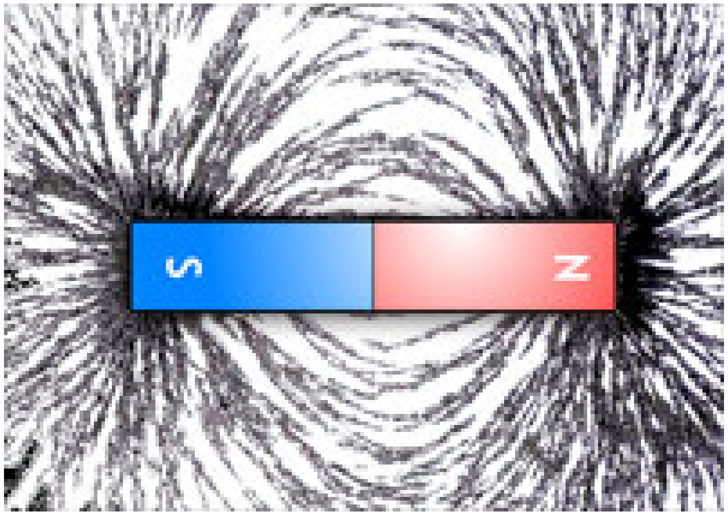
Chapter 28

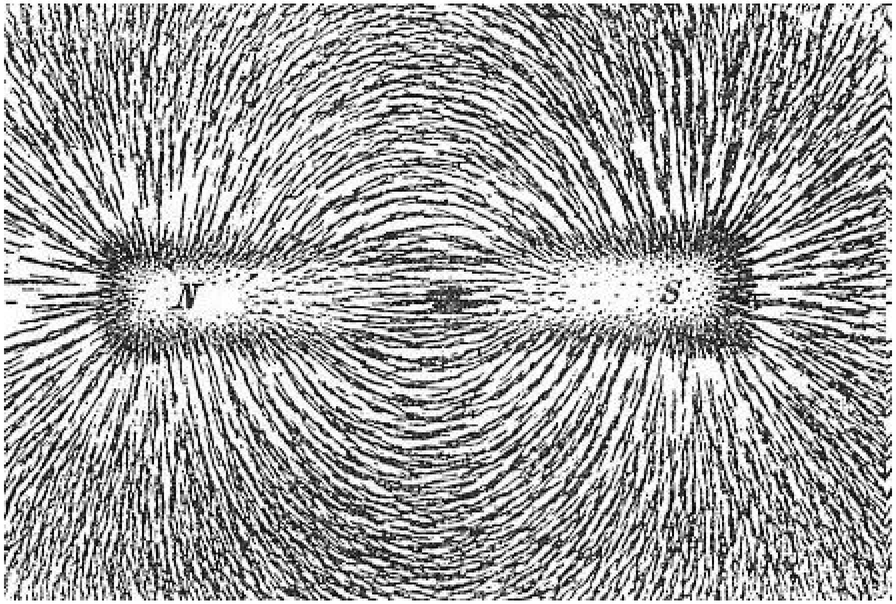
The Magnetic Field

Magnetic Field

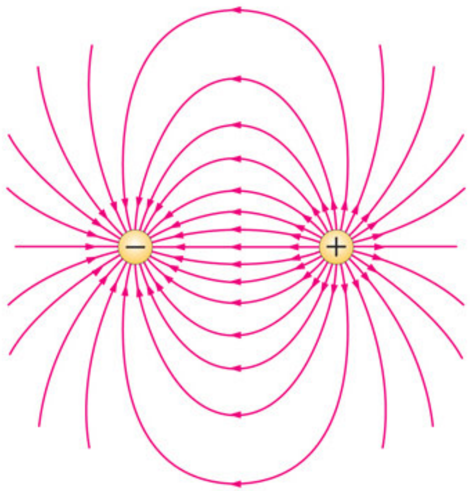
- Force Exerted by a Magnetic Field
- Point Charge in a Magnetic Field
- Torques on Current Loops

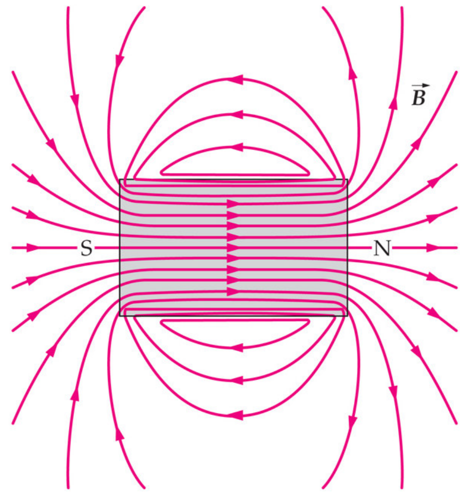
Magnetic Field

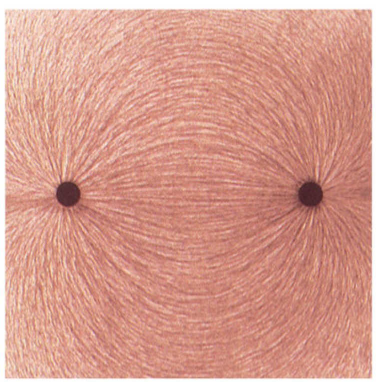


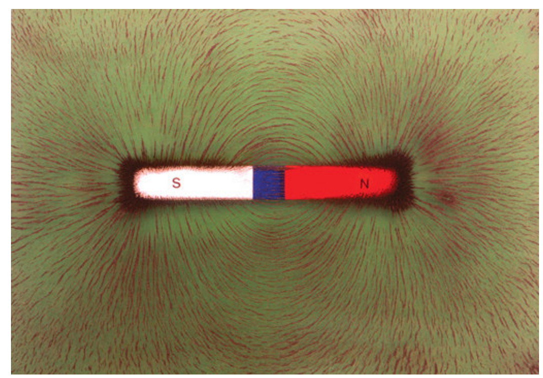


Electric and Magnetic Field Analogy





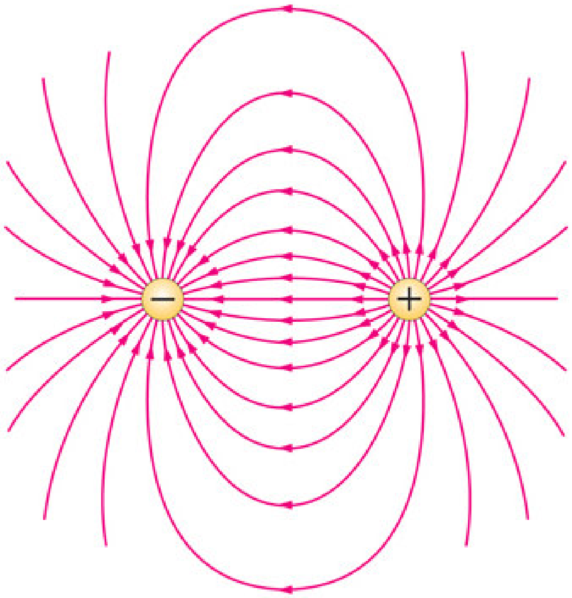


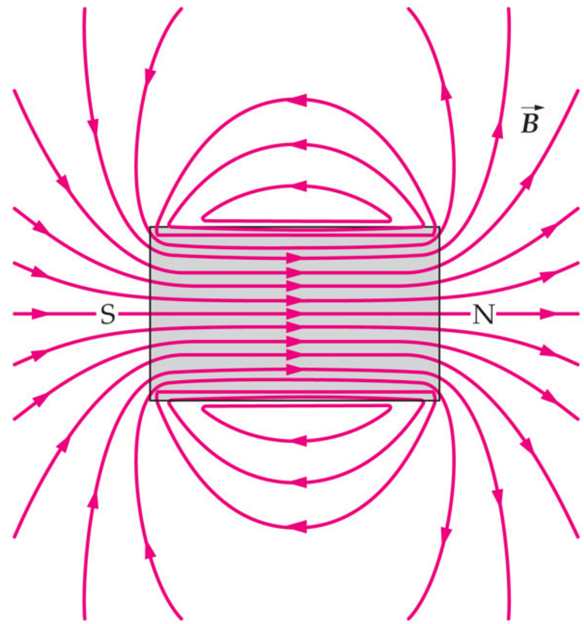


Electric Dipole

Magnetic Dipole

Electric and Magnetic Dipole Fields



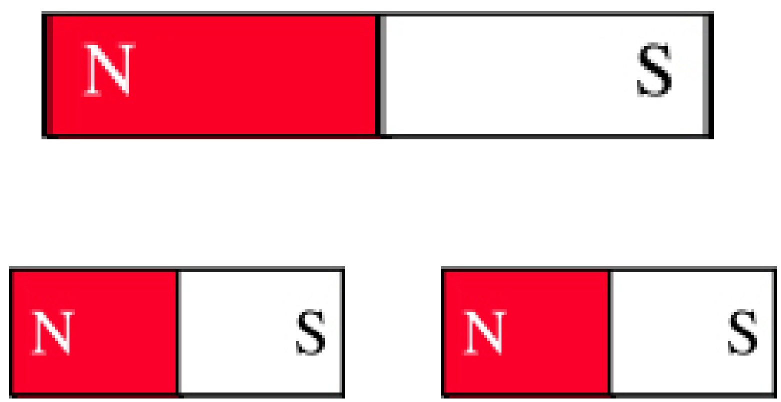


Electric field lines start on a positve charge and terminate on a negative charge.

Magnetic field lines have no beginning or end, they form continuous loops. There are no discrete magnetic charges.

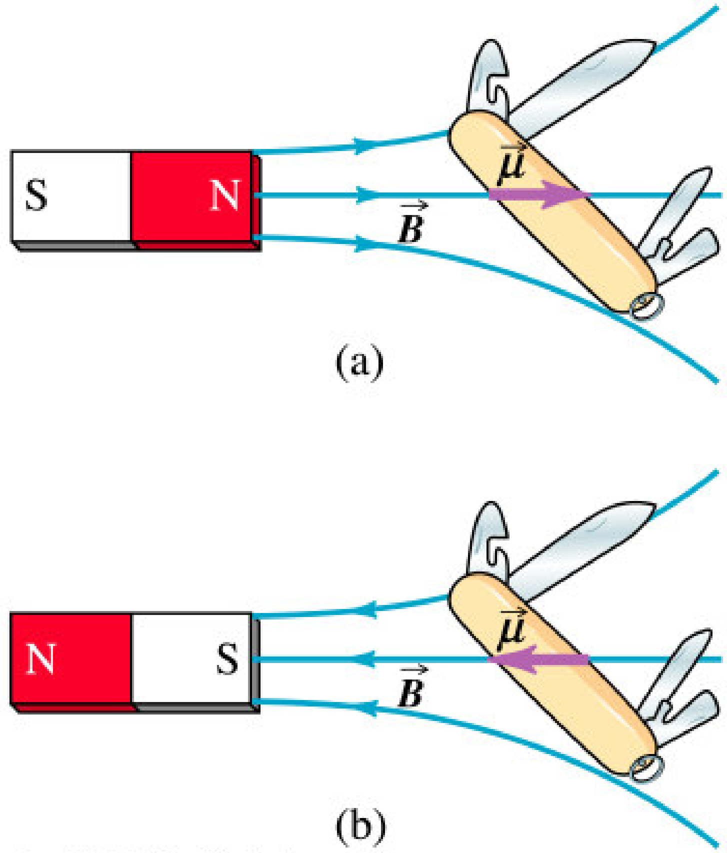
Magnetic Monopoles Don’t Exist

Isolated North and South poles cannot be created by breaking a bar magnet in half



Breaking a bar magnet in half will only create two smaller bar magnets. Each with a north and south pole.

