE 16.1 Common Electric Potential Differences (Voltages)

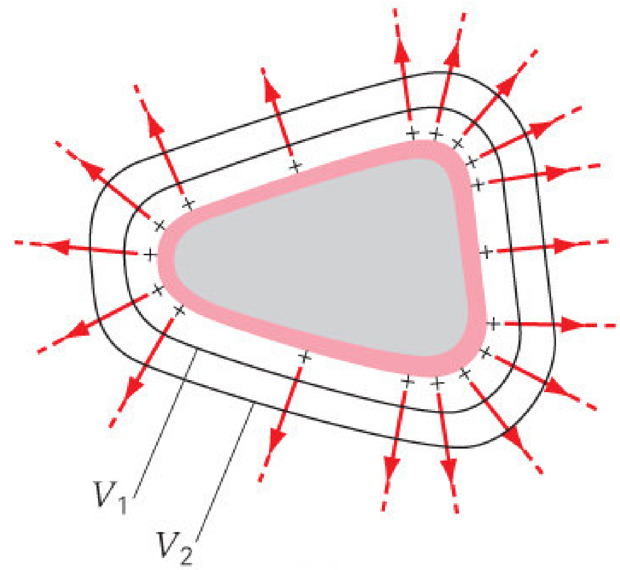
Source	*Approximate Voltage* (ΔV)
Across nerve membranes	100 mV
Small-appliance batteries	1.5 to 9.0 V
Automotive batteries	12 V
Household outlet (United States)	110 to 120 V
Houschold outlets (Europe)	220 to 240 V
Automotive ignitions (spark plug firing)	10 000 V
Laboratory generators	25 000 V
High-voltage electric power delivery lines	300 kV or more
Cloud-to-Earth surface during thunderstorm	100 MV or more

Equipotential Surfaces and the Electric Field



Equipotential surfaces outside a conductor





In practice draw V lines first and then the E-Field.

Equipotential Surfaces and the Electric Field

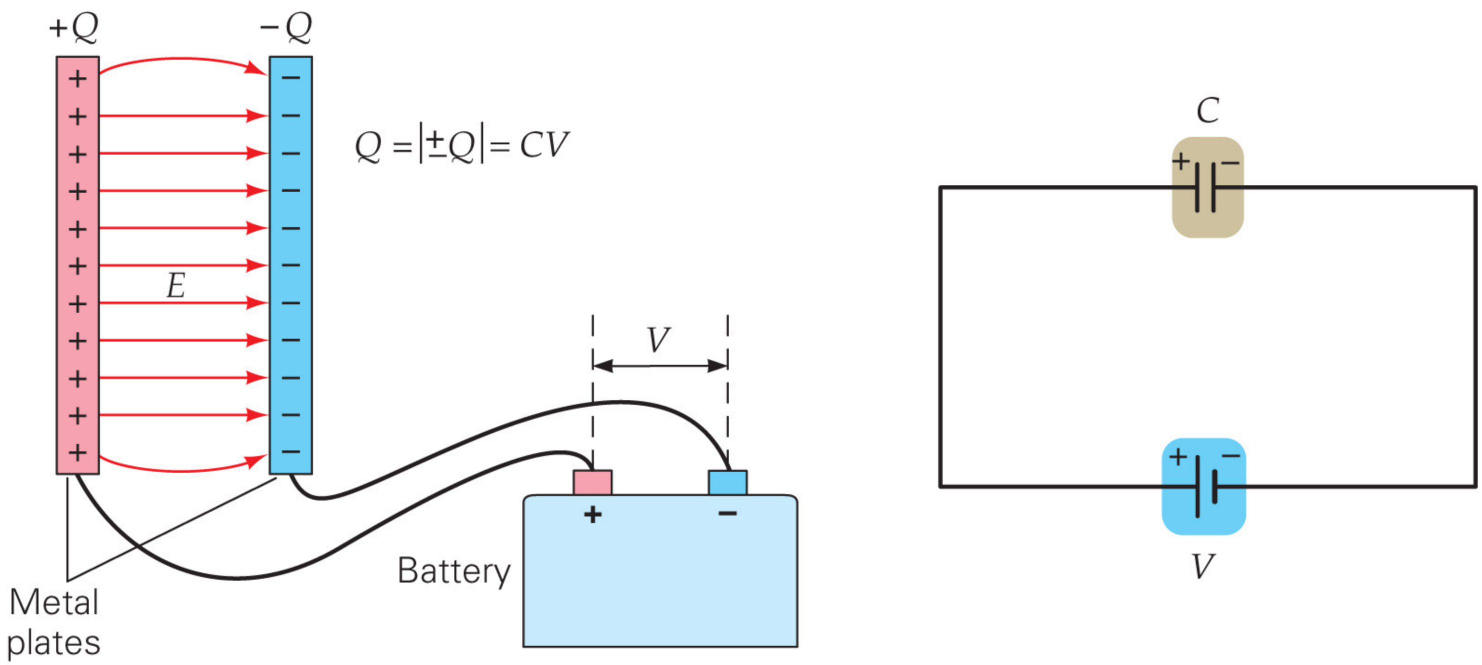
The electron-volt (eV) is the amount of energy needed to move an electron through a potential difference of one volt.

$$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$$

The electron-volt is a unit of energy, not voltage, and is not an SI standard unit. It is, however, quite useful when dealing with energies on the atomic scale.

Capacitance

A pair of parallel plates will store electric energy if charged oppositely; this arrangement is called a capacitor.



