Chapter 26 & 27

Electric Current and Direct-Current Circuits

Electric Current and Direct-Current Circuits

Current and Motion of Charges

Resistance and Ohm’s Law

Energy in Electric Circuits

Combination of Resistors

Kirchhoff’s Rules

RC Circuits

Electric Current

- “Under steady state conditions electrons may be fed into the metal at one point and removed at another, producing a current, but the metal as a whole is electrostatically neutral. Strong electrostatic forces keep excess electrons from accumulating at any part of the metal.”
- “Similarly a deficiency of electrons is remedied by electrostatic forces of the opposite sign.”
- “Excess charge is dissipated extremely rapidly in the conductor.”

Electric Current

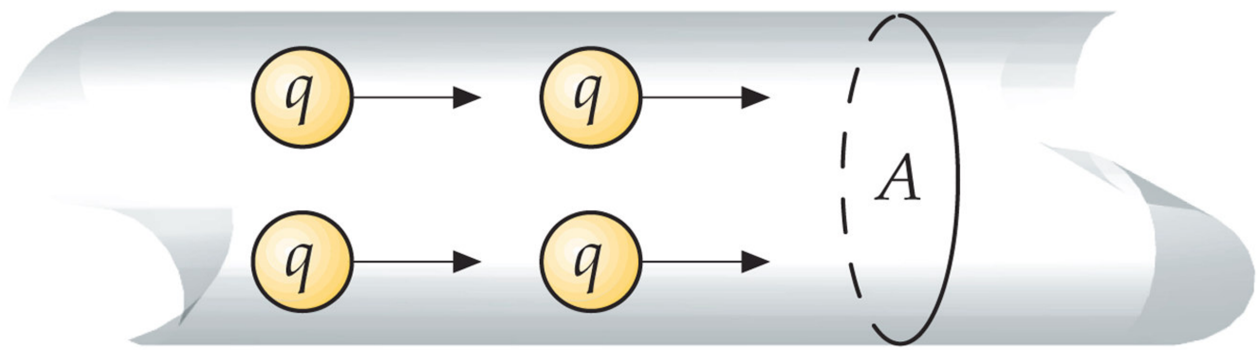
- "The basic laws describing the equilibrium behavior of conductors are altered when static equilibrium is disturbed, i.e. when there is motion or flow of electric charge in the conductor.
- When the charges in a conductor are in motion, the electric field intensity just at the conductor surface need not be perpendicular to the surface.
- The conducting body is not an equipotential, as it is for static equilibrium."

In A Conductor

- A sea of electrons (i.e. A Fermi gas) shields the lattice ions from one another. The density of these electrons for Copper (Cu) is $8.5 \times 10^{22}/cm^3$
- These electrons are contributed to the conduction band - one electron per atom.

The Definition of Current

The definition of current $$I = \frac{\Delta Q}{\Delta t}$$



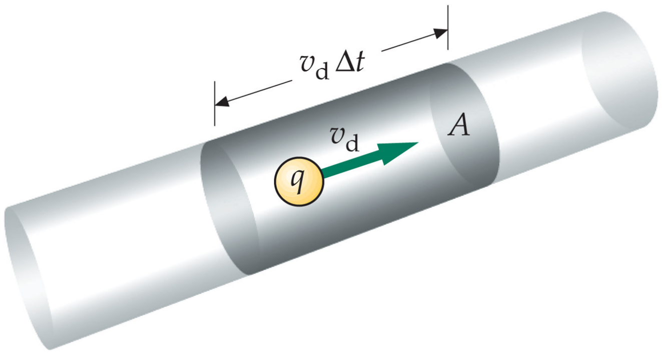
where ΔQ is the amount of charge that flows through the cross sectional area in a time Δt.

The units of current are Amperes (A) or coulombs/sec.

The Definition of Current

- In solids, only the electrons flow.
- For historical reasons, since we didn’t know that the electron existed, the conventional current flows opposite to that of the electron current.
- Conventional current is considered a flow of positive electricity that comes out of the positive terminal of the battery and flows from high potential to low potential.

Current and Electron Drift



$$\Delta Q = qnAv_d \Delta t$$

$$I = \frac{\Delta Q}{\Delta t} = qnAv_d$$

Individual electrons have high speeds (10^6 m/s) but their many collisions with the lattice atoms cause them to execute a “random walk” through the conductor which is described by a drift velocity v_d .

When you turn on a light that first electron out of the wall doesn’t make it to the light bulb for hours.

Wire Dimensions

Table 25-2 Wire Diameters and Cross-Sectional Areas for Commonly Used Copper Wires

AWG* Gauge Number	Diameter† at 20°C, mm	Area, mm^2
4	5.189	21.15
6	4.115	13.30
8	3.264	8.366
10	2.588	5.261
12	2.053	3.309
14	1.628	2.081
16	1.291	1.309
18	1.024	0.8235
20	0.8118	0.5176
22	0.6438	0.3255

<u>10 Gauge Cu Wire</u>

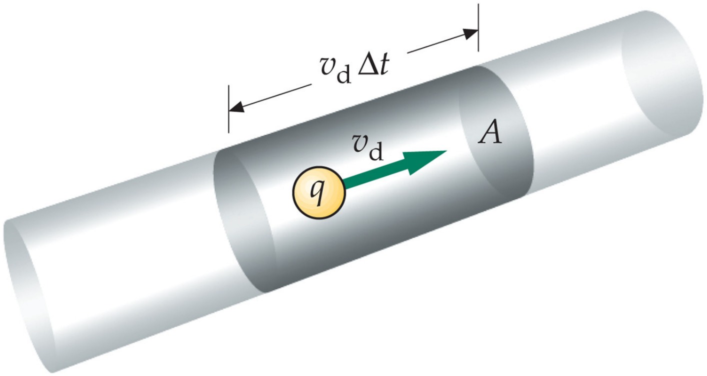
2.588 mm Diameter

5.261 mm^2 Area

$I_{max} = 30A$

The diameter is related to the gauge number n by $d = 0.127x92^{\left[\frac{(36-n)}{39}\right]}$

Current and Motion of Charges



$$I = nqv_d A = n\left[\frac{carriers}{m^3}\right] q \left[\frac{coulombs}{carrier}\right] v_d \left[\frac{m}{s}\right] A\left(m^2\right)$$

Assume 1 carrier per atom: $n = n_A$

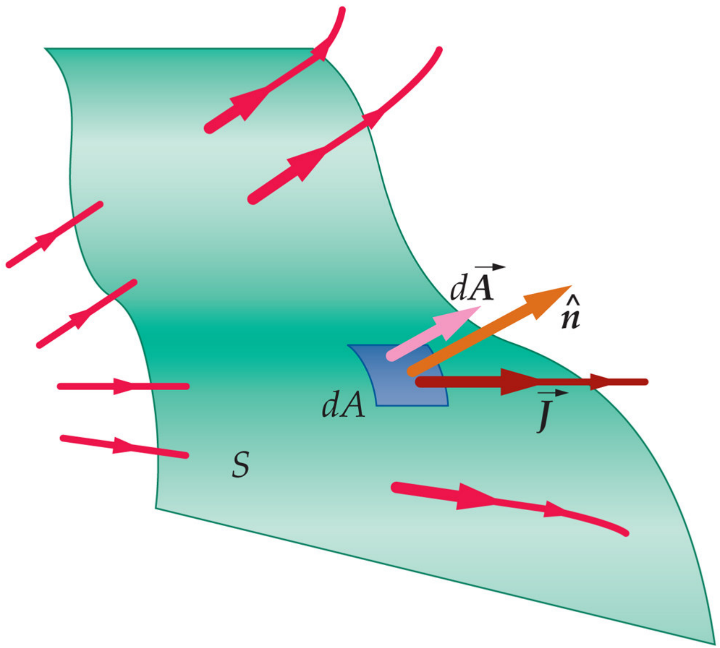
$$n_A = \frac{atoms}{m^3} = \frac{\rho_m \left[\frac{gm}{m^3}\right] N_A \left[\frac{atoms}{mole}\right]}{M\left[\frac{gm}{mole}\right]}$$

$$n_A = \frac{8.93\left[\frac{gm}{cm^3}\right]\left[\frac{10^2\,cm}{m}\right]^3 6.02x10^{23}}{63.5\left[\frac{gm}{mole}\right]} = 8.47x10^{28}\,\frac{atoms}{m^3}$$

$$v_d = \frac{I}{n_A qA} = \frac{30}{8.47x10^{28}\left(1.602x10^{-19}\right)\left(5.261x10^{-6}\right)} = 0.420x10^{-3}\,\frac{m}{s}$$

Time to travel 100m = $\Delta t = \frac{L}{v_d} = \frac{100}{0.420x10^{-3}} = 2.38x10^5\,sec = 66.1\,hrs$

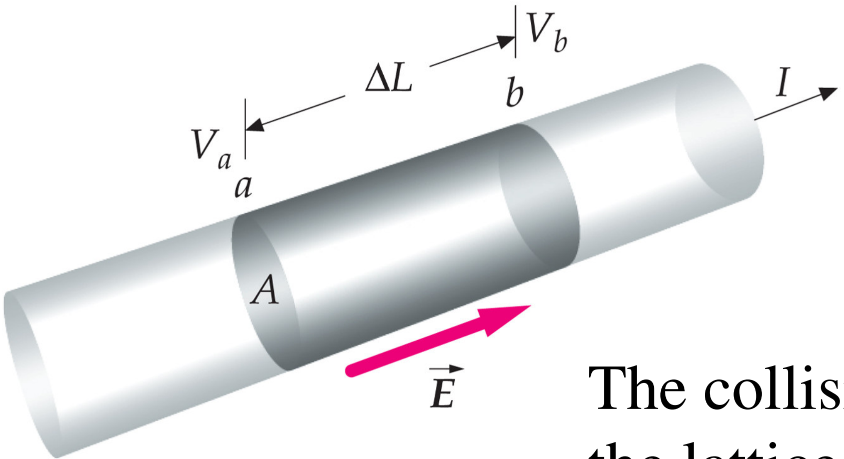
Current Density Mumbo Jumbo



$$I = \int_S \vec{J} \bullet d\vec{A} = \int_S \vec{J} \bullet \hat{n}\, dA$$

Skip this for now - we will pick it up when we get to magnetism

Resistance and Ohm's Law



Our assumed positive charges will flow in the direction of the electric field in the conductor.