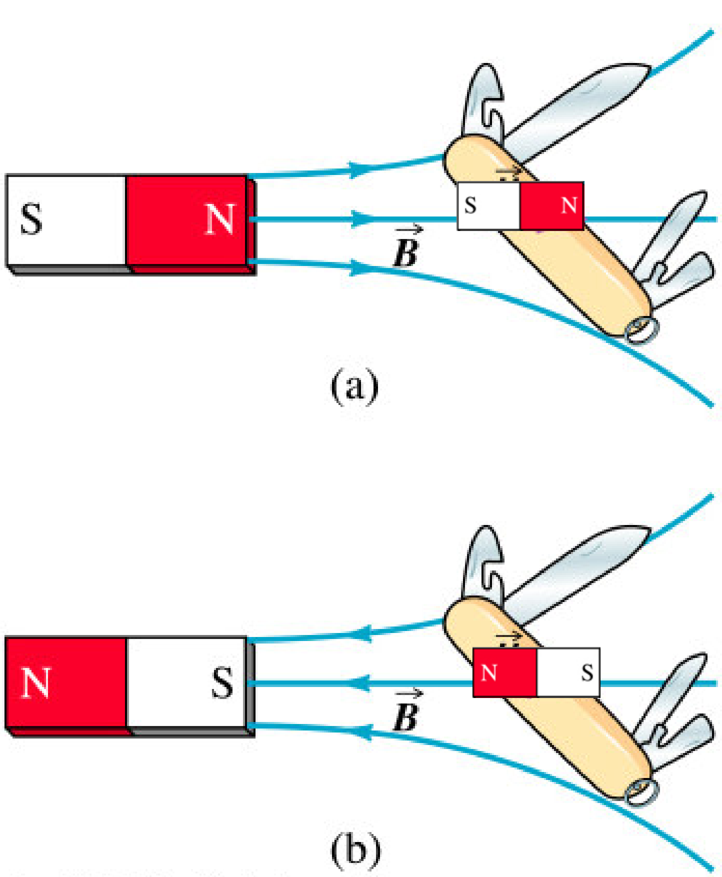
Attraction by a Magnetic Field



In a <u>*uniform magnetic field*</u> there would be the magnetization of the object characterized by μ. But there would not be any attractive force.

The <u>*magnetic field gradient*</u> is required for the magnetic moment μ to experience an attractive force.

Attraction by a Magnetic Field



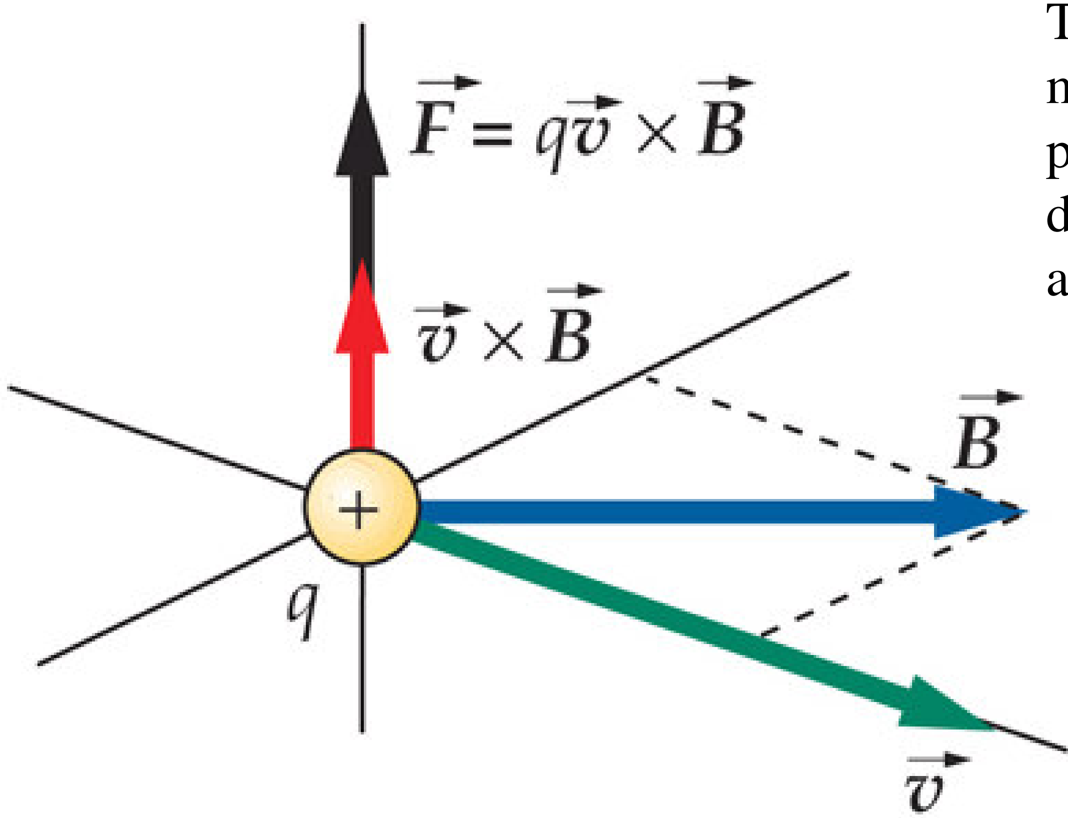
The magnetic moment of the jack knife acts as if it was a small bar magnet.

In each case the opposite poles are facing each other and the knife experiences an attractive force.

<u>Never point at a magnet!</u>

Force Exerted by a Magnetic Field on a Charged Particle

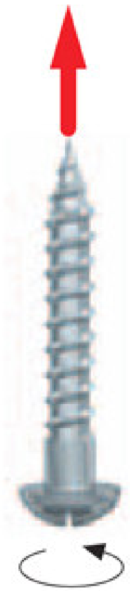
Force Exerted by a Magnetic Field

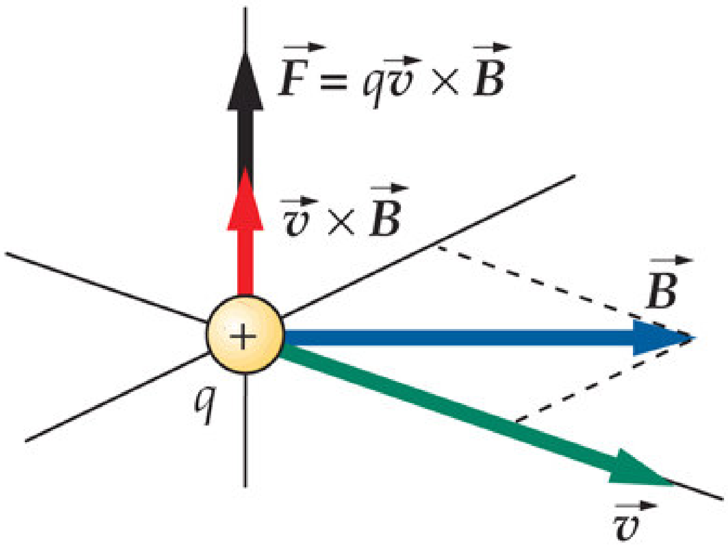


The direction of the magnetic force is perpendicular to the plane determined by the velocity and magnetic field vectors.

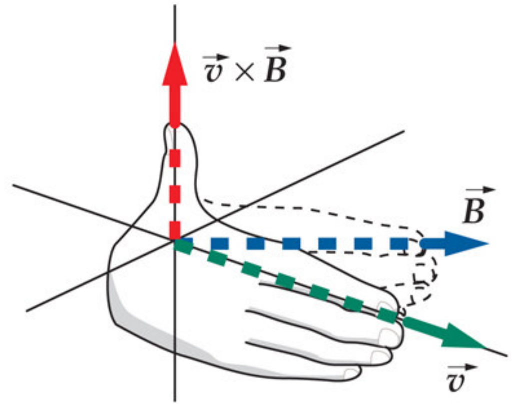
This force is sometimes called the Lorentz force