(a) Parallel-plate capacitor **(b) Schematic diagram**

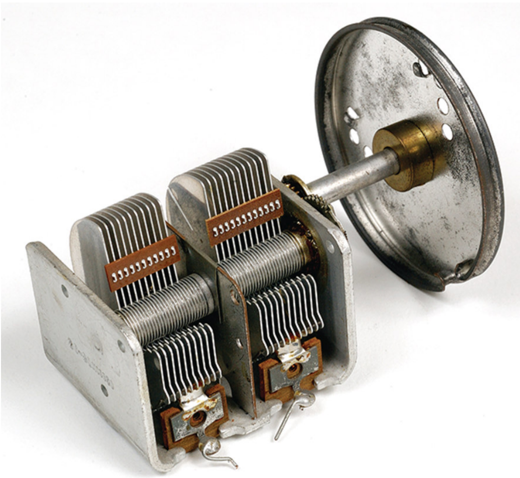
Capacitance

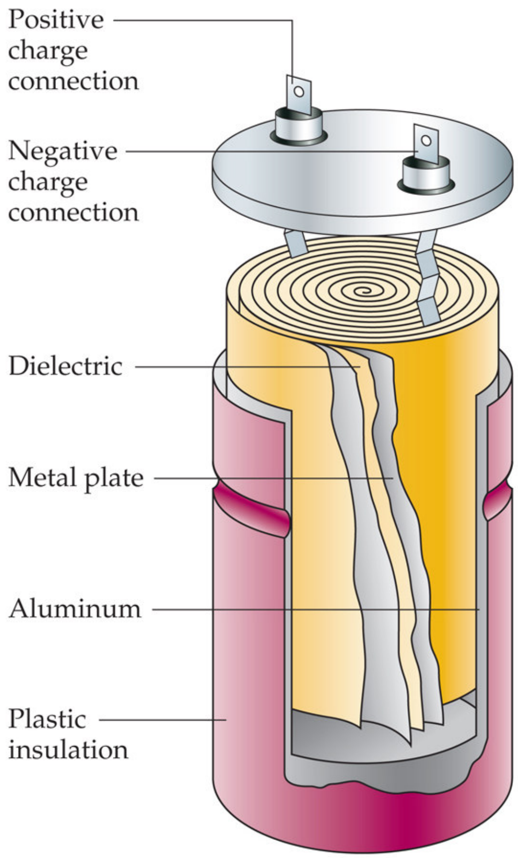
The charge on one plate of the capacitor is related to the potential difference; the ratio is called the capacitance.

$$Q = CV \qquad \text{or} \qquad C = \frac{Q}{V}$$

SI unit of capacitance: the farad, F which is a coulomb/volt.

Real Capacitors





Capacitance

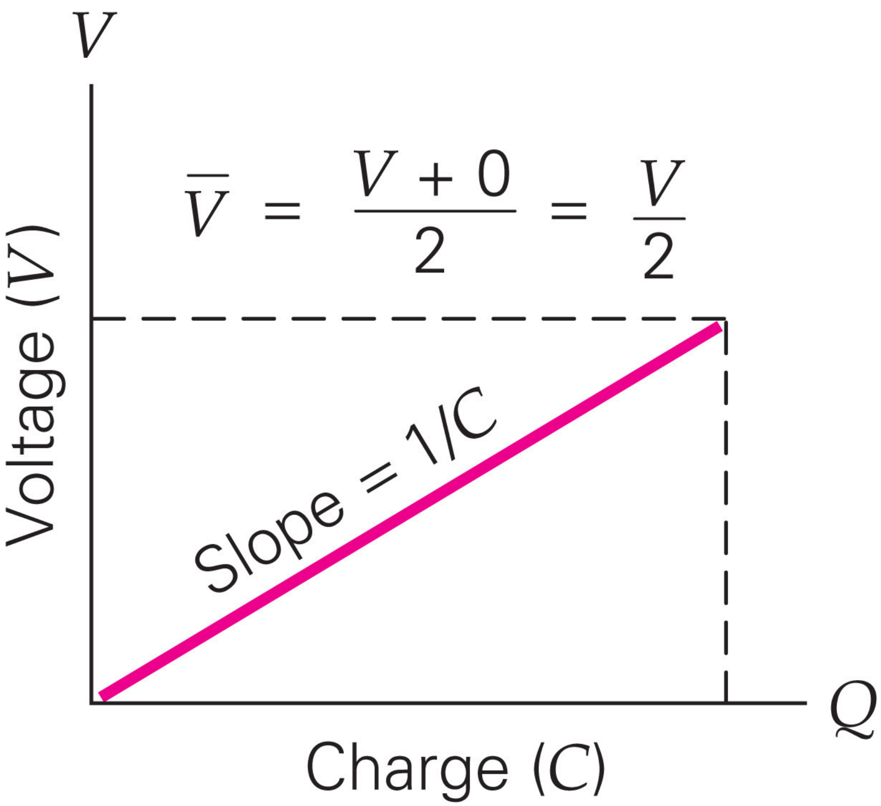
For a parallel-plate capacitor,

$$C = \frac{Q}{V} = \left(\frac{1}{4\pi k}\right)\frac{A}{d} = \frac{\varepsilon_0 A}{d}$$

The quantity inside the parentheses is called the permittivity of free space, and is represented by ε_0.

$$\varepsilon_o = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$$

Capacitance



The energy stored in a capacitor is the energy required to charge it:

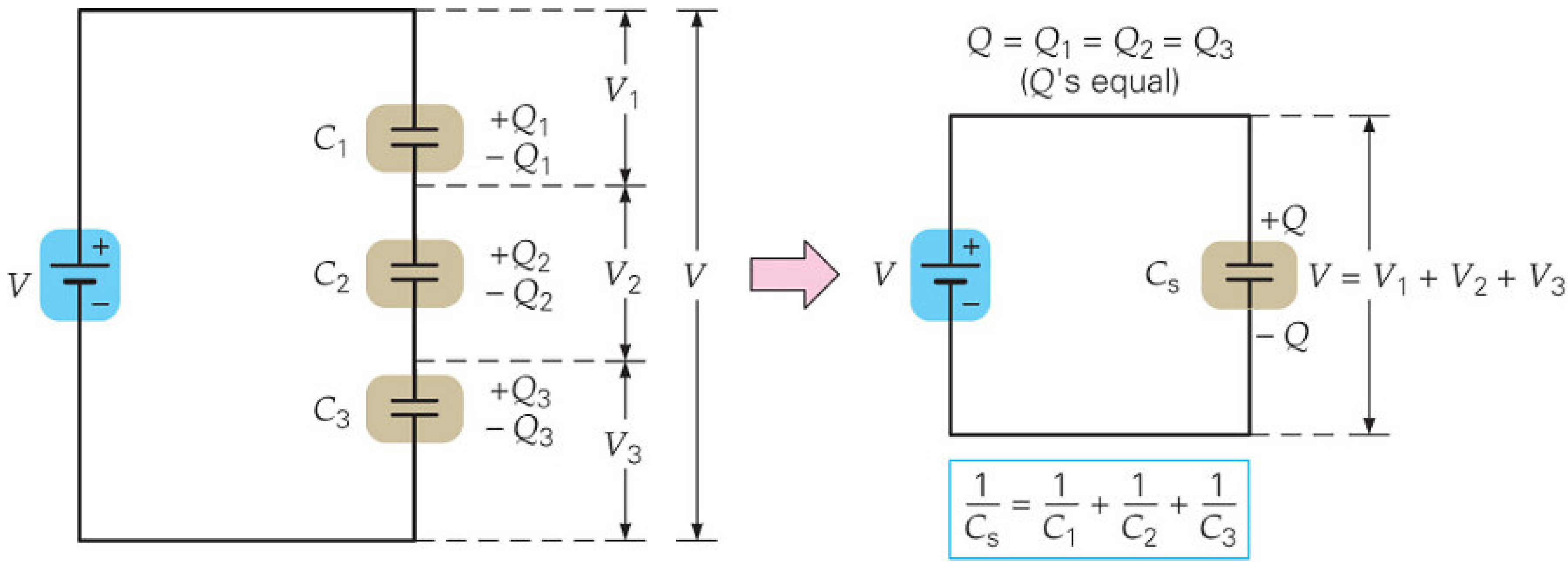
$$U_C = \tfrac{1}{2}QV = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2$$

Capacitors in Series

Capacitors in series all have the same charge; the total potential difference is the sum of the potentials across each capacitor.

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

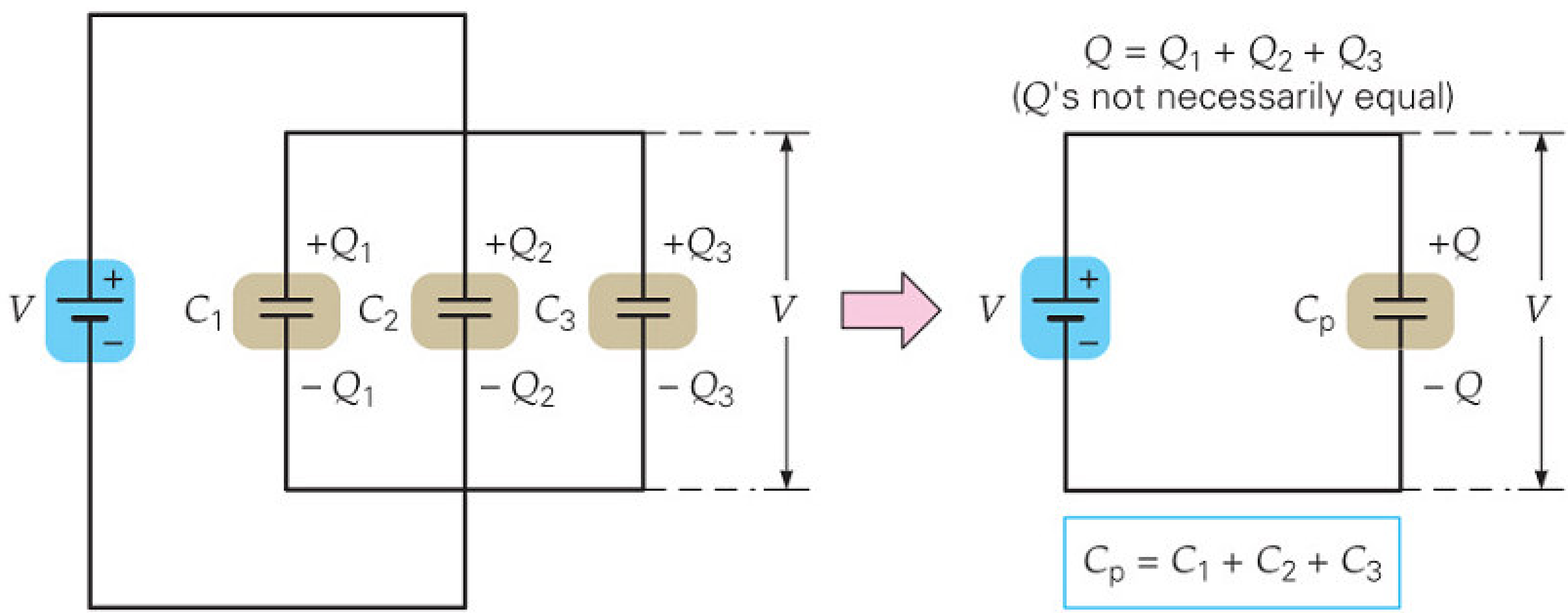
Note that this gives the *inverse* of the capacitance.