The collisions of the moving charges with the lattice ions gives rise to a resistance to the flow of current.

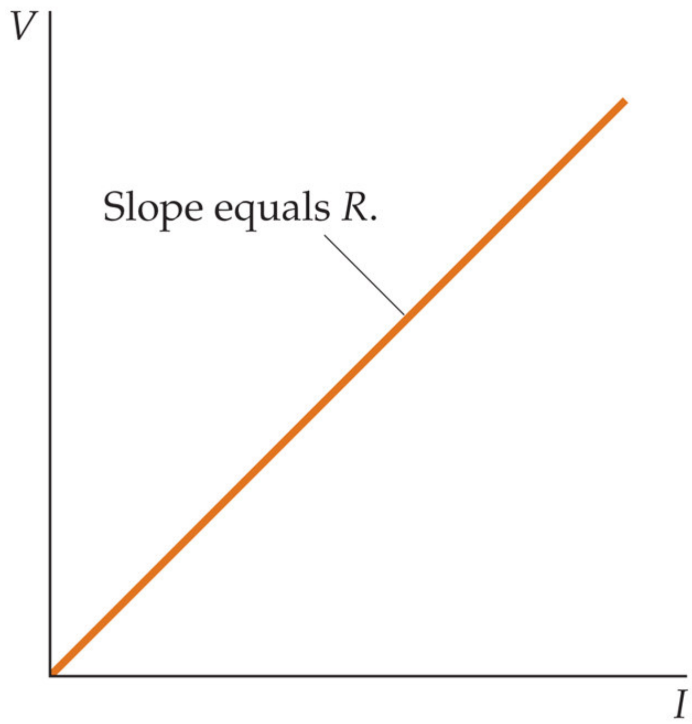
$$R = \frac{V}{I}$$

The units of resistance are volts/amp or ohms with the symbol Ω.

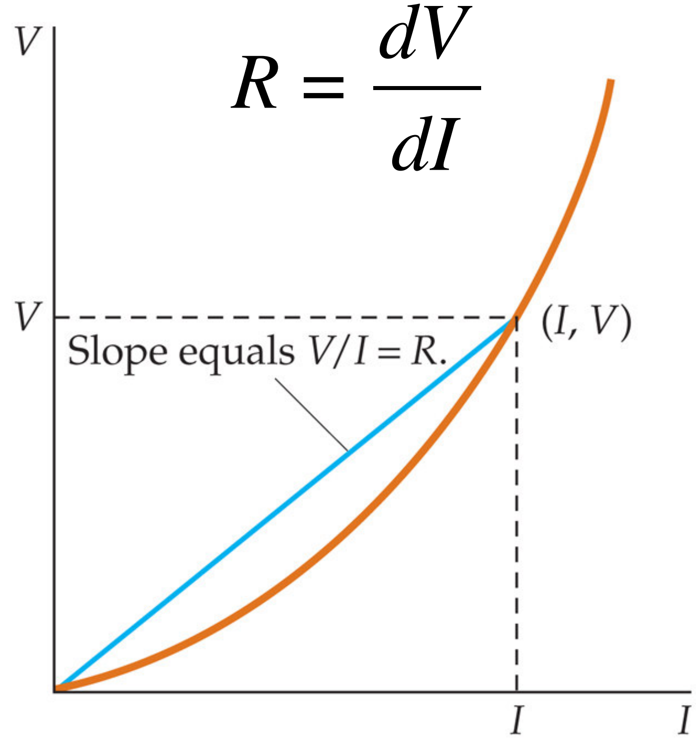
This is no longer electrostatics so electric fields are allowed inside the conductors.

Ohm's Law

$$V = IR$$



$$R = \frac{dV}{dI}$$



Ohm's Law is strictly defined only for materials of constant R but the V-I characteristic is useful even if R is not constant.

Resistance and Resistivity

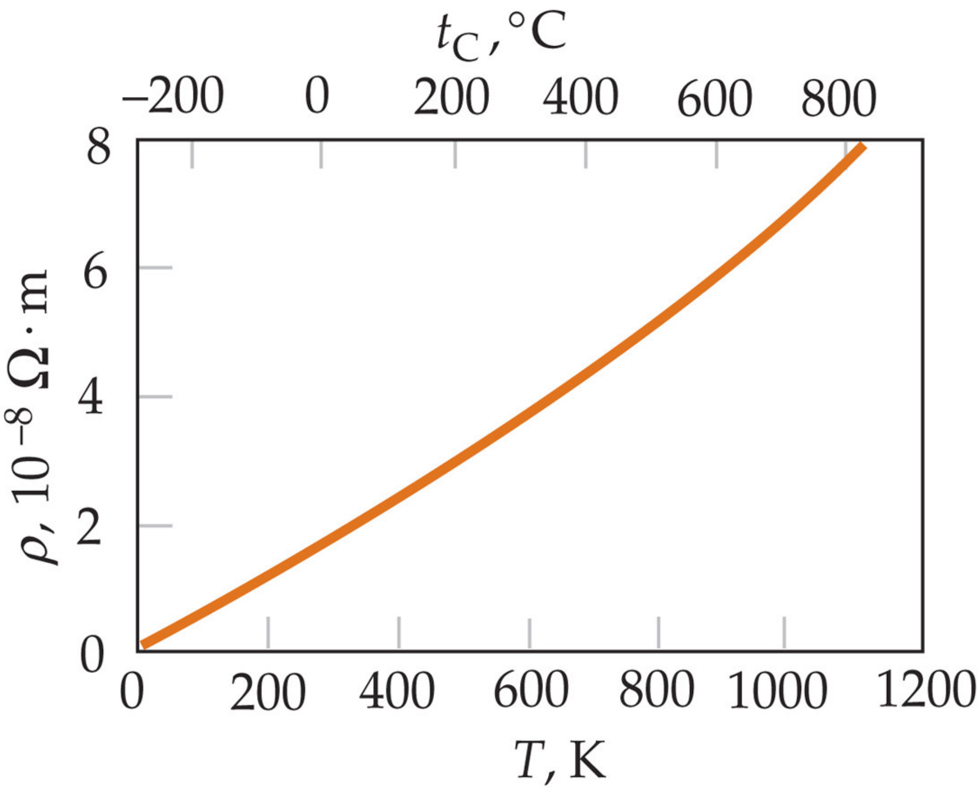
The resistance of a material depends upon the material from which it is made and its physical dimensions.

It is customary to separate these two aspects into a material part described by the resistivity ρ and a geometric part.

$$R = \rho \frac{L}{A}$$

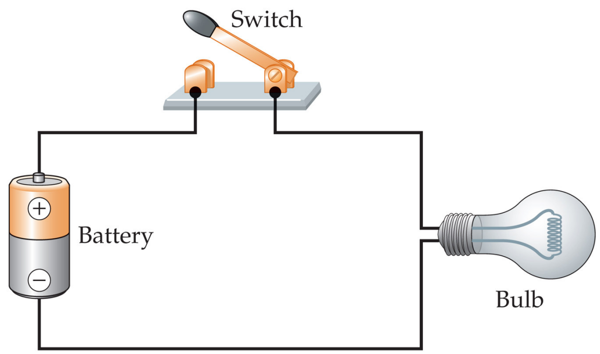
ρ is the resistivity in Ω-m, L is the length of the conductor in meters and A is the cross sectional area in m^2

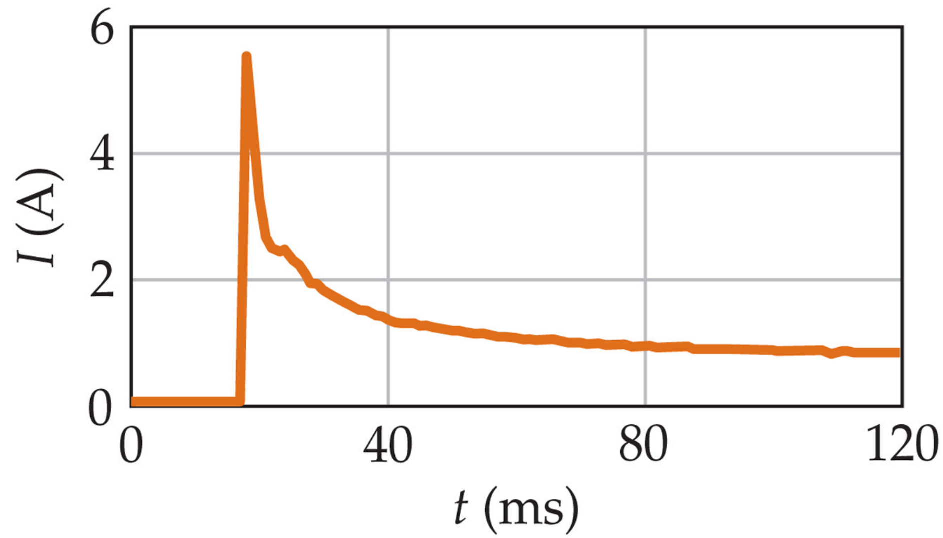
Resistivity Temperature Dependence



The change of the resistivity with temperature is not too important for what we will be doing in this course but it is an important consideration for designing sensitive circuit applications.