Right Hand Rule Yields Force Direction



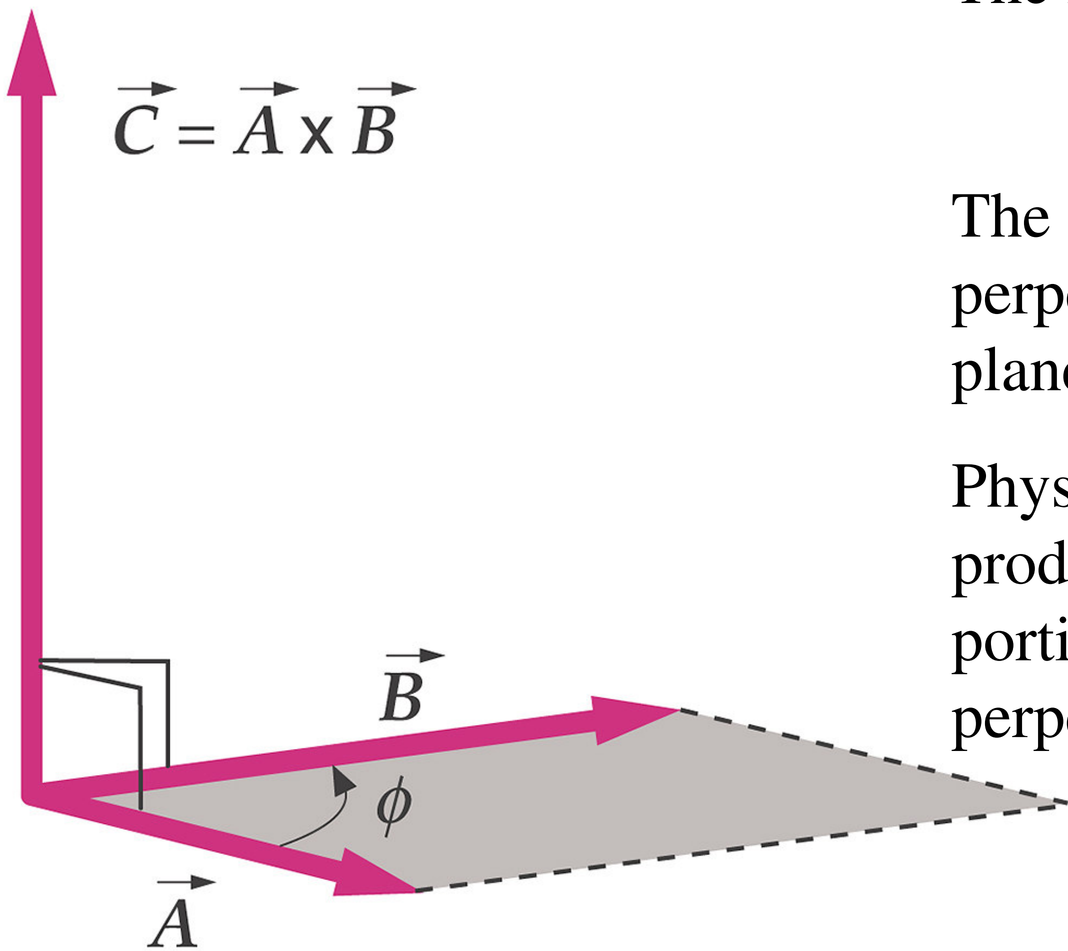


In vector language this is a vector cross product.



The direction is determined by turning the v vector into the B vector and the force direction proceeds in the direction that a right handed screw would move.

The Vector Cross Product



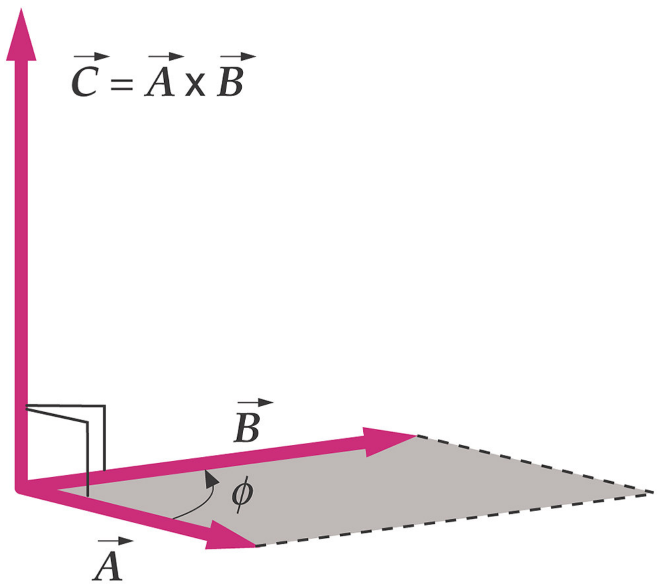
The magnitude of C

$$\mathbf{C = ABsin(\Phi)}$$

The direction of C is perpendicular to the plane of A and B.

Physically it means the product of A and the portion of B that is perpendicular to A.

The Cross Product by Components



$$\boldsymbol{A} \times \boldsymbol{B} = \text{vector}$$

$$(\boldsymbol{A} \times \boldsymbol{B})_z = A_x B_y - A_y B_x$$

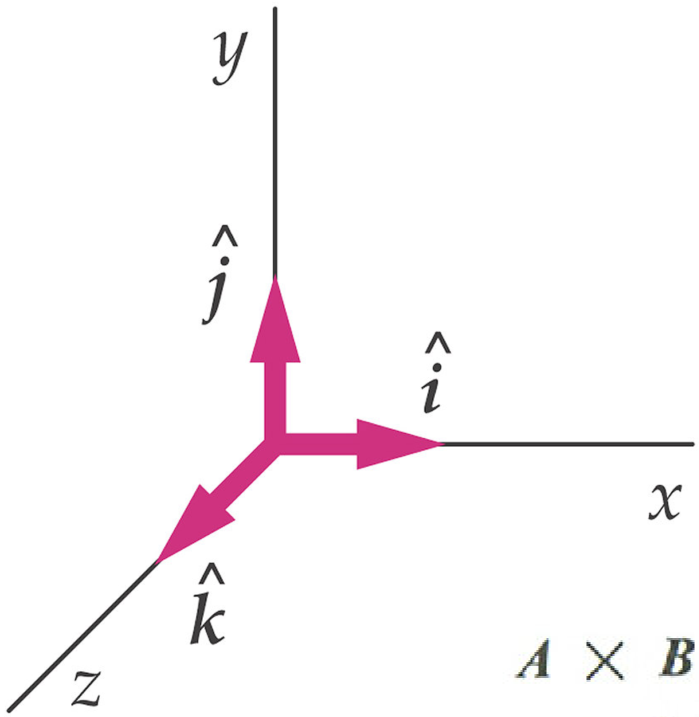
$$(\boldsymbol{A} \times \boldsymbol{B})_x = A_y B_z - A_z B_y$$

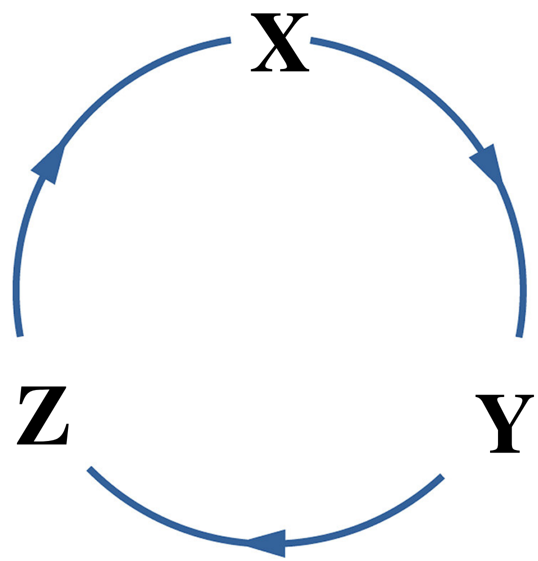
$$(\boldsymbol{A} \times \boldsymbol{B})_y = A_z B_x - A_x B_z$$

Since A and B are in the x-y plane A x B is along the z-axis.

$$\boldsymbol{A} \times \boldsymbol{B} = (\boldsymbol{A} \times \boldsymbol{B})_z = A_x B_y - A_y B_x$$

Memorizing the Cross Product





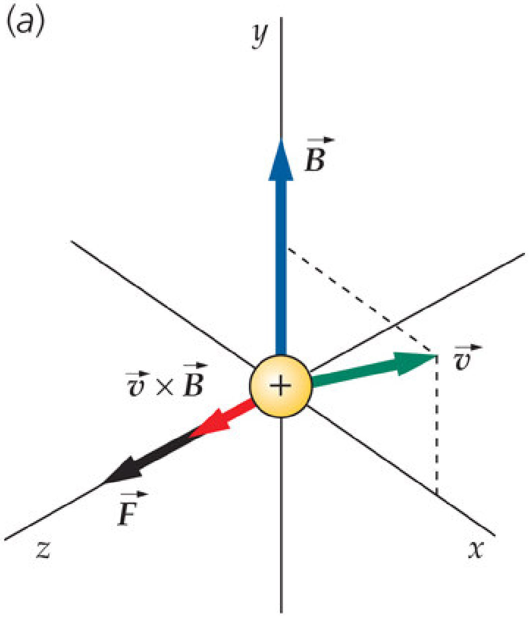
$$\boldsymbol{A} \times \boldsymbol{B} = \text{vector}$$

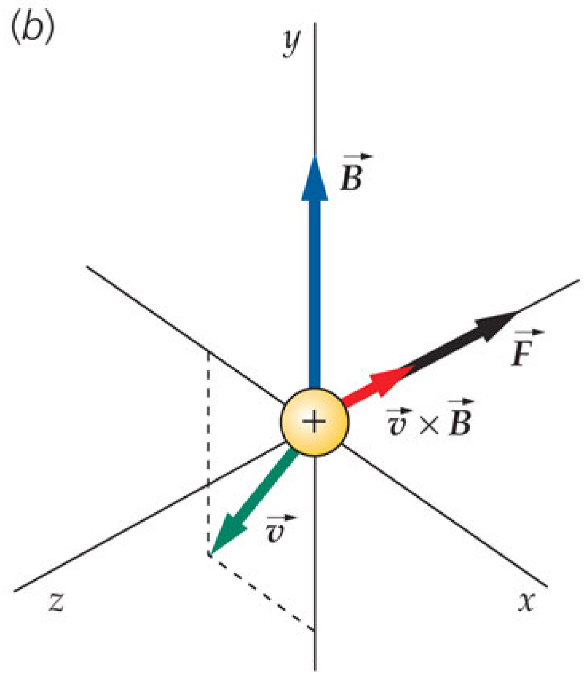
$$(\boldsymbol{A} \times \boldsymbol{B})_z = A_x B_y - A_y B_x$$