Capacitors in Parallel

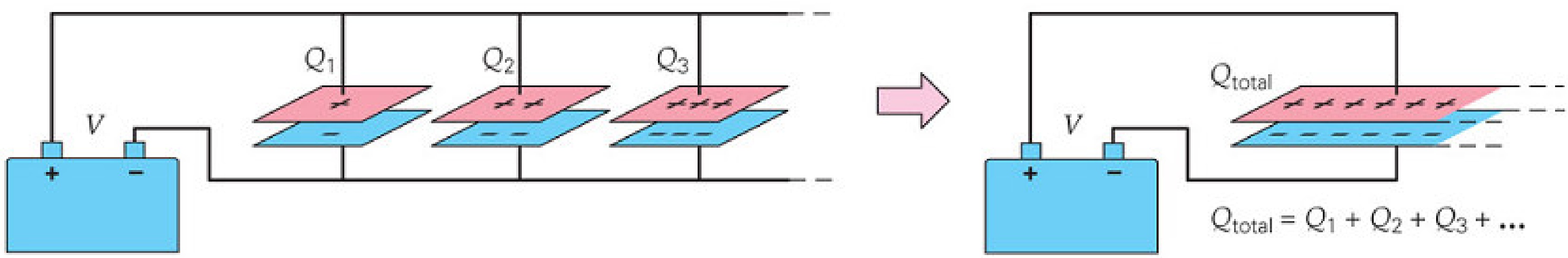
Capacitors in parallel all have the same potential difference; the total charge is the sum of the charge on each.

$$C_{\mathrm{p}} = C_1 + C_2 + C_3 + \cdots$$



Capacitors in Parallel

We can picture capacitors in parallel as forming one capacitor with a larger area:



Added Material

Potential of an isolated sphere

Electric Energy Density

Field in a Capacitor: $\frac{\sigma}{\varepsilon_0}$ or $\frac{2\sigma}{\varepsilon_0}$?

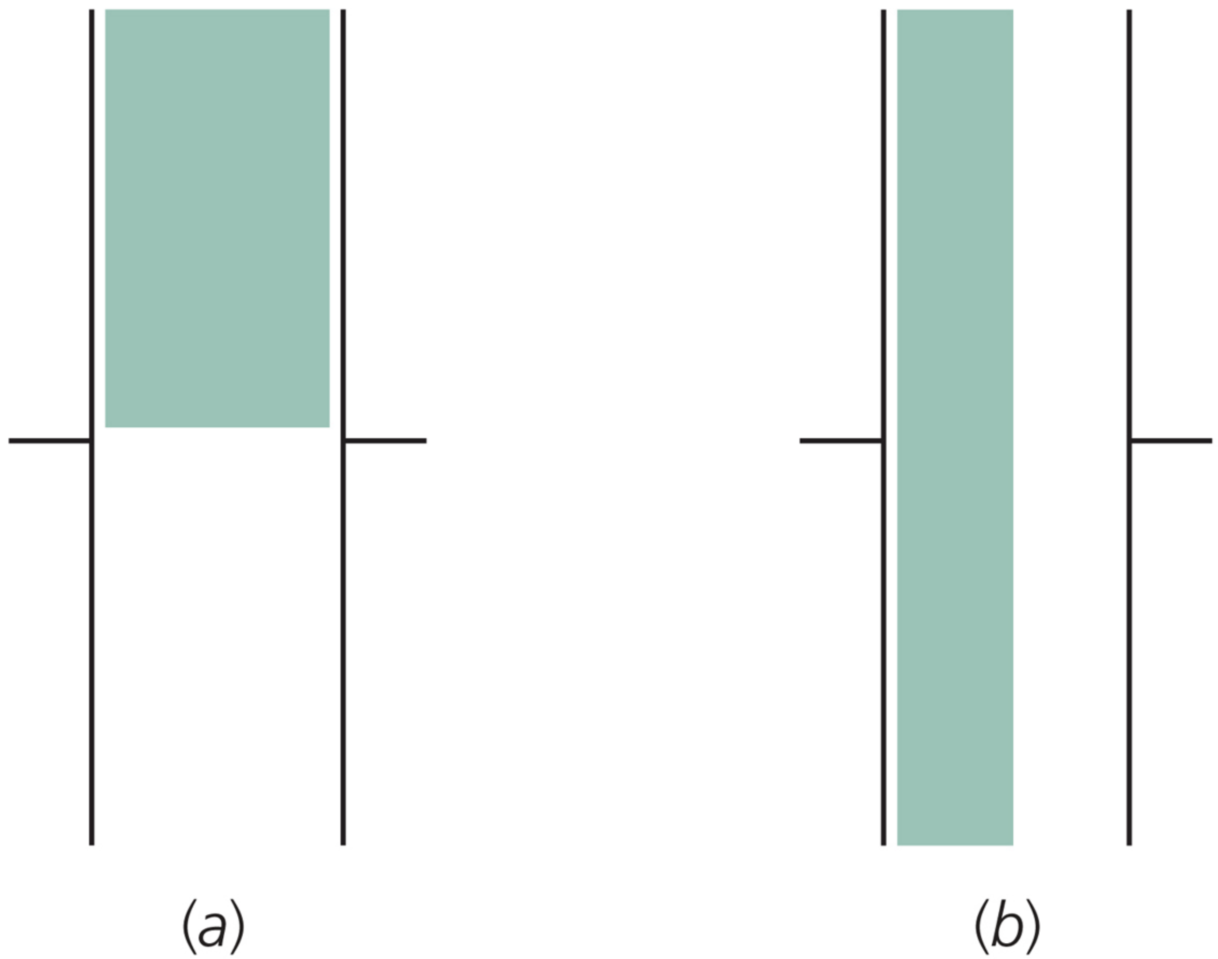
Dielectrics

The capacitance of a capacitor containing a dielectric is increased:

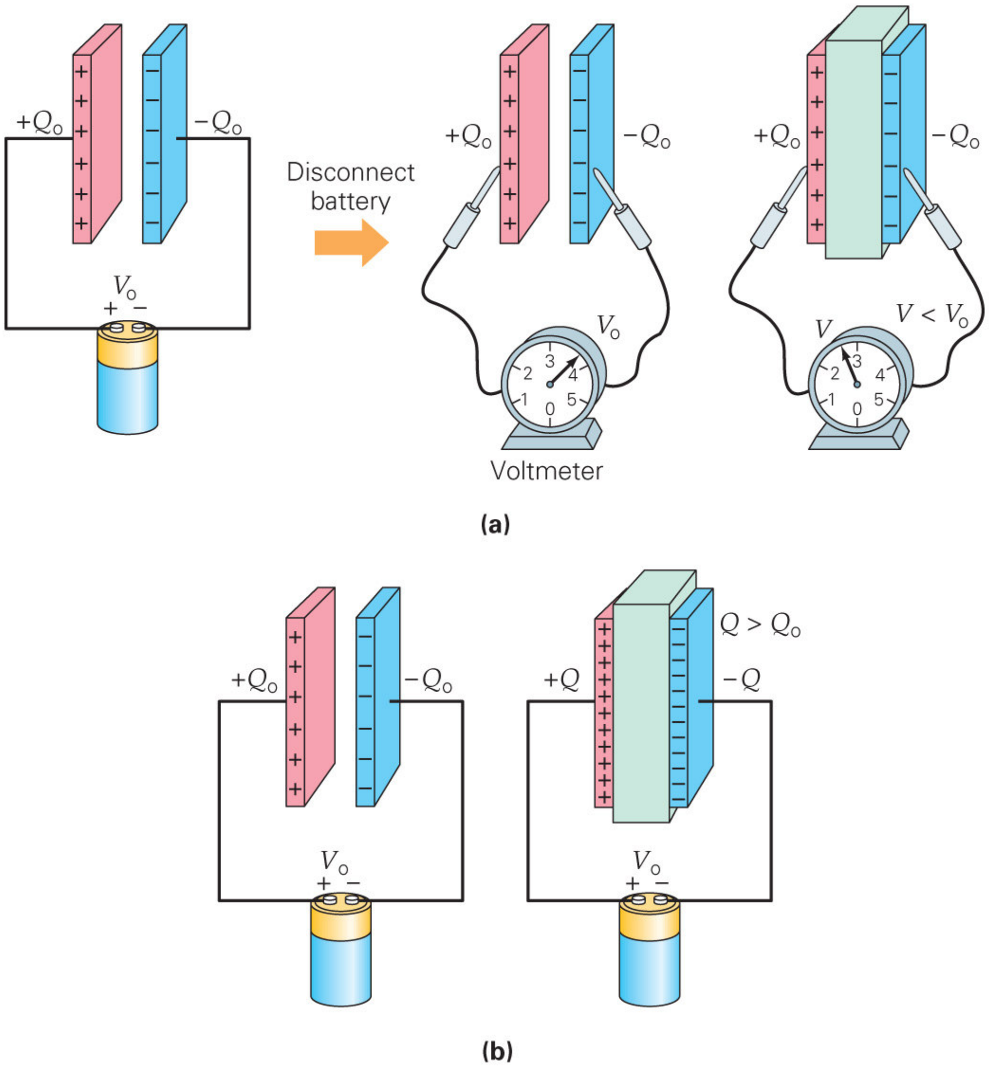
$$C = \frac{Q}{V} = \frac{Q_o}{(V_o/\kappa)} = \kappa\left(\frac{Q_o}{V_o}\right)$$