Resistivity Temperature Dependence





When the switch is first closed the filament in the light is at room temperture and a large current passes through the circuit.

After the current has been flowing for a very short time the filament heats up to a high temperature (it's glowing) and the resistance increases. This causes the circuit current to decrease.

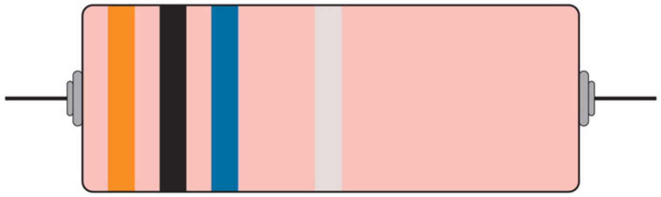
Resistivities & Temperature Coefficients

Material	Resistivity ρ at 20°C, $\Omega \cdot m$	Temperature Coefficient α at 20°C, K^{-1}
Conducting Elements		
Aluminum	2.8×10^{-8}	3.9×10^{-3}
Copper	1.7×10^{-8}	3.93×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Lead	22×10^{-8}	4.3×10^{-3}
Mercury	96×10^{-8}	0.89×10^{-3}
Platinum	100×10^{-8}	3.927×10^{-3}
Silver	1.6×10^{-8}	3.8×10^{-3}
Tungsten	5.5×10^{-8}	4.5×10^{-3}
Carbon	3500×10^{-8}	-0.5×10^{-3}
Conducting alloys		
Brass	$\sim 8 \times 10^{-8}$	2×10^{-3}
Constantin (60% Cu, 40% Ni)	$\sim 44 \times 10^{-8}$	0.002×10^{-3}
Manganin (~84% Cu, ~12% Mn, ~4% Ni)	44×10^{-8}	0.000×10^{-3}
Nichrome	100×10^{-8}	0.4×10^{-3}

Resistivities & Temperature Coefficients

Material	Resistivity ρ at 20°C, $\Omega \cdot m$	Temperature Coefficient α at 20°C, K^{-1}
Semiconductors		
Germanium	0.45	-4.8×10^{-2}
Silicon	640	-7.5×10^{-2}
Insulators		
Neoprene	$\sim 10^{9}$	
Polystyrene	$\sim 10^{8}$	
Porcelain	$\sim 10^{11}$	
Wood	$10^{8} - 10^{14}$	
Glass	$10^{10} - 10^{14}$	
Hard rubber	$10^{13} - 10^{16}$	
Amber	5×10^{14}	
Sulfur	1×10^{15}	
Teflon	1×10^{14}	
Body material		
Blood	1.5	
Fat	25	

Resistor Color Codes



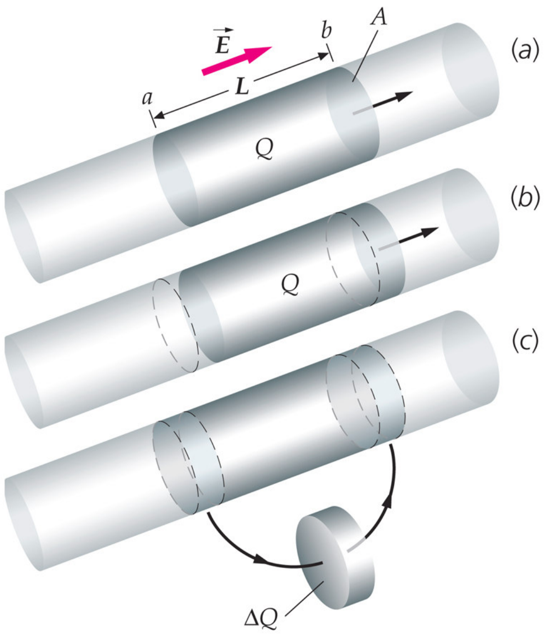
Colors	Numeral		Tolerance	
Black	= 0		Brown	= 1%
Brown	= 1		Red	= 2%
Red	= 2		Gold	= 5%
Orange	= 3		Silver	= 10%
Yellow	= 4		None	= 20%
Green	= 5			
Blue	= 6			
Violet	= 7			
Gray	= 8			
White	= 9			

306   10%

$30 \times 10^6\,\Omega$ at 10% precision

$(30 \pm 3) \times 10^6\,\Omega$

Energy in Electric Circuits



$$\Delta U = \delta Q\left(V_b - V_a\right)$$

$$\Delta U = -\delta Q\left(V_a - V_b\right)$$

$$-\Delta U = \delta Q\left(V_a - V_b\right)$$

$$-\Delta U = \delta Q \bullet \Delta V$$

$$-\frac{\Delta U}{\Delta t} = \frac{\delta Q}{\Delta t}\ \Delta V$$

The rate of loss of potential energy

$$-\frac{\Delta U}{\Delta t} = I\Delta V$$

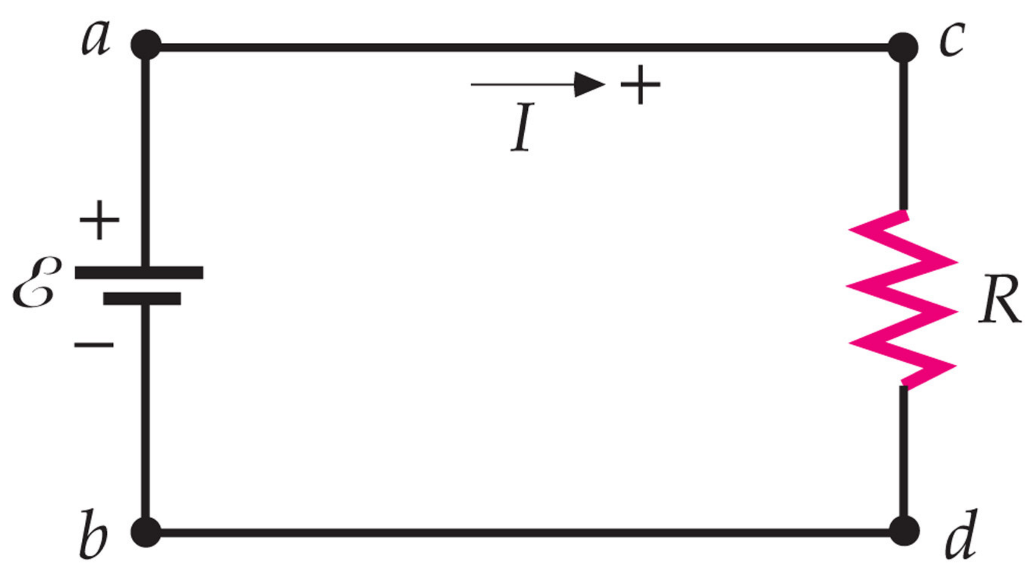
Power delivered to conducting segment

Energy in Electric Circuits

$$P = IV = I^2 R = \frac{V^2}{R}$$

In these equations V is understood to be a potential difference.

The Most Basic Electric Circuit

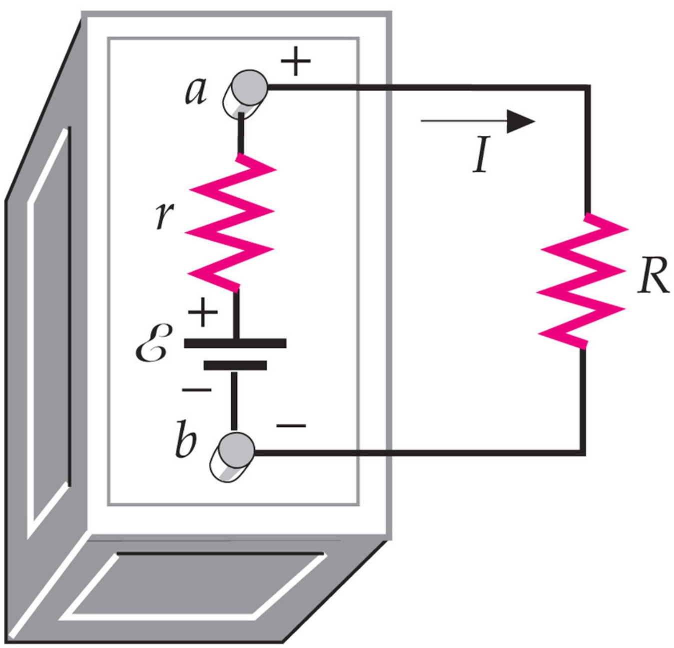


$\mathcal{E}$ is called the emf or Electromotive Force. This is an antiquated term but it is still useful in some discussions.

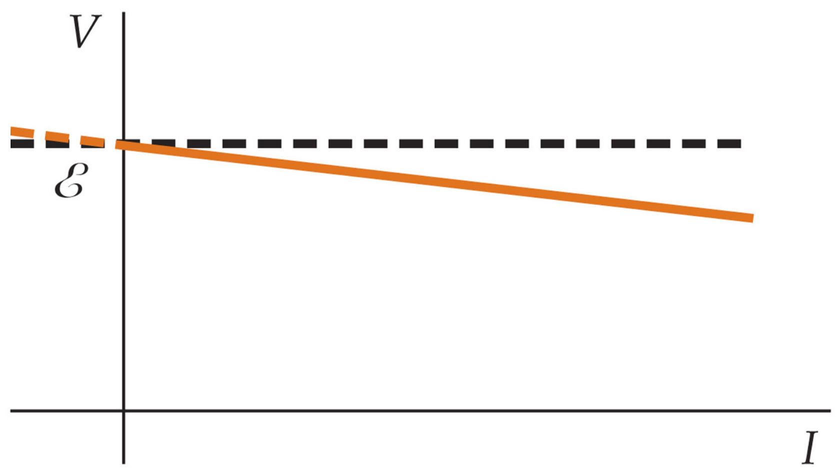
The emf represents the ideal voltage available from the battery.