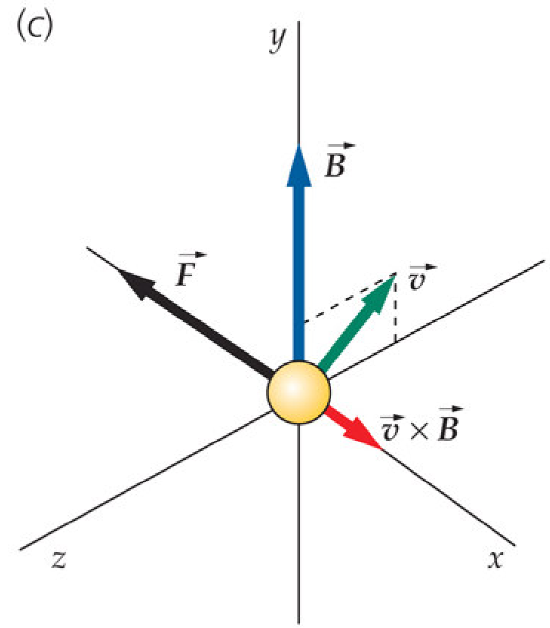
$$(\boldsymbol{A} \times \boldsymbol{B})_x = A_y B_z - A_z B_y$$

$$(\boldsymbol{A} \times \boldsymbol{B})_y = A_z B_x - A_x B_z$$

Magnetic Force Examples







The Units of Magnetic Field

$$1\,Tesla(T) = 1\frac{N}{C(m/s)} = 1\frac{N}{A\bullet m}$$

$$1\,Gauss = 10^{-4}\,T$$

$$B_{Earth} \approx 0.5\,G$$

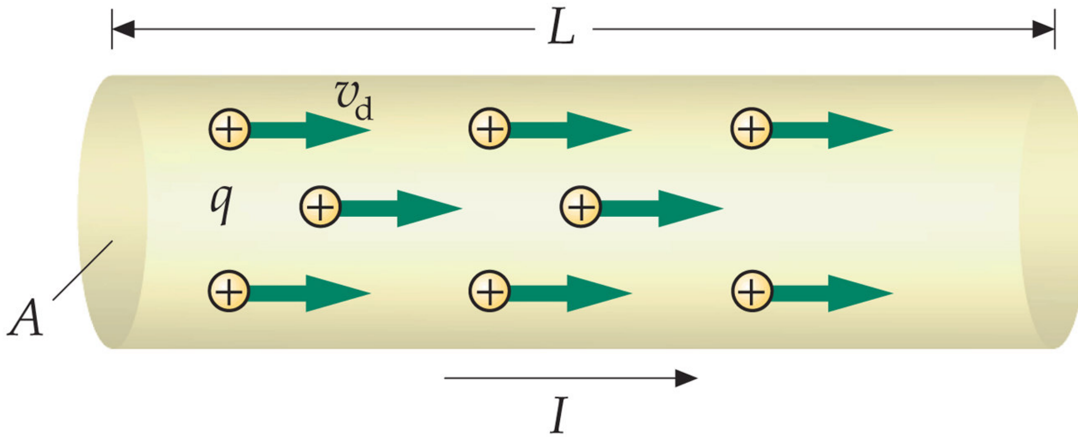
Range of Magnetic Field Strengths

Smallest value in a magnetically shielded room	10^-14 Tesla	10^-10 Gauss
Interstellar space	10^-10 Tesla	10^-6 Gauss
Earth's magnetic field	0.00005 Tesla	0.5 Gauss
Small bar magnet	0.01 Tesla	100 Gauss
Within a sunspot	0.15 Tesla	1500 Gauss
Small NIB magnet	0.2 Tesla	2000 Gauss
Big electromagnet	1.5 Tesla	15,000 Gauss
Strong lab magnet	10 Tesla	100,000 Gauss
Surface of neutron star	100,000,000 Tesla	10^12 Gauss
Magstar	100,000,000,000 Tesla	10^15 Gauss

Source: http://www.coolmagnetman.com/magflux.htm

Magnetic Force on Current Carrying Wires

Magnetic Force on Current Carrying Wires



The magnetic force on a wire segment is the sum of the magnetic force on all charge carrying particles in the wire.

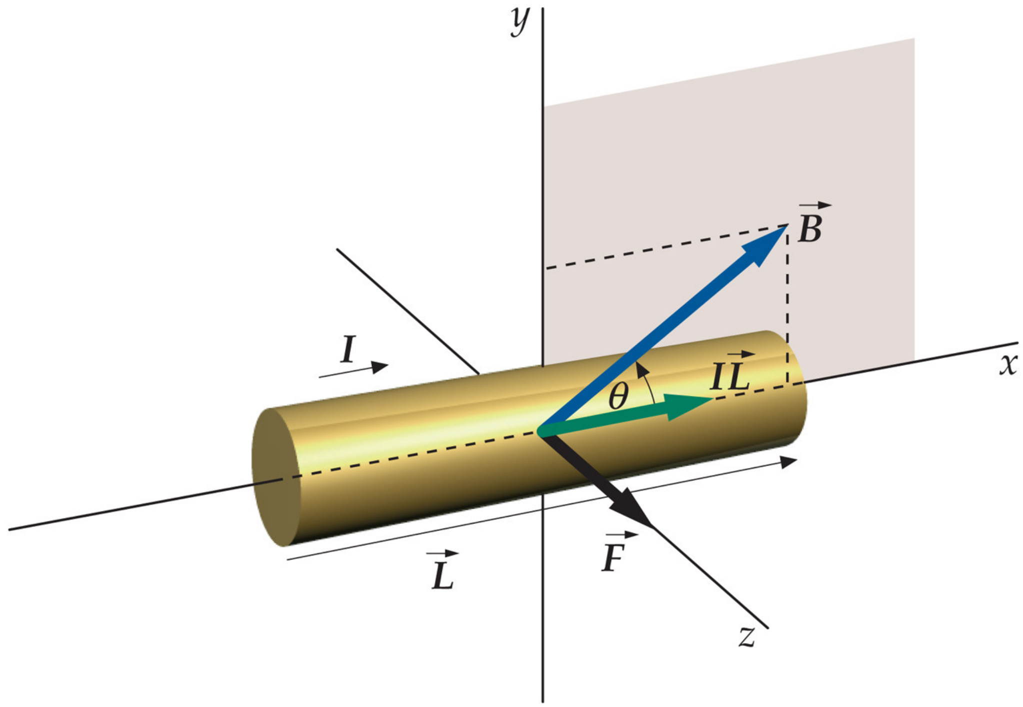
$$\vec{F} = (q\vec{v}_d \times \vec{B})nAL$$

$$I = nqv_d L$$

$$\vec{F} = I\vec{L} \times \vec{B}$$

$\vec{L}$ is a vector whose magnitude is the length of the wire segment and whose direction is that of the current.

B and ***L*** form a plane and ***F*** is perpendicular to this plane



The wire segment feels a magnetic force perpendicular to the motion of the current and also perpendicular to the direction of the magnetic field.

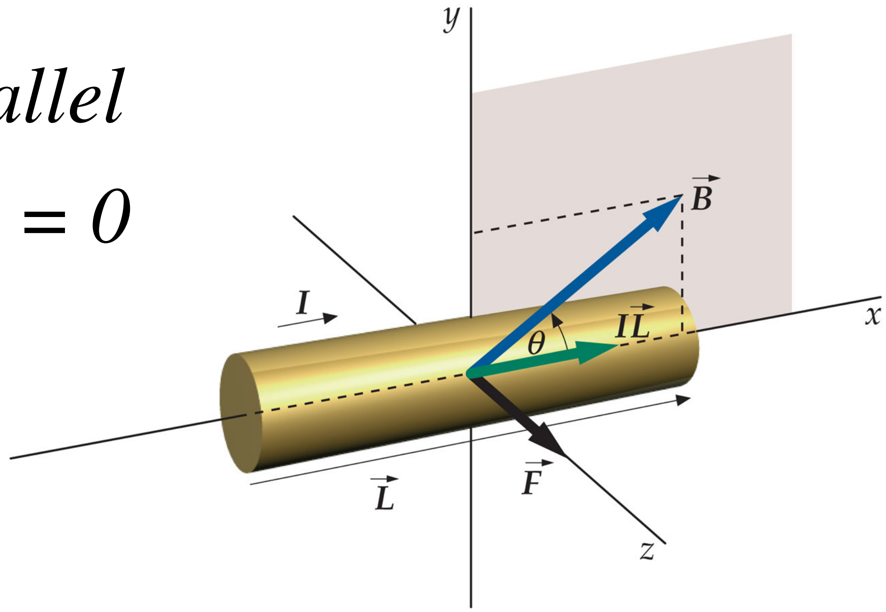
B and ***L*** form a plane as long as they are not parallel.