$$C = \kappa C_o$$

Partially Filled Capacitors



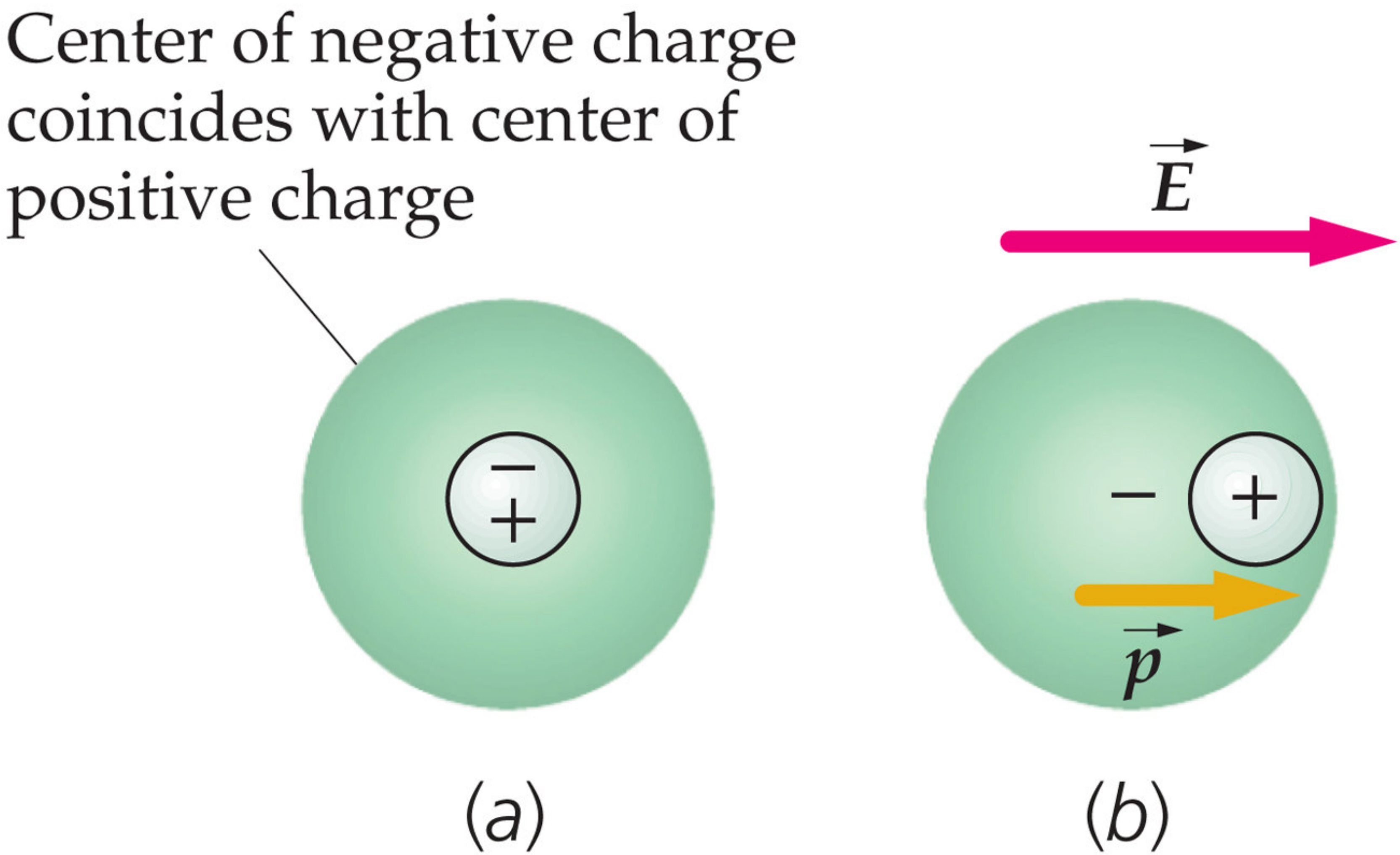
Dielectrics



Inserting a dielectric into a capacitor while either the voltage or the charge is held constant has the same effect – the ratio of charge to voltage increases.

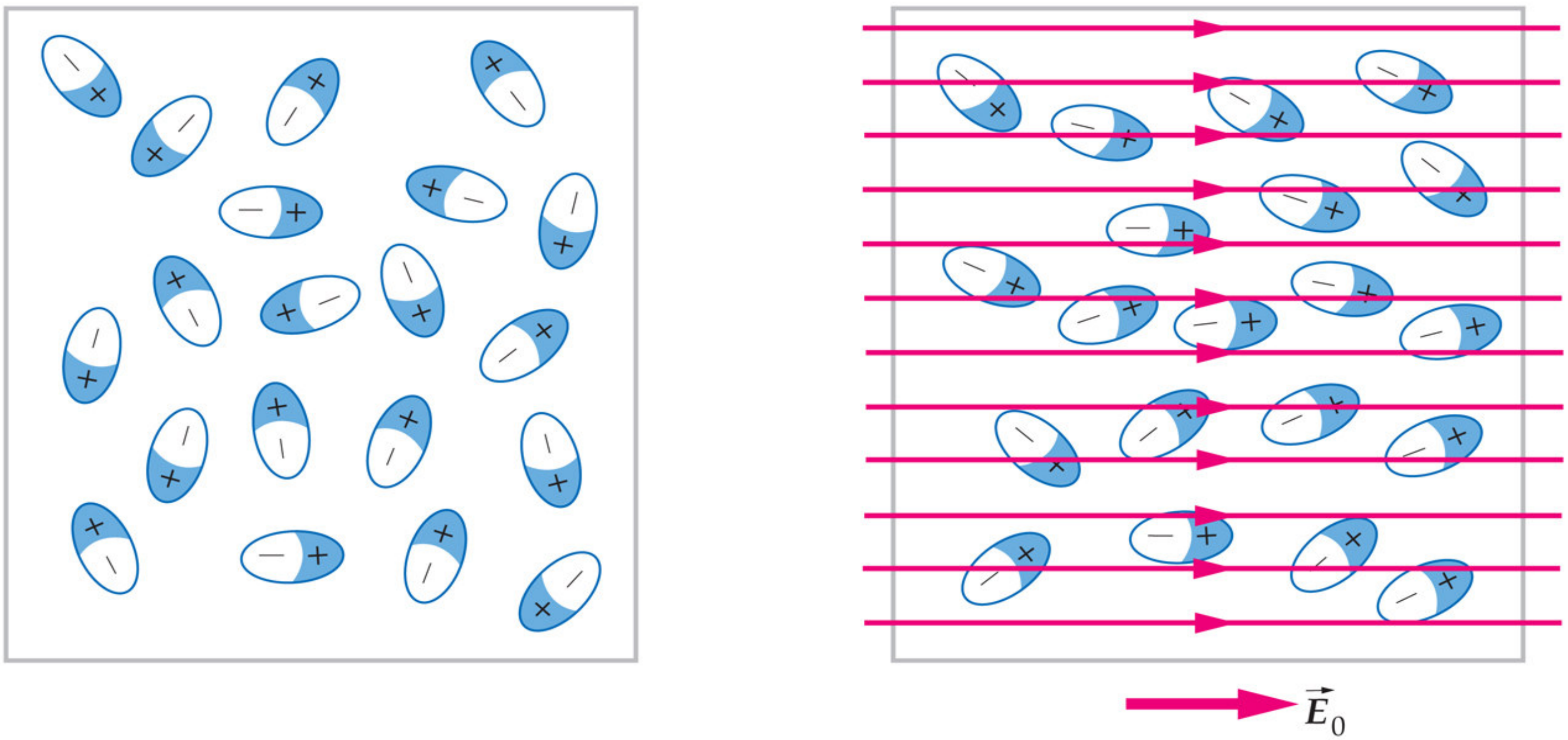
The capacitance increases.