Internal Resistance

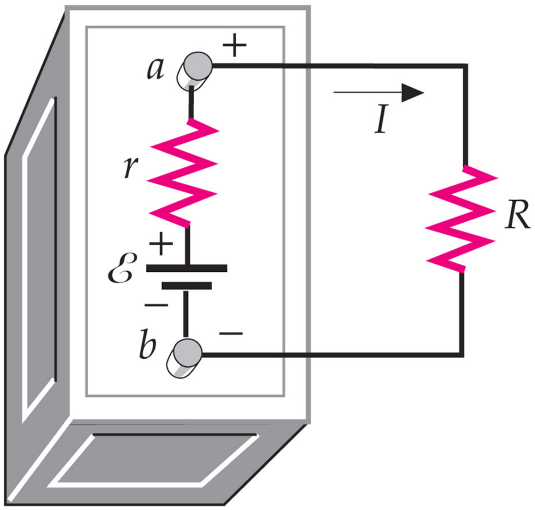
In an ideal battery the internal resistance, represented by r, would be equal to zero and the voltage delivered by the battery would be constant as shown by the dashed line.



The internal resistance causes the voltage delivered by the battery to be load dependent as shown by the solid line.



Internal Resistance & Power Transfer



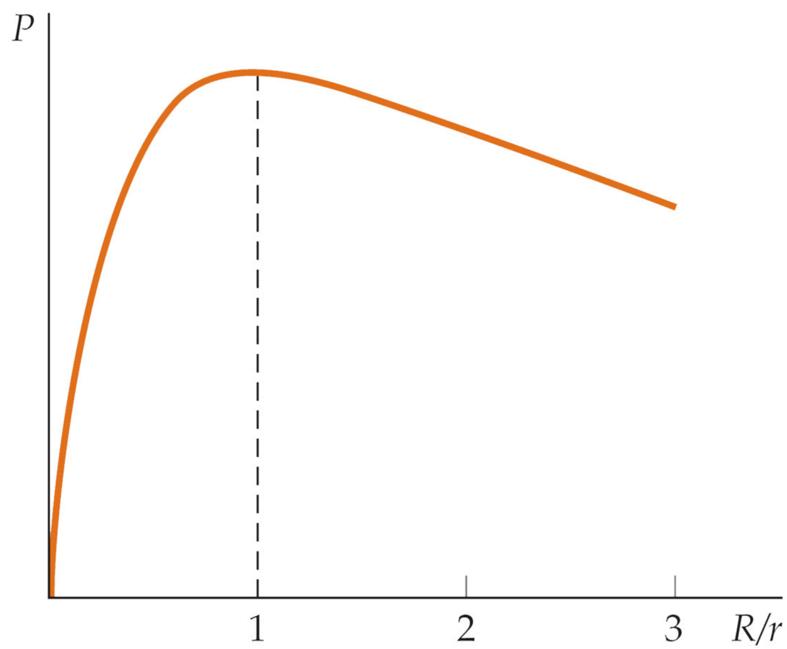
Question: What should be the value of R, the load resistor, to maximize the power delivered by the battery?

The power delivered to R is I^2R.

Therefore we want R to be large. But R is part of the circuit that determines I. As R increases the total resistance in the circuit goes up and the current coming out of the battery goes down.

Small R means large I but small I^2R. Large R means small I and also small I^2R. Somewhere in between is the max power solution.

Internal Resistance & Power Transfer



$$I = \frac{E}{(R + r)}$$

$$P = I^2 R = \frac{E^2 R}{(R + r)^2}$$

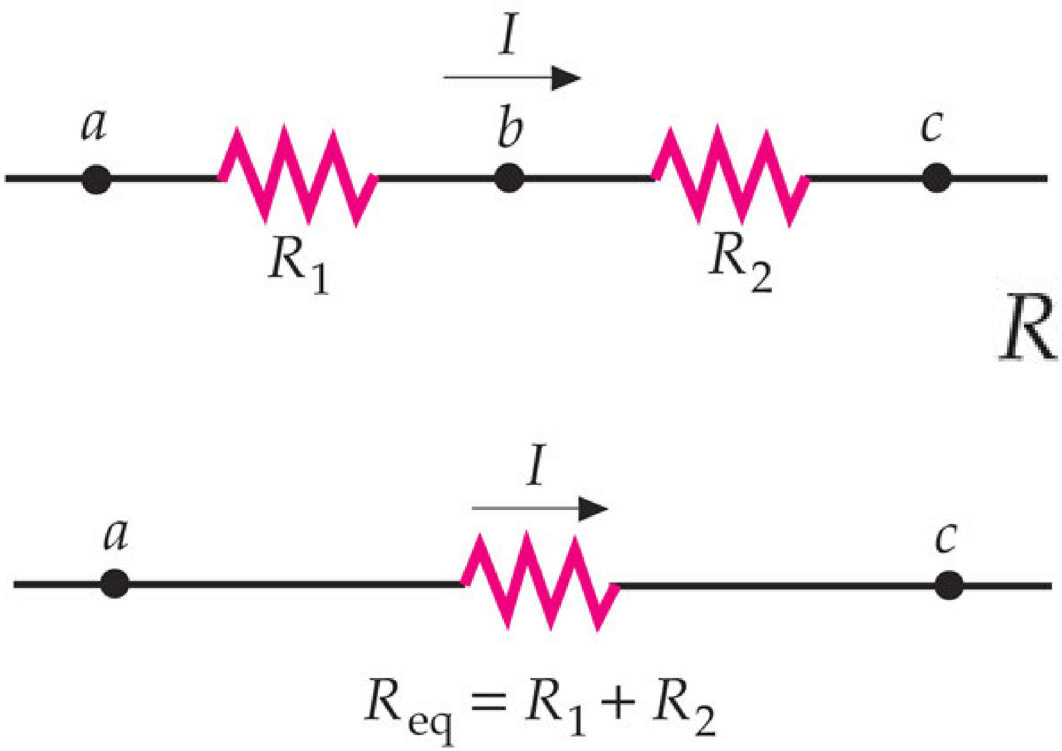
$$\frac{dP}{dR} = \frac{E^2 (r - R)}{(R + r)^2} = 0$$

$$R = r$$

This is a concept known as impedance matching.