$$\vec{F} = I\vec{L} \times \vec{B}$$

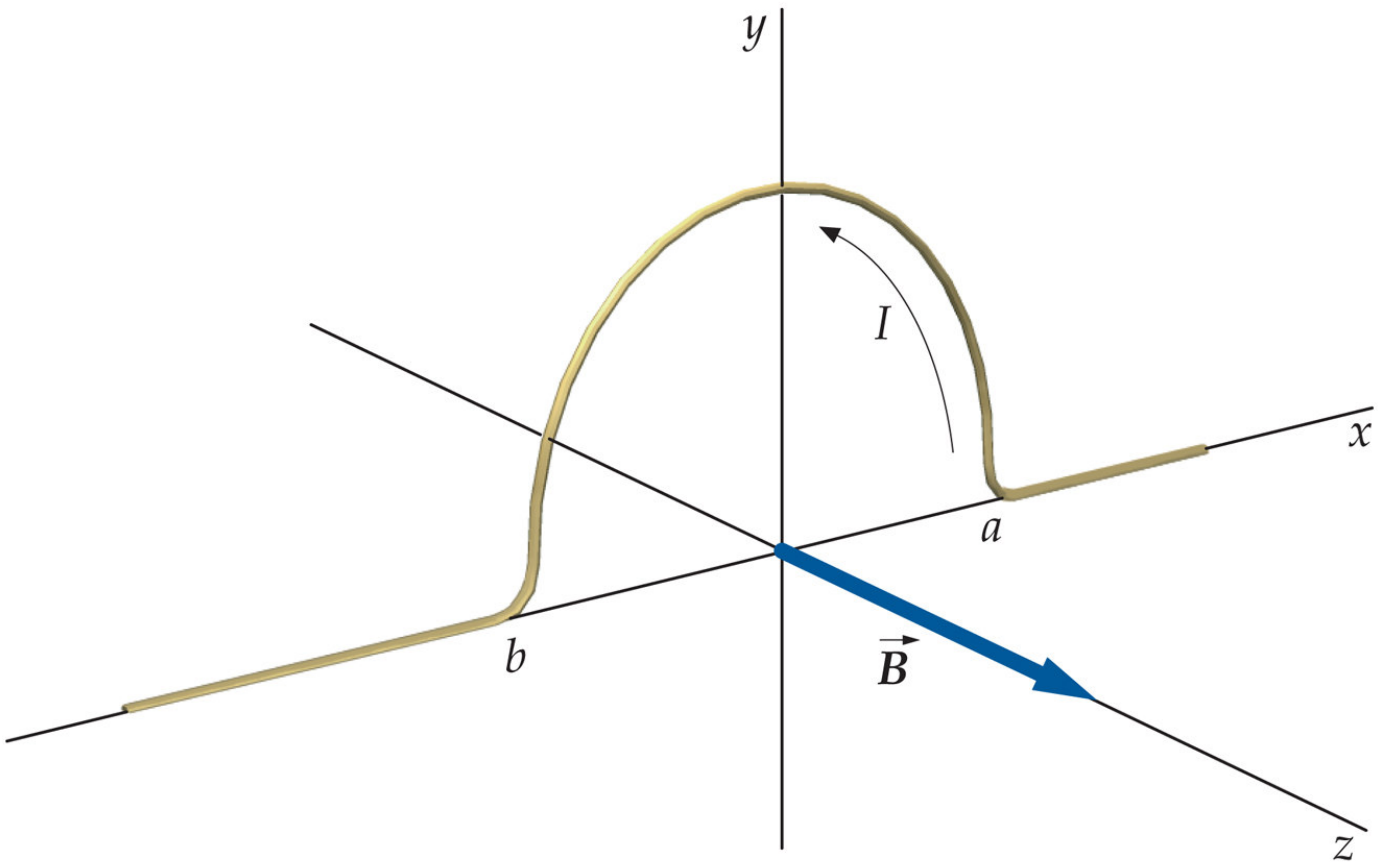
$$|F| = ILB\sin\theta$$

If L and B are parallel

then $\sin\theta = 0$ *and* $F = 0$

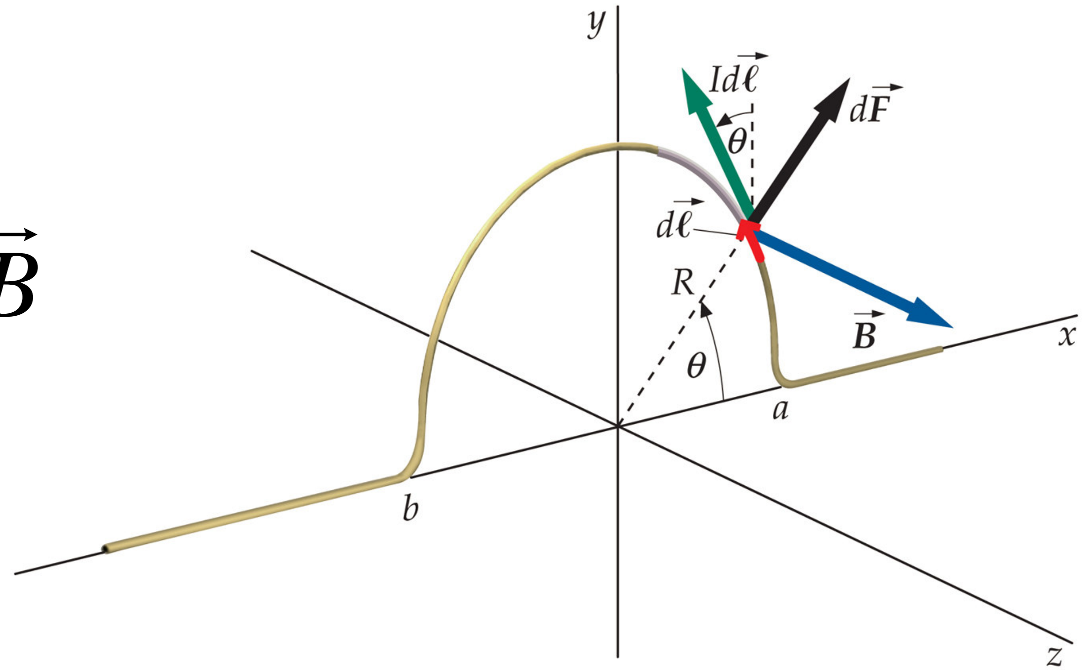


Force on a Semicircular Wire Loop



Force on a Semicircular Wire Loop

$$d\vec{F} = Id\vec{L} \times \vec{B}$$



You need to understand and be able to demonstrate this calculation.

Force on a Semicircular Wire Loop

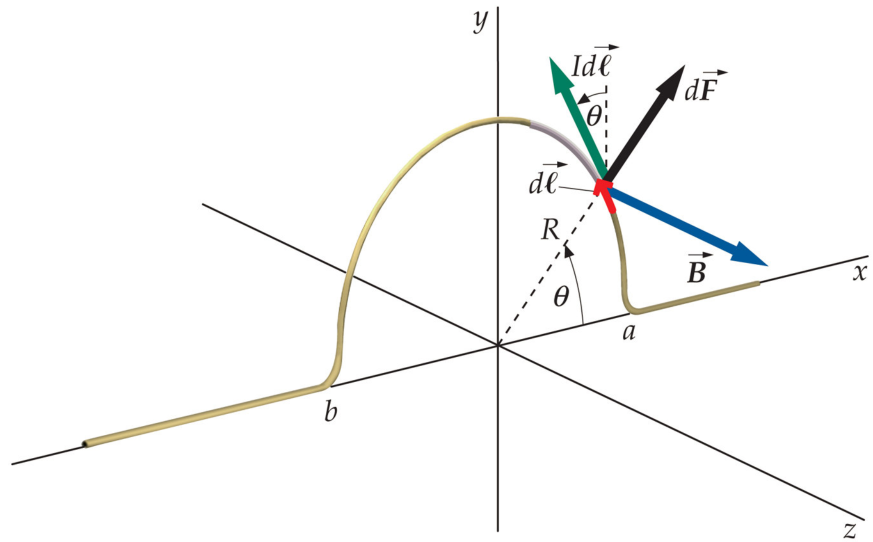
$$d\vec{F} = Id\vec{L} \times \vec{B}$$

$$d\vec{L} = dL\hat{u} = Rd\theta\hat{u}$$

$$B = B_0\hat{k}$$

$$\theta = 0; \quad \hat{u} = \hat{j}$$

$$\theta = \tfrac{\pi}{2}; \ \hat{u} = -\hat{i}$$



$$\hat{u} = -sin(\theta)\hat{i} + cos(\theta)\hat{j}$$

$$\vec{F} = \int_0^{\pi} IRd\theta(-sin(\theta)\hat{i} + cos(\theta)\hat{j}\ ) \times B_0\hat{k}$$

$$\vec{F} = \int_0^{\pi} IRd\theta(-sin(\theta)\hat{i} + cos(\theta)\hat{j}\ )\times B_0\hat{k}$$

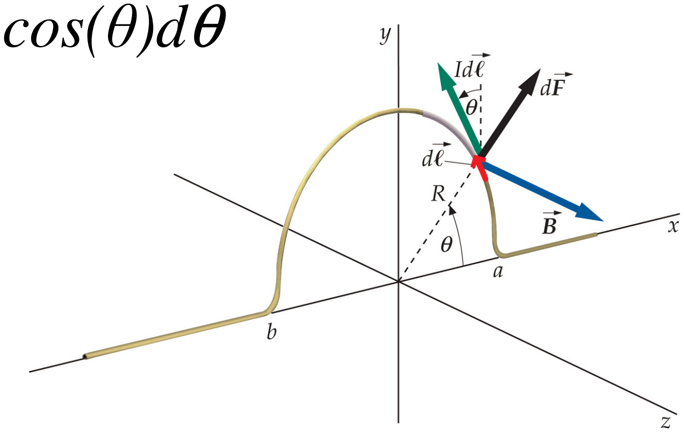
$$\vec{F} = \int_0^{\pi} IRB_0(-sin(\theta)\hat{i} + cos(\theta)\hat{j})\times \hat{k}d\theta$$

$$\vec{F} = IRB_0\int_0^{\pi}(-sin(\theta)\hat{i})\times \hat{k}d\theta\ + IRB_0\int_0^{\pi}cos(\theta)\hat{j}\times \hat{k}d\ \theta$$

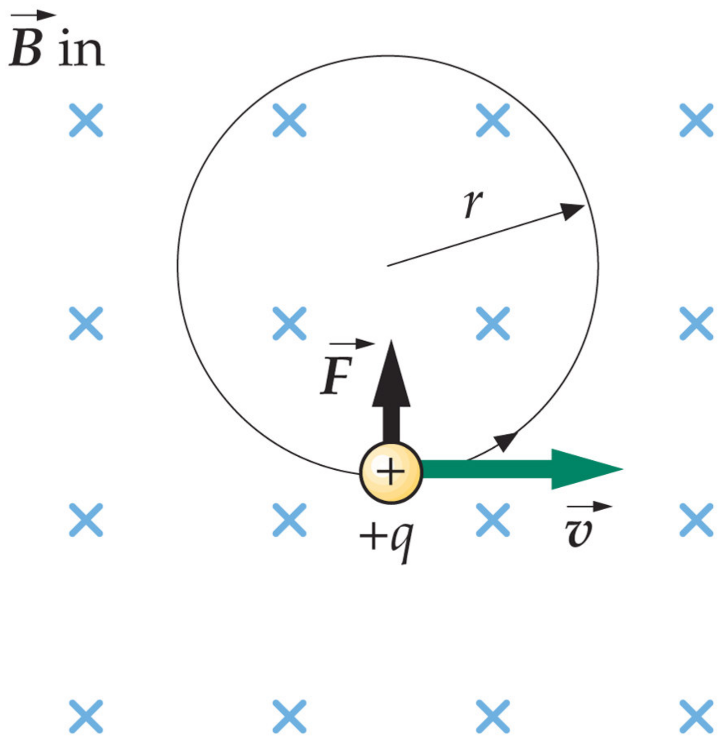
$$\vec{F} = IRB_0\hat{j}\int_0^{\pi} sin(\theta)d\theta\ + IRB_0\hat{i}\int_0^{\pi}cos(\theta)d\theta$$

$$\vec{F} = IRB_0\hat{j}(2) + IRB_0\hat{i}(0)$$

$$\vec{F} = I(2R)B_0\hat{j}$$



Motion of a Point Charge in a Magnetic Field



This is a central acting force that gives rise to cyclotron motion.

This is an example of a centripetal force.

The magnetic force can't change the energy of the particle because it can do no work. Why?