Dielectric Polarization



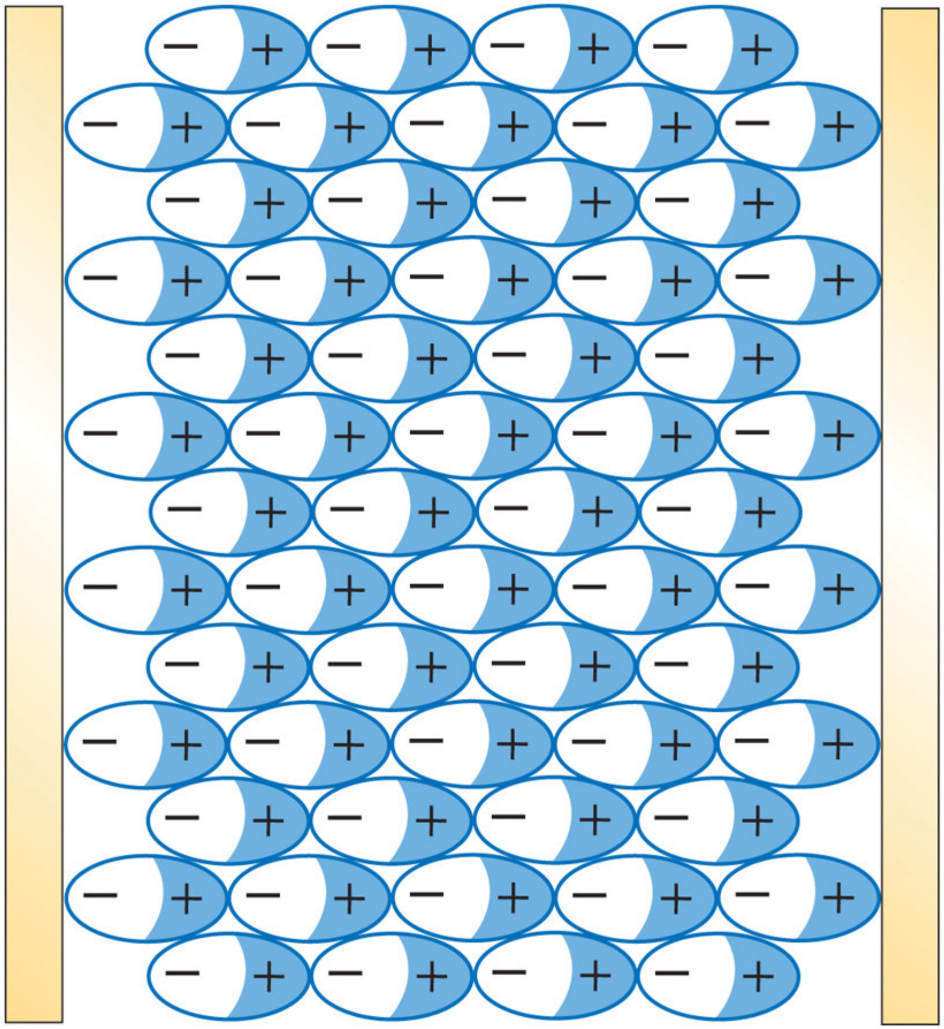
Dielectric Polarization

Permanent Dipole Moments

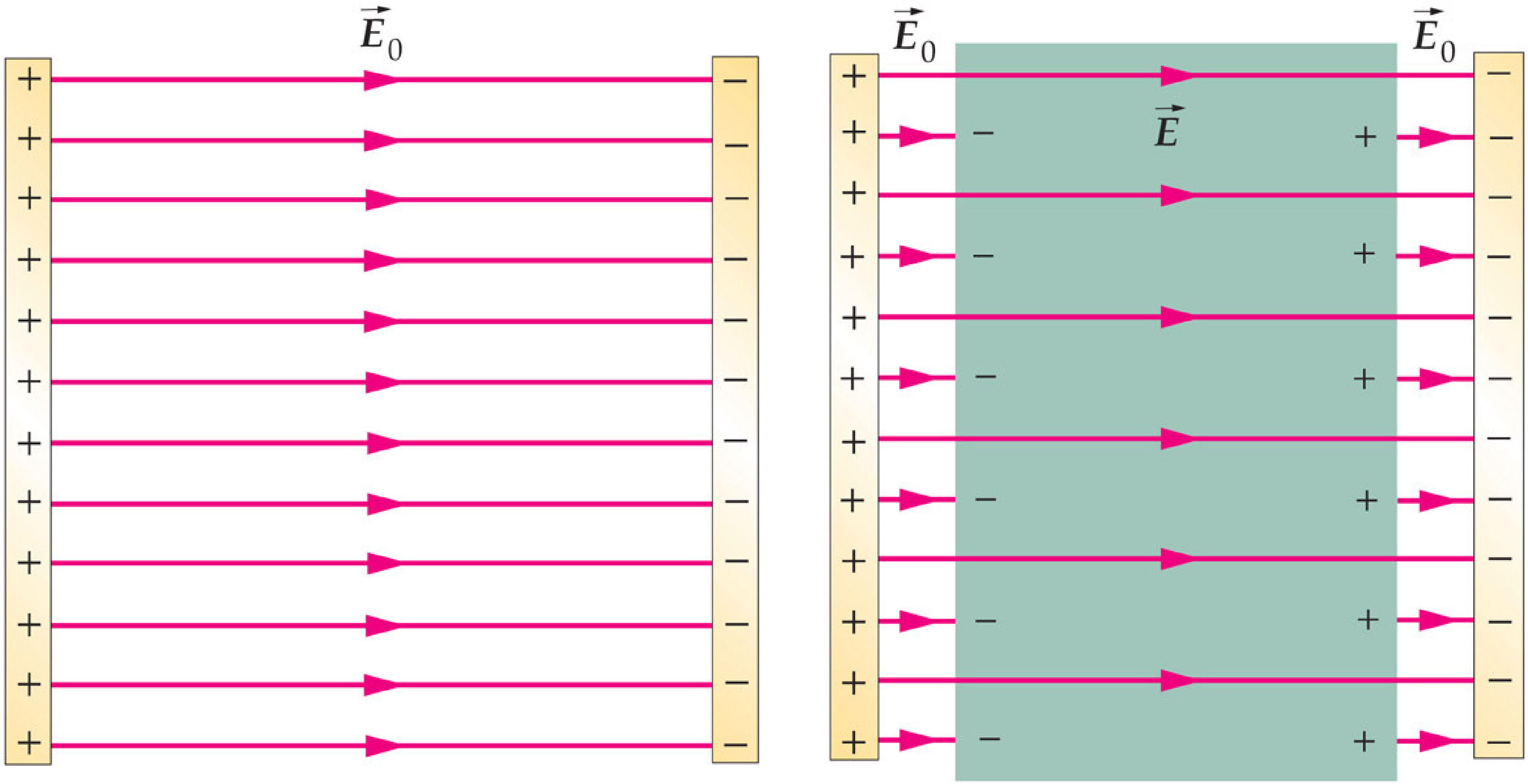


(a) (b)

Dielectric Polarization

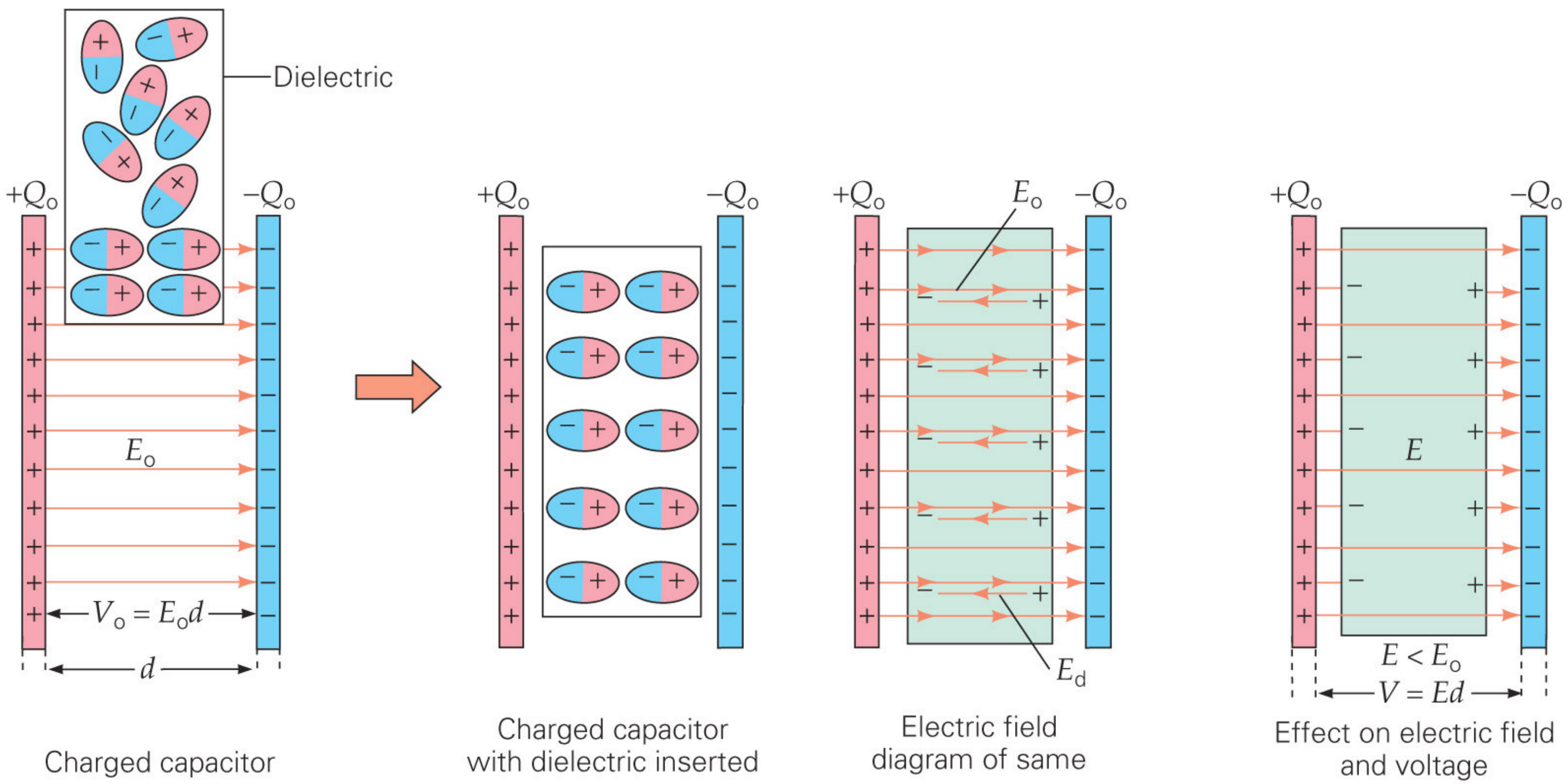


Dielectric Polarization



Dielectric Polarization Summary

A dielectric in an electric field becomes polarized; this allows it to sustain a larger electric field for the same potential difference.



Dielectrics

"Dielectric" is another word for insulator. A dielectric inside a capacitor increases the capacitor's energy storage by an amount characterized by the dielectric constant, κ.

TABLE 16.2 Dielectric Constants for Some Materials

Material	*Dielectric Constant (κ)*	*Material*	*Dielectric Constant (κ)*
Vacuum	1.0000	Glass (range)	3–7
Air	1.00059	Pyrex glass	5.6
Paper	3.7	Bakelite	4.9
Polyethylene	2.3	Silicon oil	2.6
Polystyrene	2.6	Water	80
Teflon	2.1	Strontium titanate	233

Extra Slides