Resistors in Series

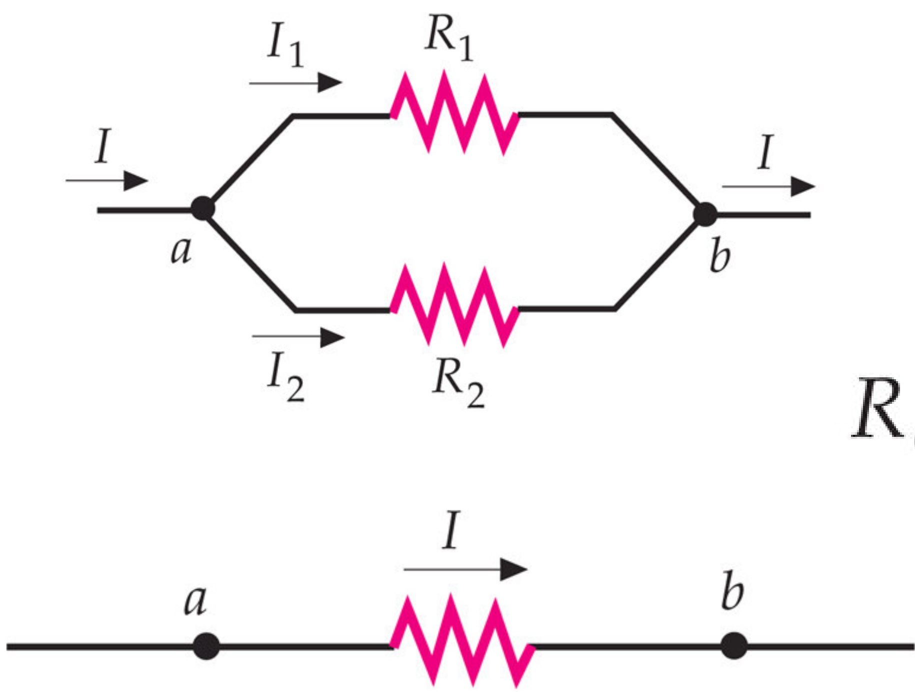
Same current - split the voltage



$$R_{eq} = R_1 + R_2$$

Resistors in Parallel

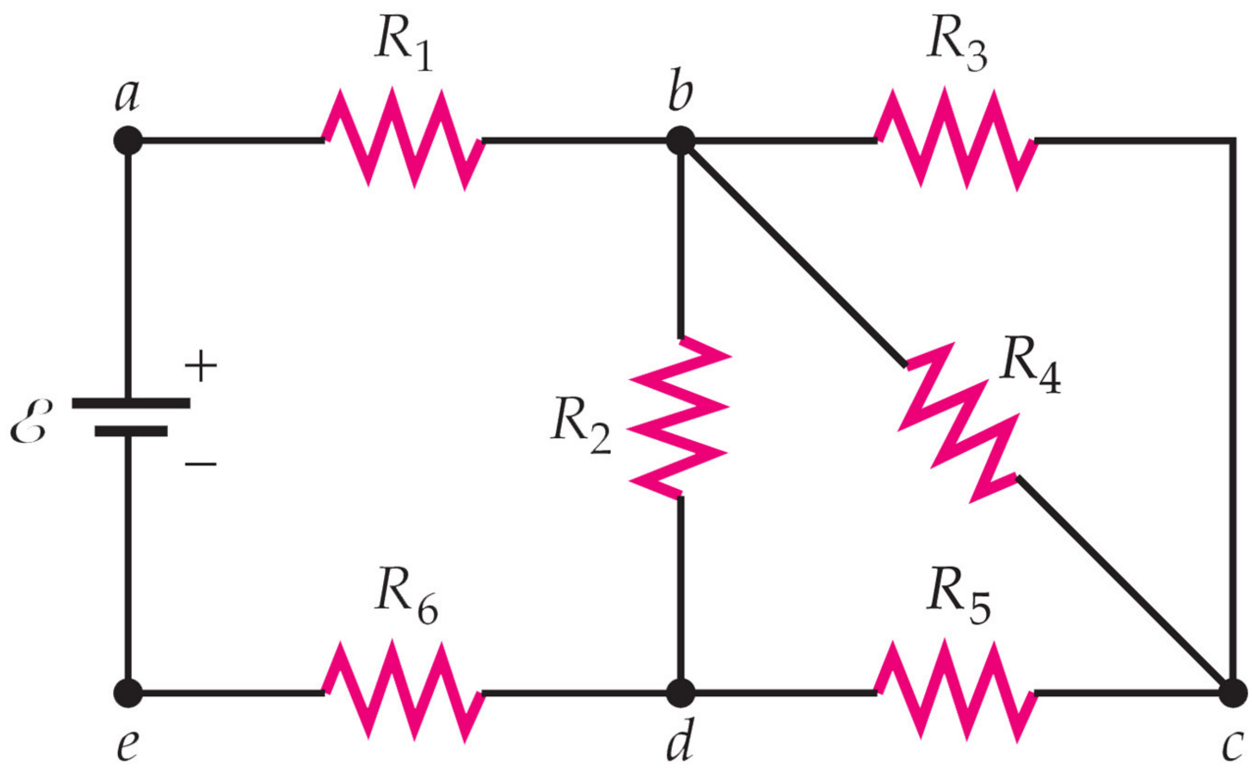
Same voltage - split the current



$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

Find the Equivalent Resistance



Kirchhoff’s Rules

Loop Rule

Whenever any closed loop is traversed, the algebraic sum of the changes in potential around the loop must be equal to zero.

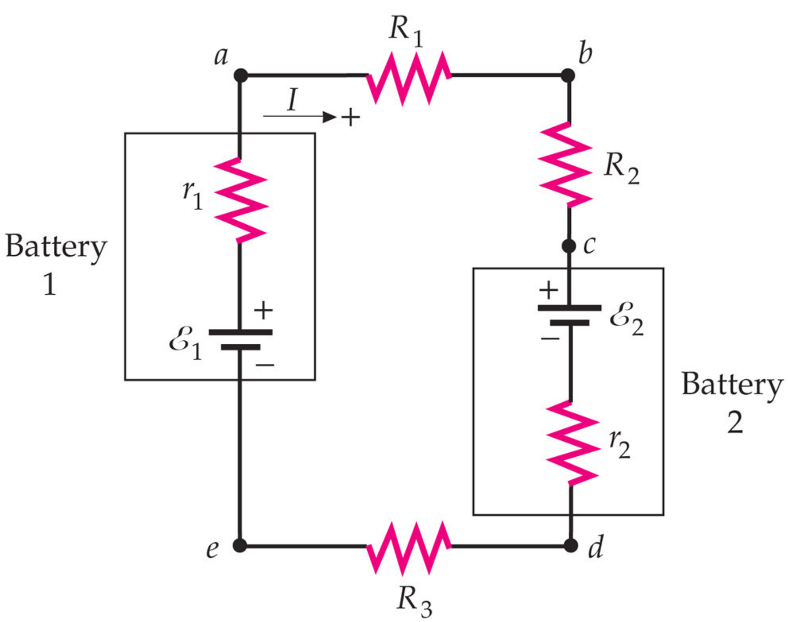
Junction Rule

At any junction (branch point) in a circuit where the current can divide, the sum of the currents into the junction must equal the sum of the currents out of the junction.

$$\sum \Delta V = -IR_1 - IR_2 - E_2 - Ir_2 - IR_3 + E_1 - Ir_1 = 0$$

$$E_1 - E_2 = I\left(R_1 + R_2 + r_2 + R_3 + r_1\right)$$

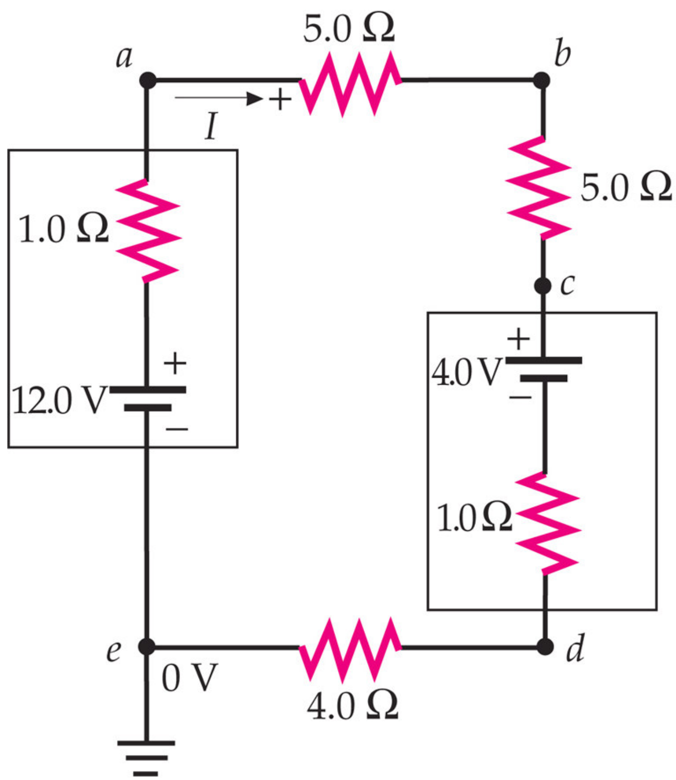
$$I = \frac{E_1 - E_2}{R_1 + R_2 + r_2 + R_3 + r_1}$$



Changes in Potential

$a \to b$	$-IR_1$
$b \to c$	$-IR_2$
$c \to d$	$-\mathcal{E}_2 - Ir_2$
$d \to e$	$-Ir_3$
$e \to a$	$+\mathcal{E}_1 - Ir_1$

The text has mistakes in this table above on page 860.



Equivalent to a series circuit with an 8 volts battery and 16 ohms of resistance.

The current is I = V/R = 8/16 = 0.5 A

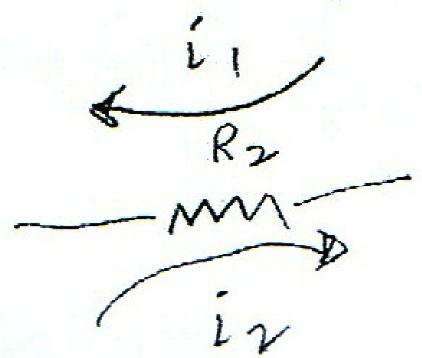
Multi-loop Circuits

Multi-loop means 2 or more circuit loops and 2 or more batteries

1. Combine all series and parallel resistors where possible.
2. Pick a current loop for each circuit loop - make them all go in the same direction.
3. Write a loop equation for each circuit loop.
    - If current through battery is - => + then ΔV is > 0.
    - If current through battery is + => - then ΔV is < 0.
    - For all resistors in a given loop ΔV is < 0.

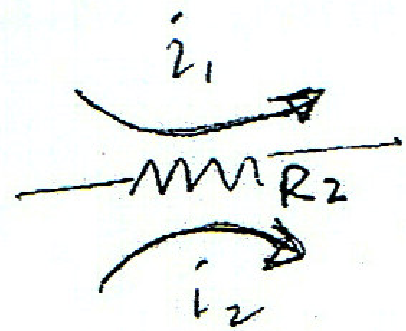
Multi-loop Circuits

4. Some circuit elements will have more than one loop current passing through them.



Opposite direction

$$-i_2R_2 + i_1R_2$$
$$= -(i_2 - i_1)R_2$$



Same direction

$$-i_2R_2 - i_1R_2$$
$$= -(i_2 + i_1)R_2$$

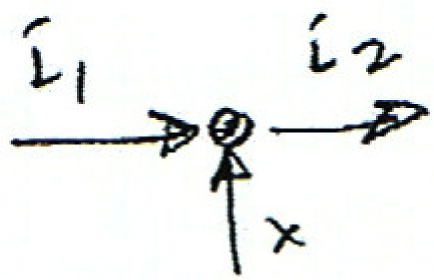
This incorporates the current node equations into the voltage loop equations.
The main loop is i_2

Multi-loop Circuits

5. There should be as many voltage equations as there are current loops.

6. Solve these equations for each of the currents.

7. Resolve the current in the shared elements using the conservation of current at the appropriate nodes.



$$-i_1 - x + i_2 = 0$$

$$x = i_2 - i_1$$

Currents going into the node are <u>negative</u>.

Currents coming out of the node are <u>positive</u>.