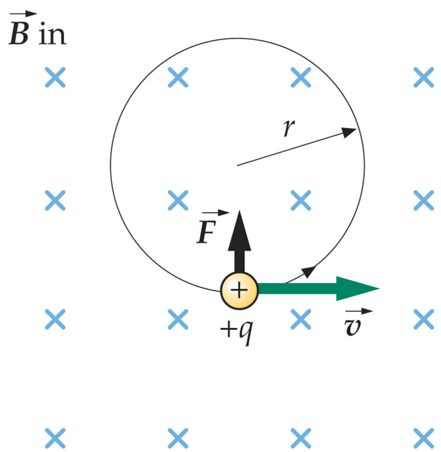
Motion of a Point Charge in a Magnetic Field

This is a central acting force that gives rise to cyclotron motion.

$$F = ma$$

$$qvB = m\frac{v^2}{r}$$

$$r = \frac{mv}{qB}$$



$$T = \frac{2\pi r}{v} = \frac{2\pi\left(mv/qB\right)}{v}$$

$$T = \frac{2\pi m}{qB}$$

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

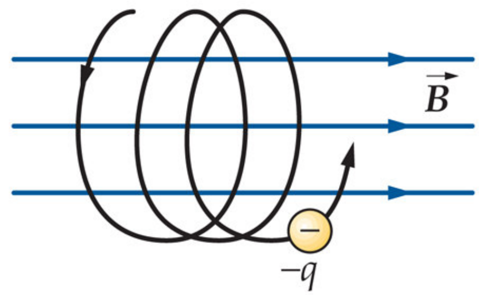
$$\omega = 2\pi f = \frac{q}{m}B$$

ω is independent of velocity

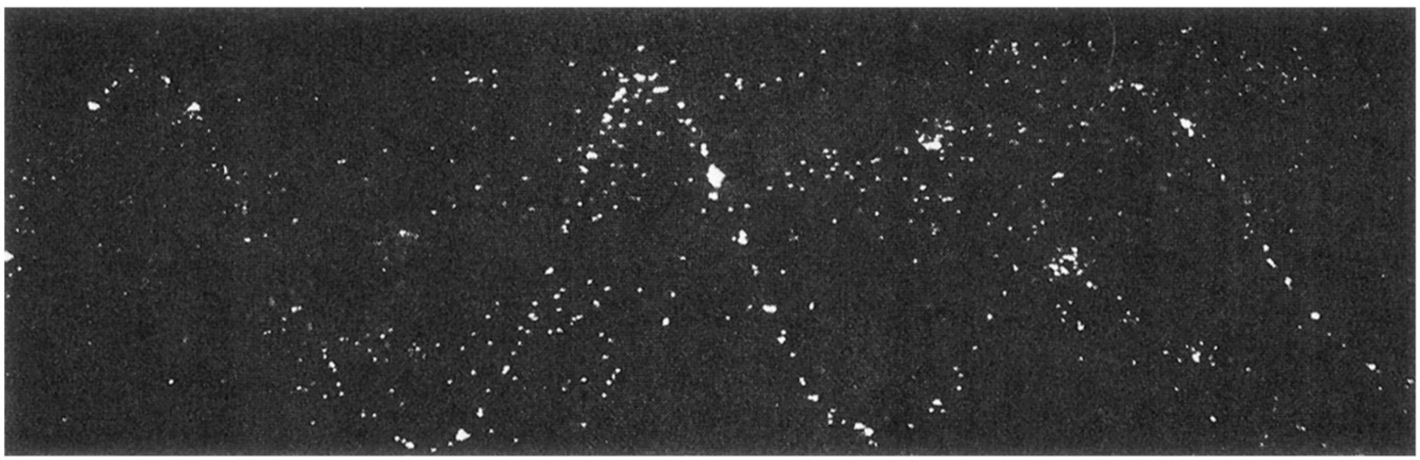
ω = Cyclotron frequency

Spiral Motion in a Magnetic Field

The component of the particle velocity in the direction of the B-field experiences no magnetic force.



(b)

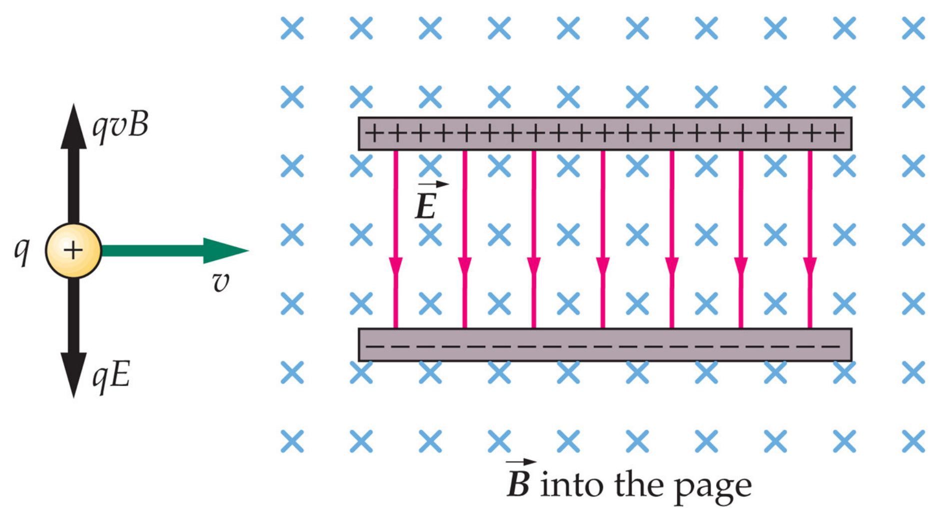


For a random velocity orientation the result is spiral motion.

Applications

Velocity Selector - Wein Filter

Objective: If the electric and magnetic forces are balanced (i.e. equal and opposite) then the charged particle's trajectory will be a straight line.



To simplify the design and the mathematics v, B and E are all perpendicular to each other.

Velocity Selector - Wein Filter

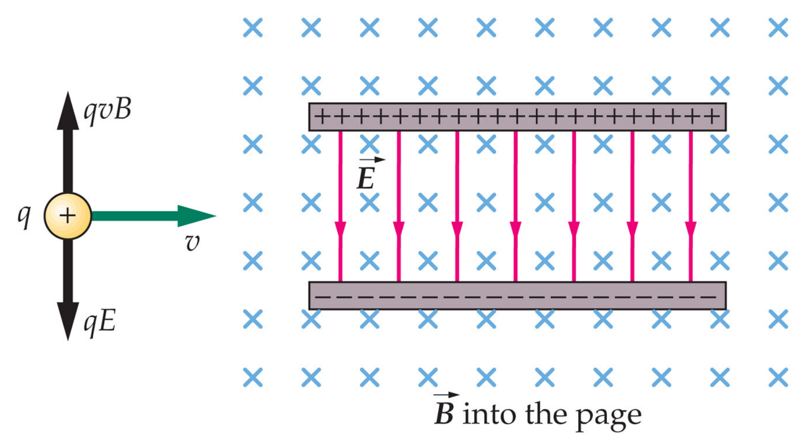
$$F_M = F_E$$

$$qvB = qE$$

$$v = \frac{E}{B}$$

To be useful the filter needs to be adjustable. Which is easier to adjust: electric field? magnetic field?

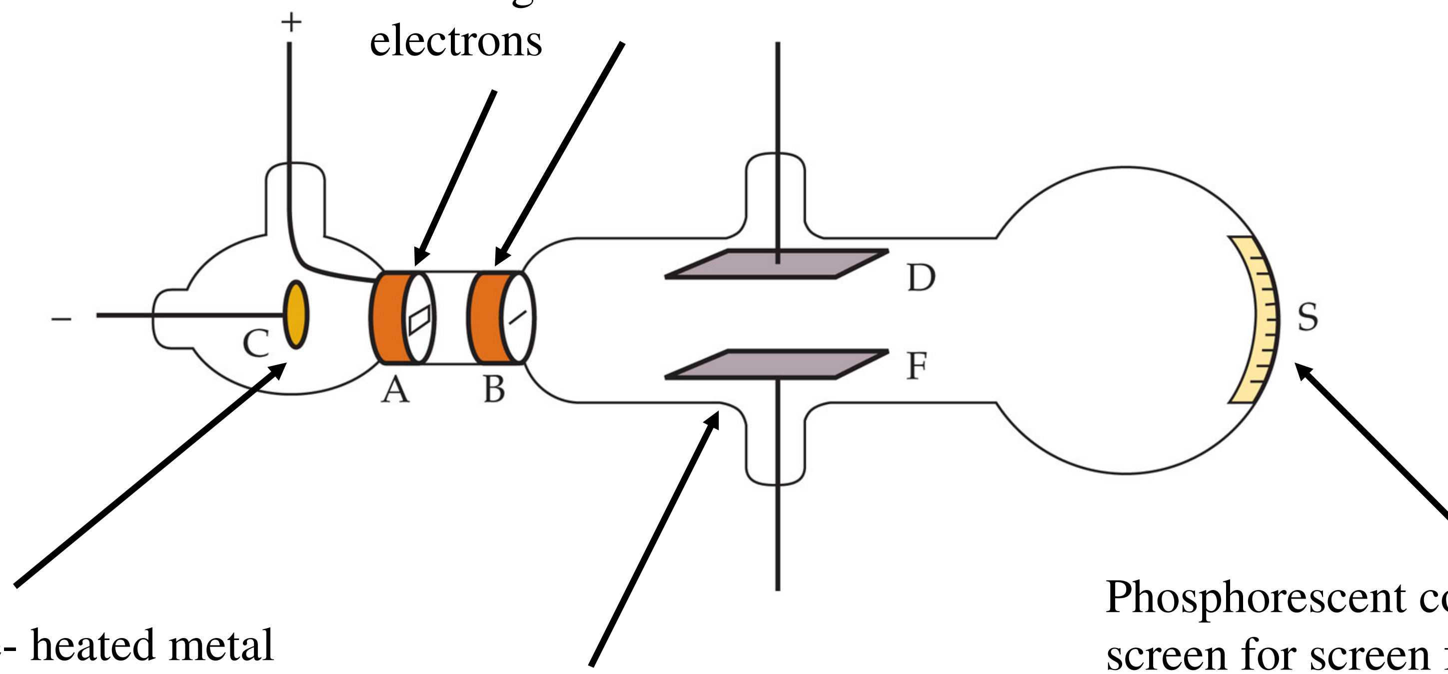
Answer: Since E is in the numerator adjusting this variable will yield linear operation. In addition, E can be changed by simply changing the voltage on the plates.



Apparatus works equally well for positive or negative charges

Apparatus for q/m Measurements

Electrodes for extracting and focusing the beam of electrons

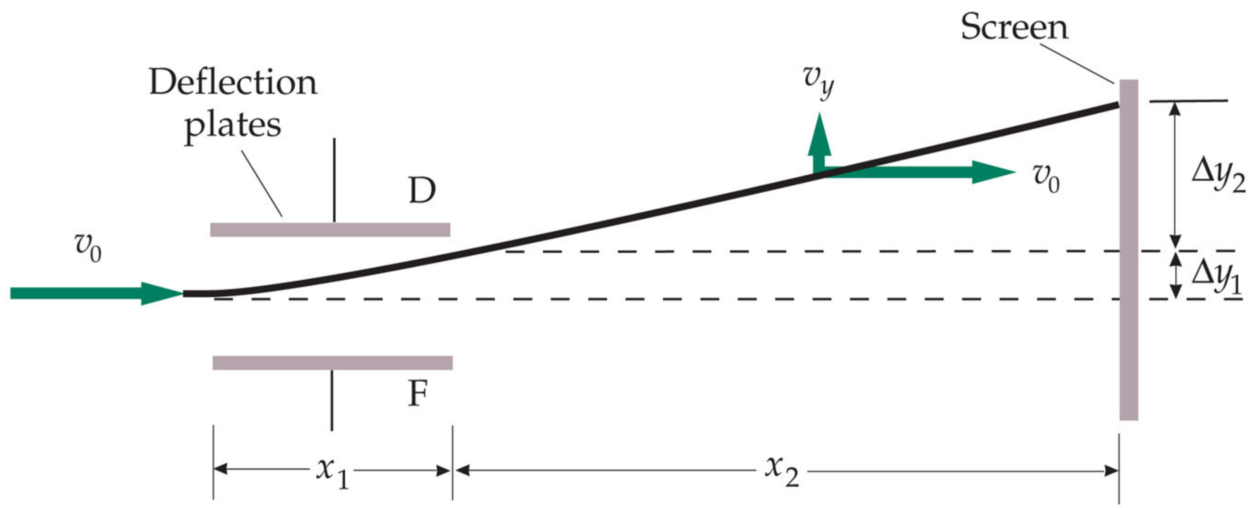


Cathode- heated metal source of electrons

Electrodes for deflecting the beam of electrons.