Multi-loop Circuits

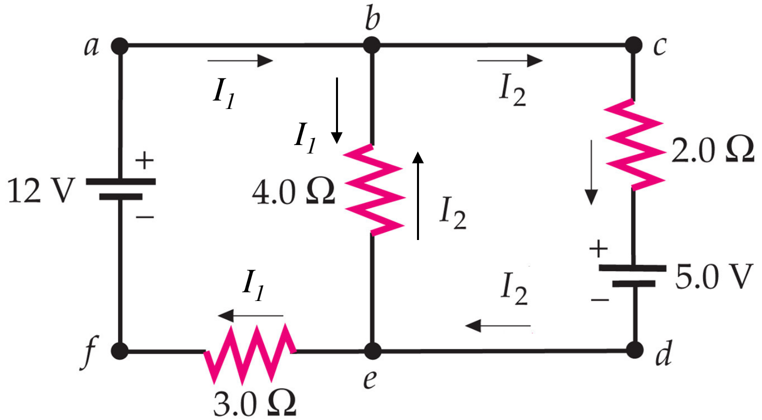
8. Label the currents in the various segments of the circuit.

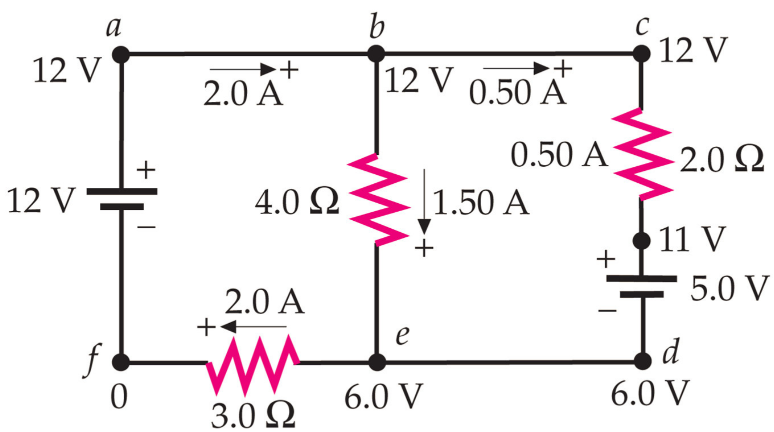
9. Label the voltage drops across each of the elements of the circuit.

10. Check the numbers. Pick a point in the circuit as ground (i.e. 0 volts). Label the voltages of all the nodes in the circuit relative to ground. Are they consistent?

11. Do the voltage drops agree with the circuit and resistance values.

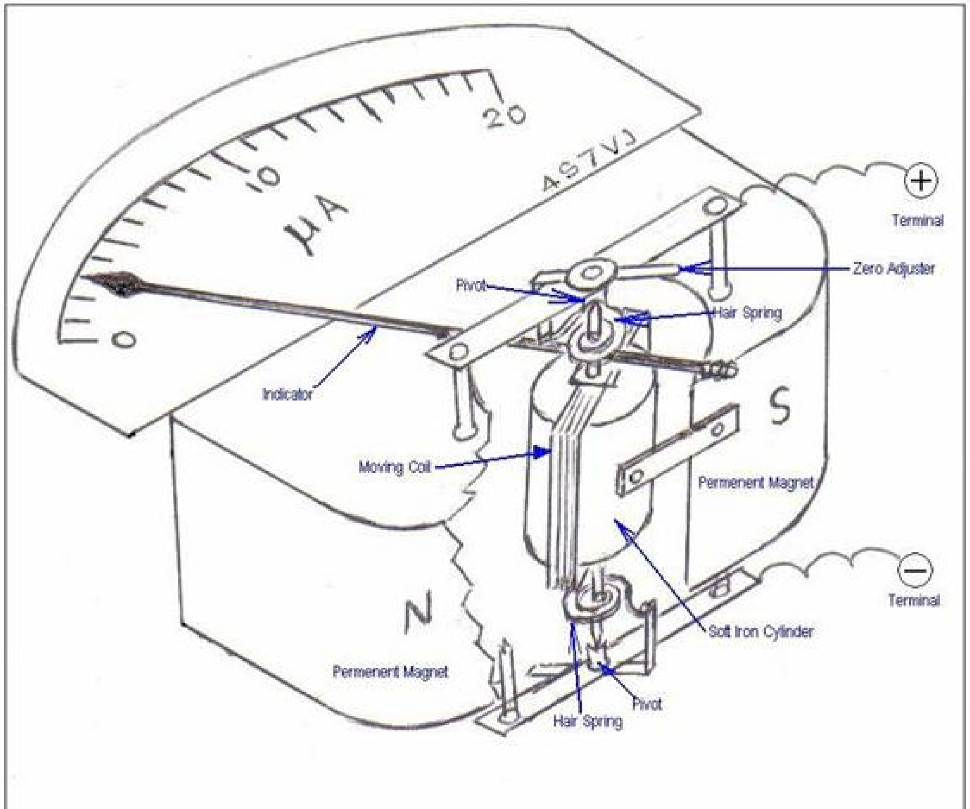
Multi-loop Circuits - Example





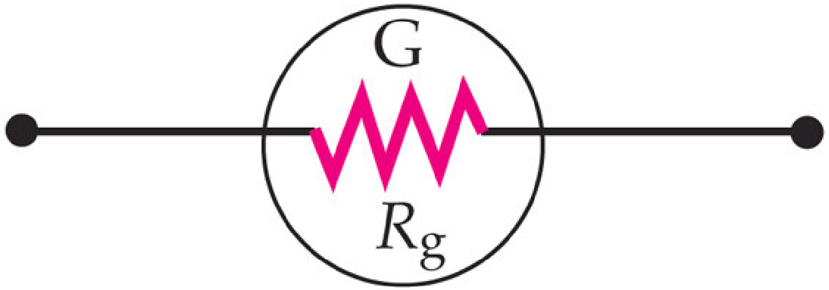
The Galvanometer

A very sensitive ammeter

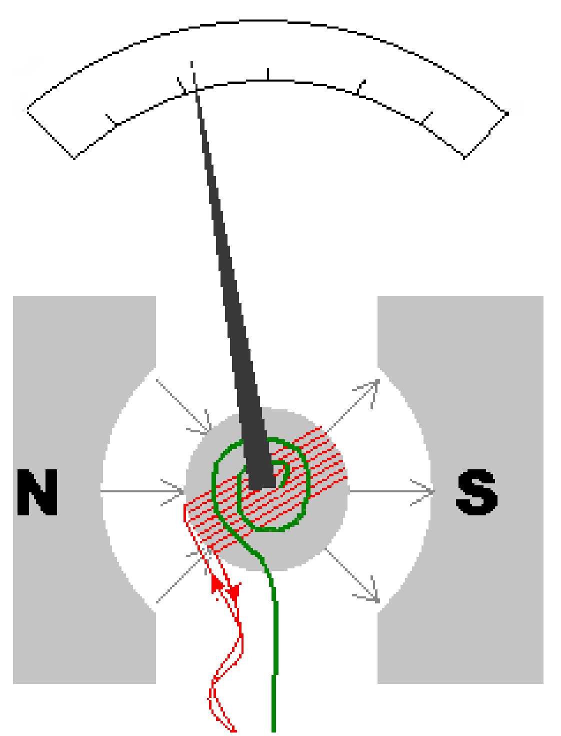


Moving Coil Micro Ammeter

The Galvanometer



Electrical Symbol



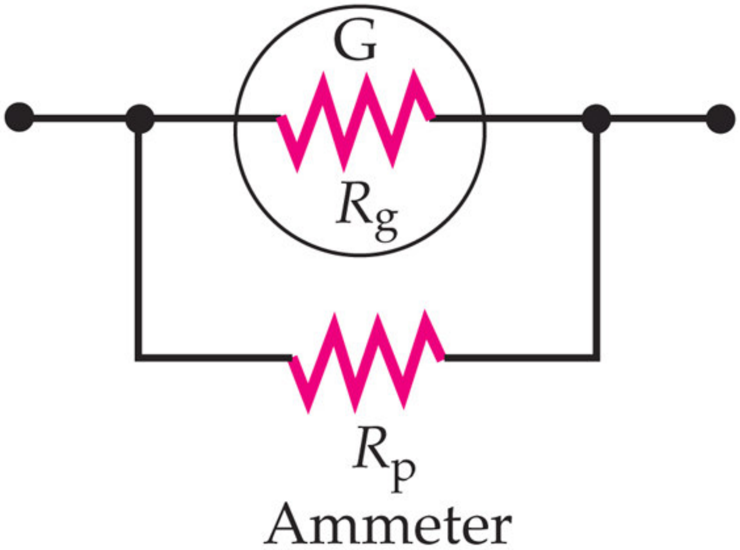
Typical Values

I_{max} = 200 μA = 0.200 mA

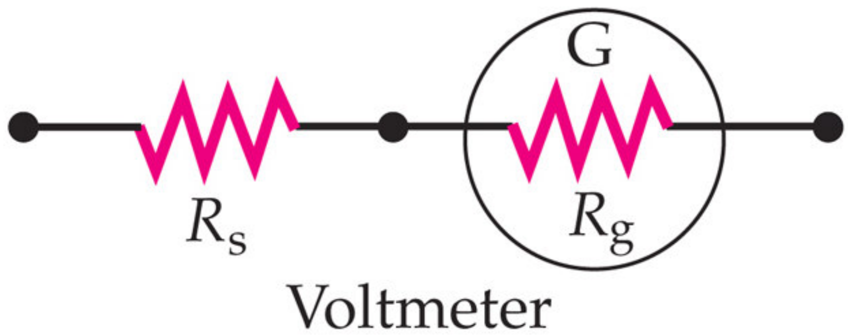
R_g = 100 Ω

The movable coil galvanometer is the basis for building an ammeter, a voltmeter and an ohmmeter.