Phosphorescent coated screen for screen for observing electron arrival

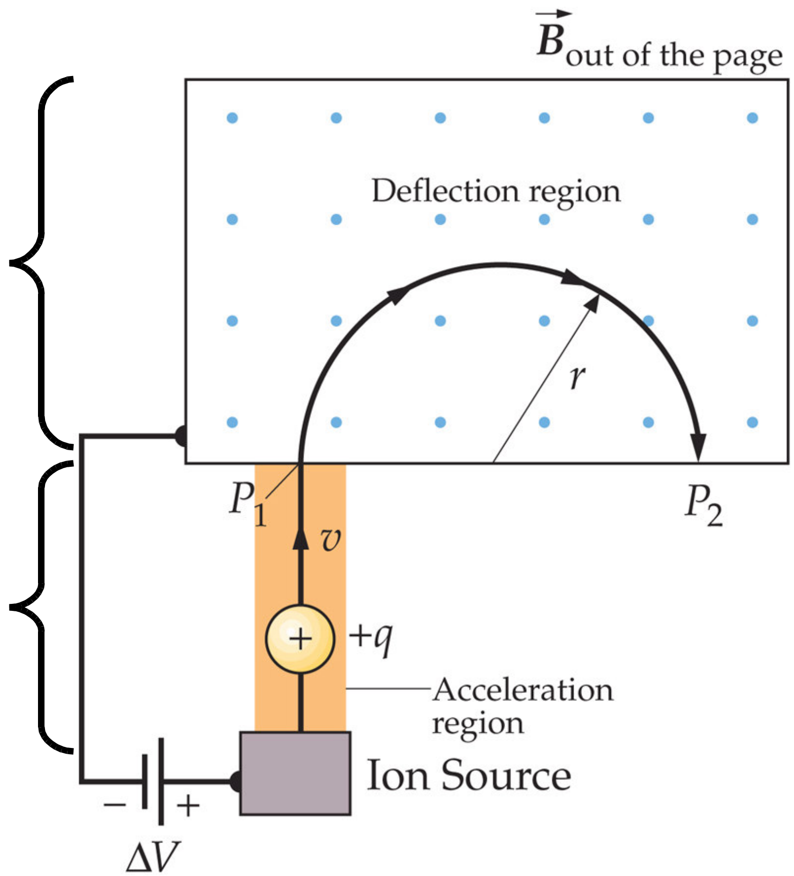
Geometry for Electron Measurements



The Mass Spectrometer

No change in kinetic energy

Increase in kinetic energy



The Mass Spectrometer

$$\Delta U_E = \Delta KE$$

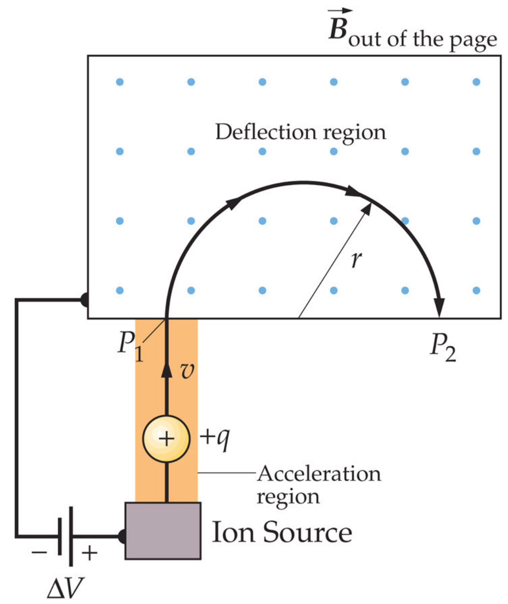
$$q\Delta V = \frac{1}{2}mv^2 \longleftarrow$$ Acceleration

$$F = qvB = \frac{mv^2}{r} \longleftarrow$$ Deflection

$$v^2 = \frac{r^2 q^2 B^2}{m^2}$$

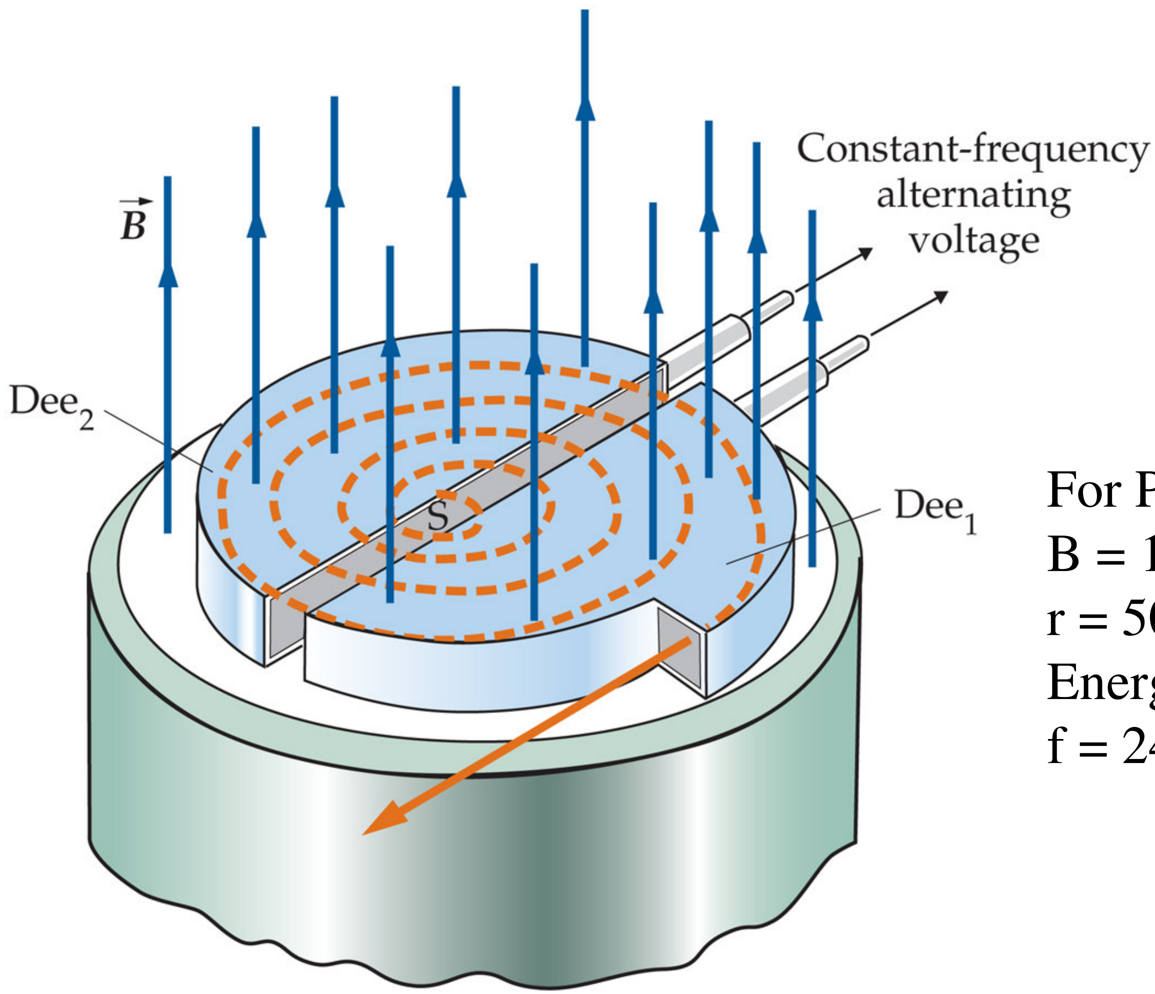
$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{r^2 q^2 B^2}{m^2}\right) = q\Delta V$$

$$\frac{q}{m} = \frac{2\Delta V}{B^2 r^2}$$



Caution: Always use is ΔV for the potential difference and v for the velocity inside the deflection region.

The Cyclotron

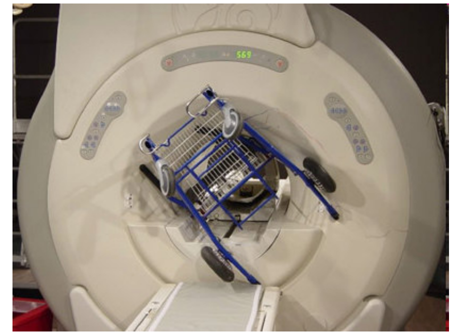


$$\omega = \frac{q}{m} B$$

$$r = \frac{mv}{qB}$$

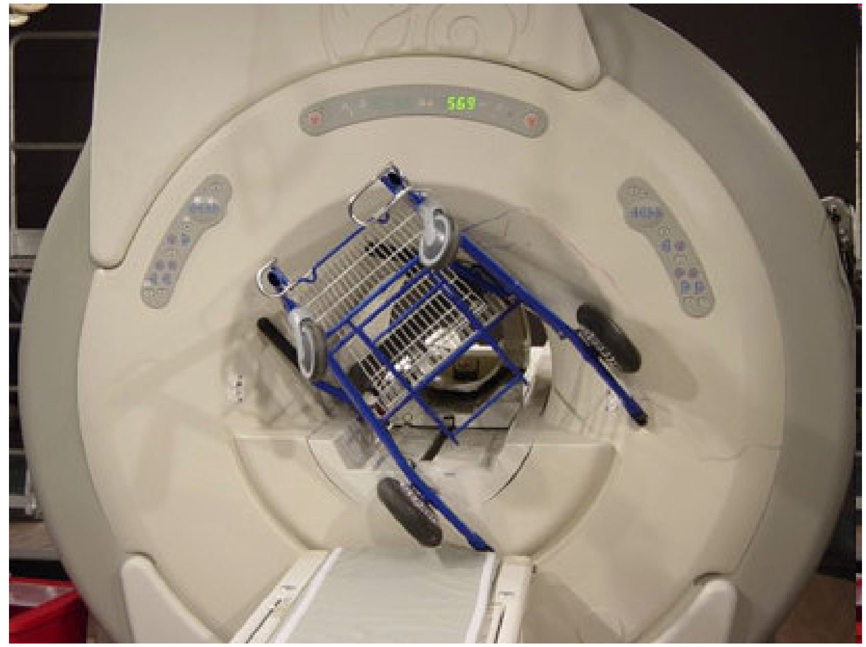
For Protons
B = 16,000 Gauss
r = 50 cm
Energy = 31 MeV
f = 24.3 MHz

MRI Machine Characteristics



- Several hundred miles of special superconducting wire windings.
- Approximately 400 amps flows through the windings to produce the magnetic field.
- MRI machines are made of superconducting wire, which is cooled to 4.2 K (-268º Celsius) using several thousand liters of liquid helium.
- Refilling the Helium in a quenched magnet costs between $10K and $20K.
- To achieve the required magnetic field strength, electrical current is sent through the windings. Due to the superconductive nature of the wire there is negligible power loss, so once at full field strength, the system is disconnected from the power source.
- An MRI machine IS ALWAYS ON. A strong magnetic field is *always* present near the machine.