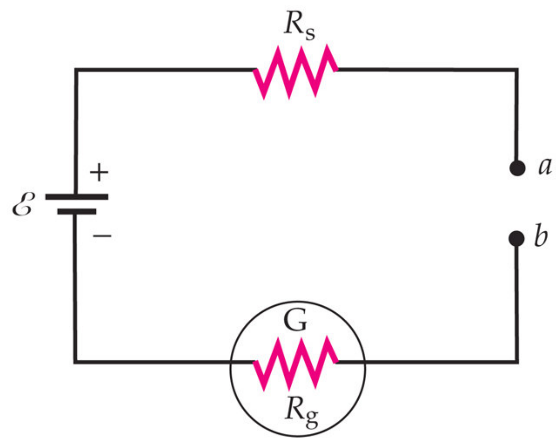
Basic Ammeter

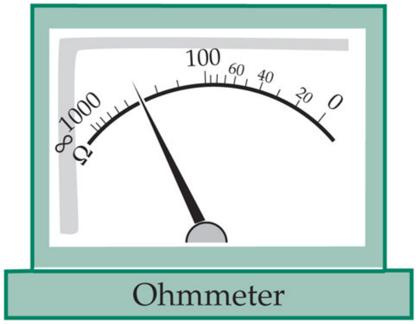


Basic Voltmeter



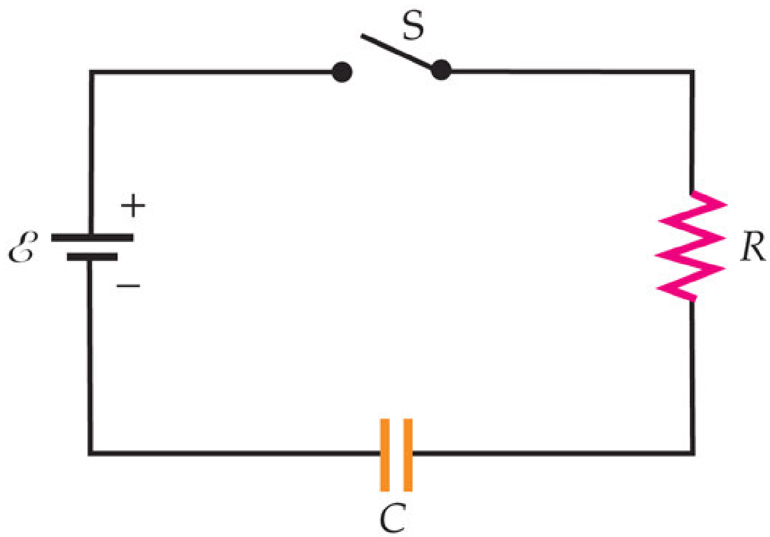
Basic Ohmmeter





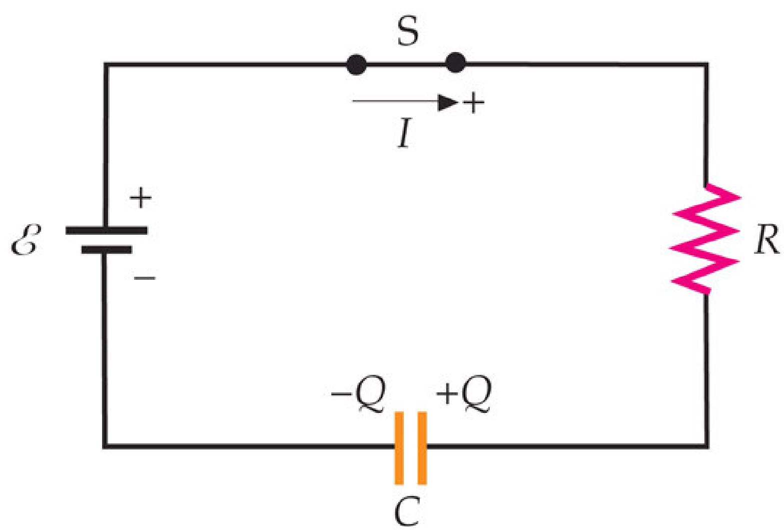
- For use as an ohmmeter the scale runs in reverse. The resistor R_s is adjusted for max current when a and b are shorted.
- The nature of the circuit requires that the scale be non linear. It is actually a logarithmic scale.
- An infinite resistance between a and b results in a zero current.

RC-Circuit

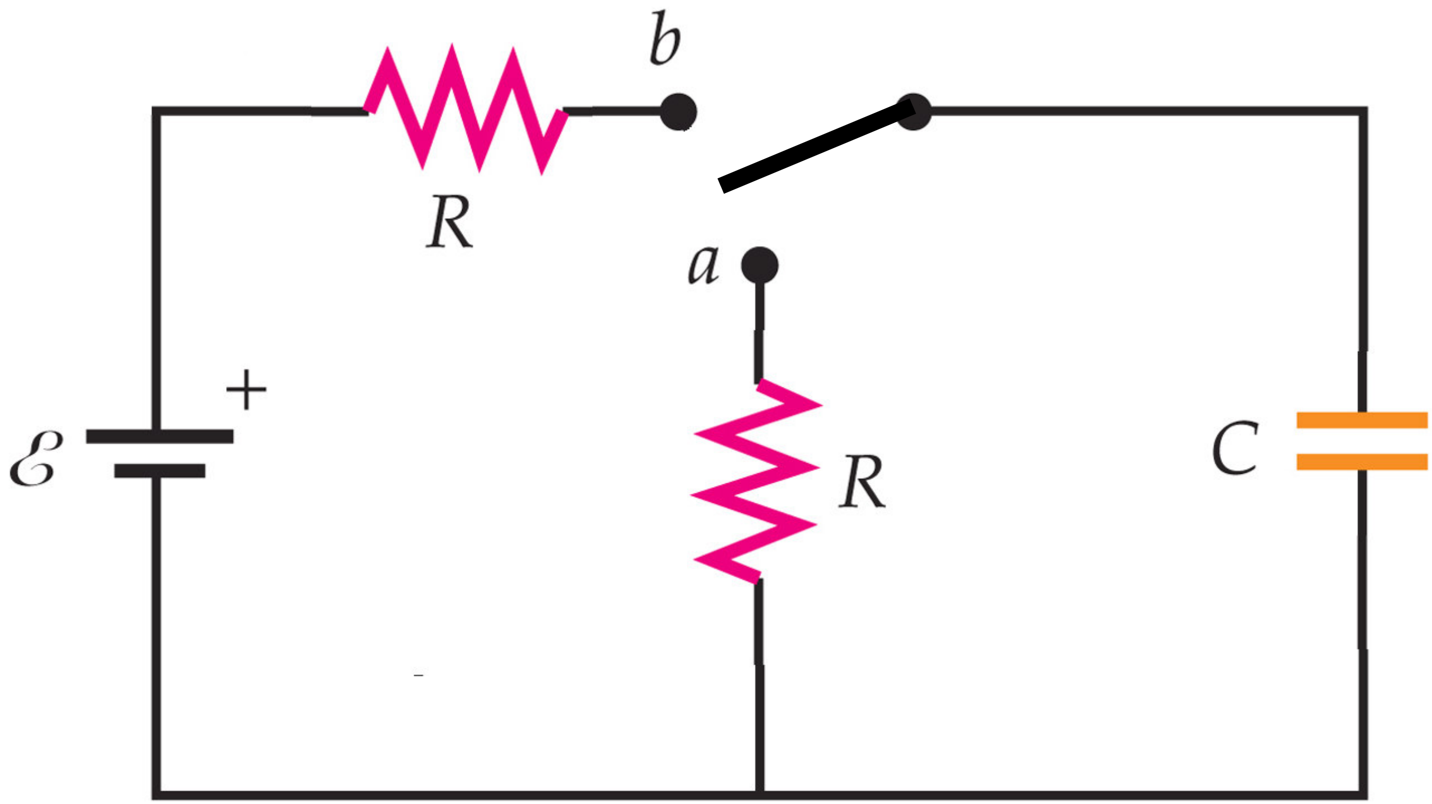


Current only flows in a series RC circuit when the capacitor is charging or discharging.

The steady state current is zero.



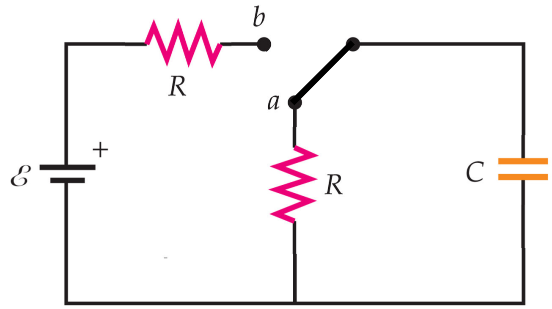
Charge-Discharge Circuit



Position “b” is the charging position

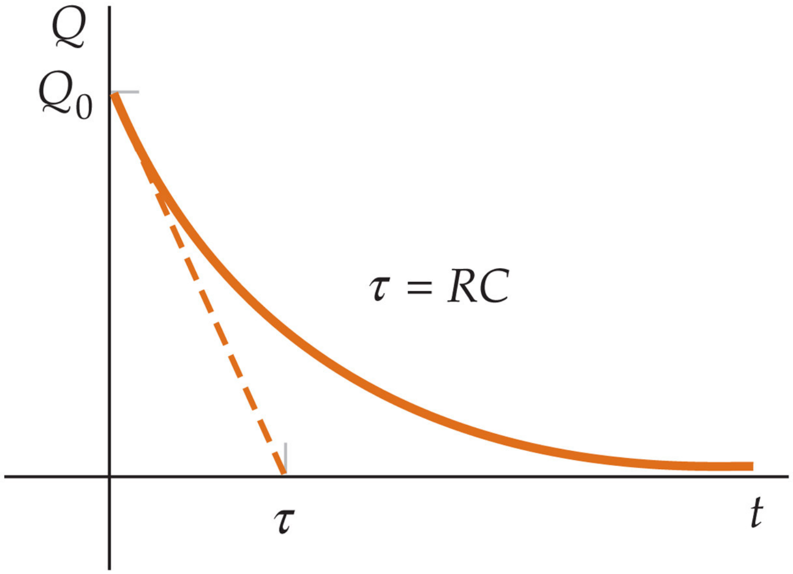
Position “a” is the discharge position

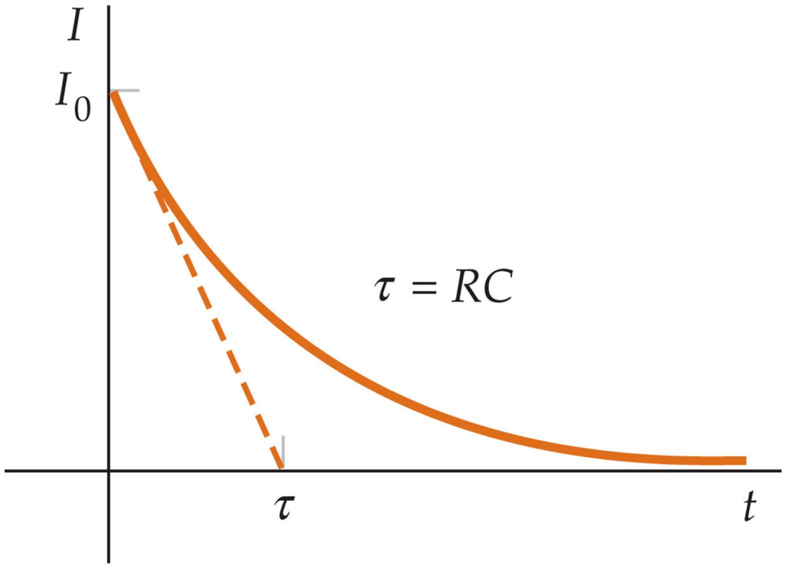
RC-Circuit - Discharging Characteristic



Initially there is a charge Q_0 on the capacitor and therefore a voltage $V_0 = Q_0 /C$ across the capacitor.

When the switch is moved to position "a" this voltage is applied across the resistor R and an initial current $I_0 = Q_0/(RC)$





RC-Circuit - Discharging Characteristic

$$I = -\frac{dQ}{dt}$$

The minus sign indicates the current flow direction is opposite the change in Q.

$$\sum \Delta V = \frac{Q}{C} - IR = 0$$

Apply Kirchhoff's loop rule

$$\frac{Q}{C} + R\frac{dQ}{dt} = 0$$

Substitute for I

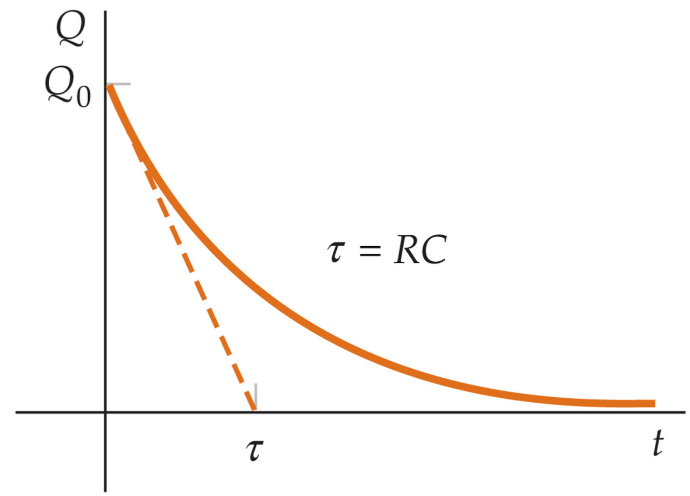
$$\frac{dQ}{dt} = -\frac{1}{RC}Q$$

$$\frac{dQ}{Q} = -\frac{1}{RC}dt$$

Get all the Q's together

RC-Circuit - Discharging Characteristic

$$\int_{Q_0}^{Q'} \frac{dQ}{Q} = -\frac{1}{RC}\int_0^{t'} dt$$



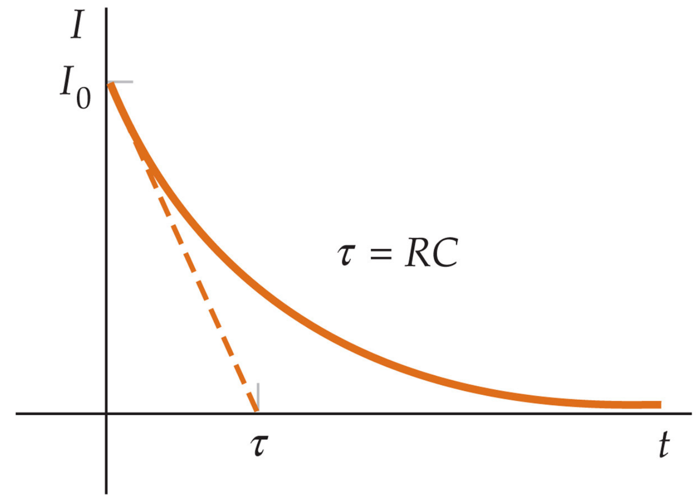
$$ln\left(\frac{Q'}{Q_0}\right) = -\frac{t'}{RC}$$

$$Q(t) = Q_0 e^{-t/(RC)} = Q_0 e^{-t/\tau}$$

$\tau = RC$ This is the time constant

RC-Circuit - Discharging Characteristic

$$I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/(RC)}$$

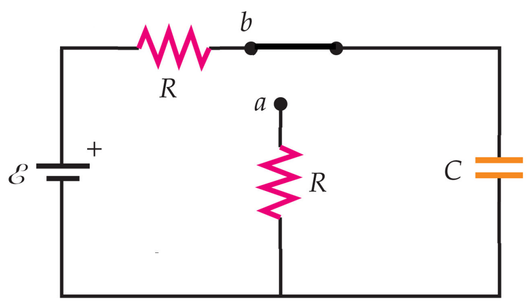


$$I = I_0 e^{-t/\tau}$$

$$I_0 = V_0/R = Q_0/(RC)$$

$$\tau = RC$$

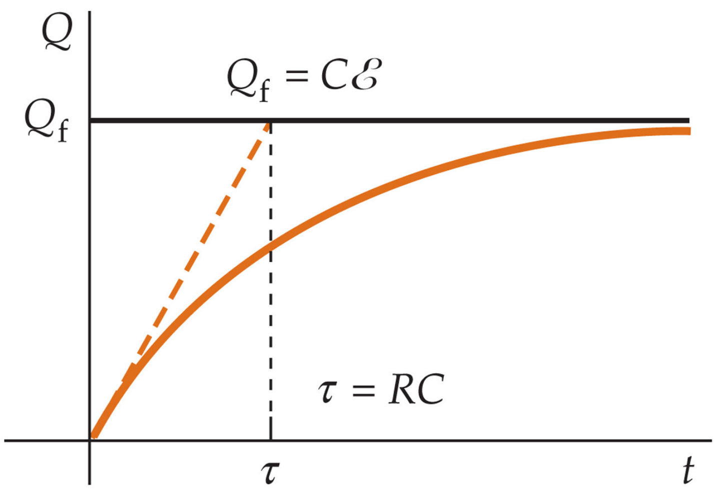
RC-Circuit - Charging

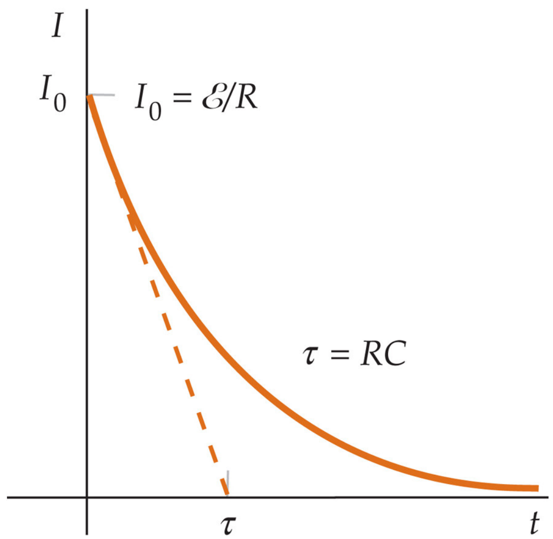


$$Q(t) = Q_f\left(1 - e^{-t/\tau}\right)$$

$$I = \frac{dQ}{dt}$$

$$I = \frac{V_0}{R}e^{-t/(RC)} = I_0 e^{-t/\tau}$$





Extra Slides

Some Characteristic Exponential Values

$t = 0$	e^{0}	1.00	$1 - e^{0}$	0.0
$t = \tau$	e^{-1}	0.368	$1 - e^{-1}$	0.632
$t = 2\tau$	e^{-2}	0.135	$1 - e^{-2}$	0.865
$t = 3\tau$	e^{-3}	0.050	$1 - e^{-3}$	0.950
$t = 4\tau$	e^{-4}	0.018	$1 - e^{-4}$	0.982
$t = 5\tau$	e^{-5}	0.007	$1 - e^{-5}$	0.993