Metal Hungry MRI Machines

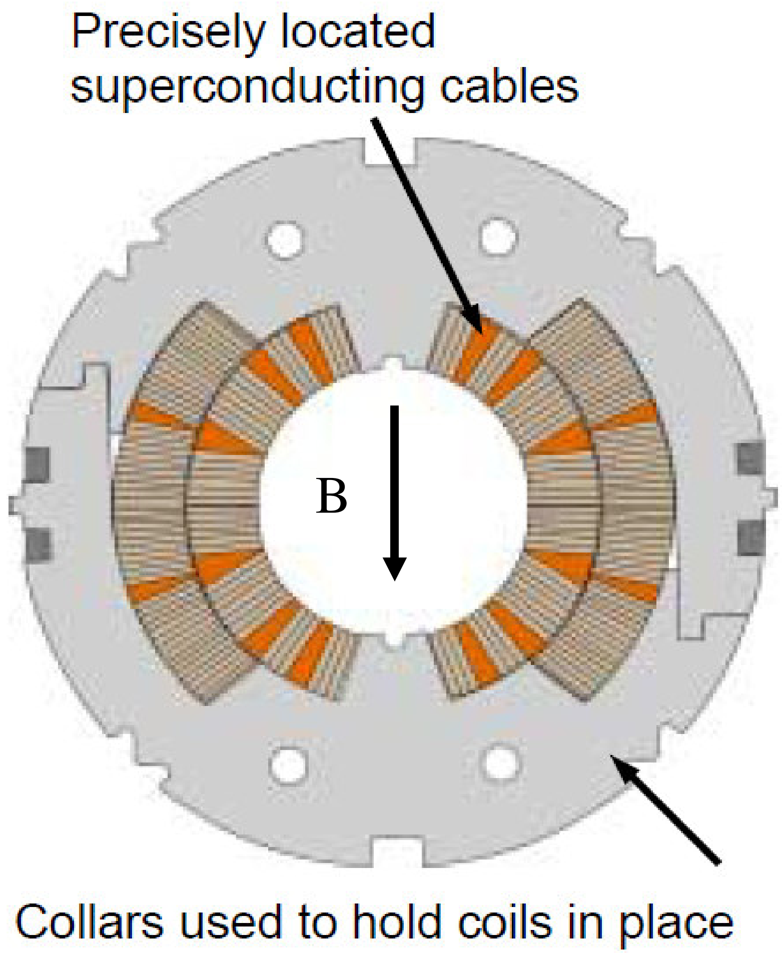


Typical Magnetic Field Strengths

1.5 T to 3.5T

15,000G to 35,000G

Superconducting Dipole Magnet



Superconducting Accelerator Magnets

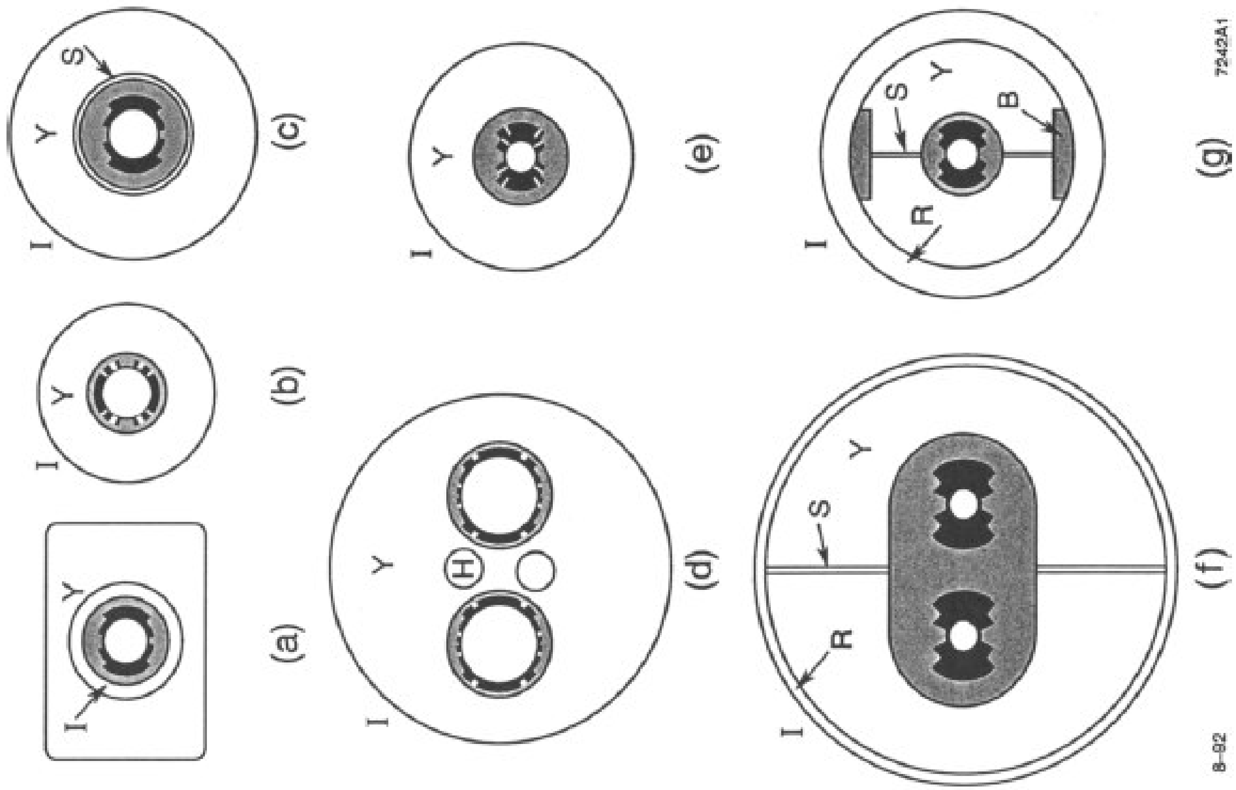


Figure 1. Magnet conceptual designs: (a) Tevatron; (b) RHIC; (c) HERA; (d) CBA Two-in-one (e) SSC; (f) LHC; (g) LBL's D19. Black areas are the coil cross sections, shaded areas are the collars or support spacers, I = thermal insulation, Y = yoke, S = space, B=block, R=ring.

SC Accelerator Magnet Parameters

Table 1. Parameters of accelerator magnets. I_q = approximate average conductor limited current; B_q = average short sample central field. For the inner layer: B_{mx} = local maximum field on conductors; n_s = number of strands; d_s = diameter of strands; R_{Cu} = copper-to-superconductor ratio; j_{Cu} = current density in nonmatrix copper; α = instability factor defined in text; and n_q = approximate average number of quenches to reach conductor limit.

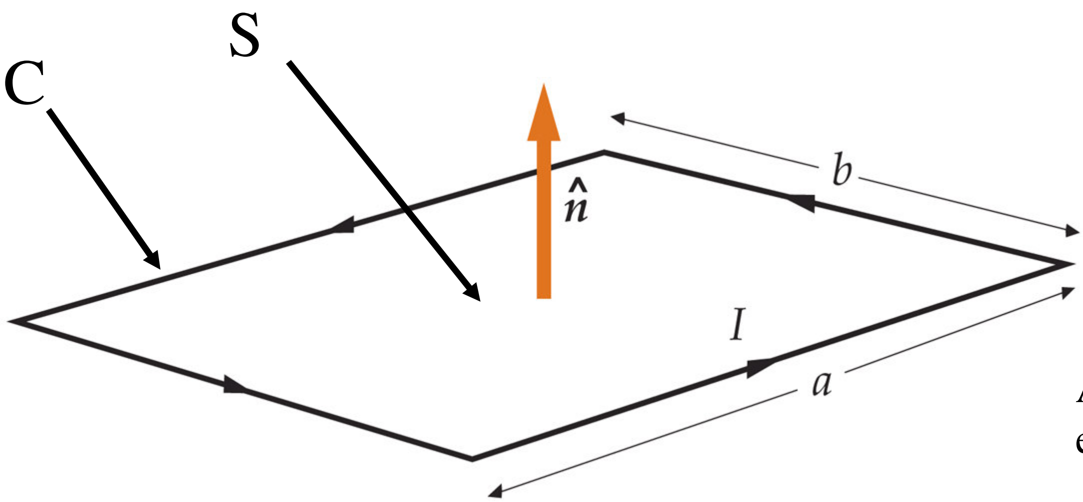
	len	bore	T	I_q	B_q	B_{mx}	n_s	d_s	R_{Cu}	j_{Cu}	α	n_q
	m	cm	deg	A	T	T	mm	d_s	A/mm^2	kA^2/mm^3		
Tevatron	6.1	7.6	4.8	4840	4.8	5.4	23	0.68	1.8	1248	.12	4
HERA	8.8	7.5	4.5	6400	5.9	6.2	24	0.84	1.8	1036	.1	0.5
Isabelle	4.5	13.0	4.6	4625	5.0	5.7	96	0.30	1.2	2142	.29*	40
CBA	4.5	13.0	4.6	4100	5.3	5.5	23	0.68	1.8	1057	.09	0.5
RHIC	9.5	8.0	4.6	7500	4.6	5.2	30	0.65	2.2	1427	.18	1.5
SSC4	15.2	4.0	4.4	6700	6.7	7.0	23	0.81	1.3	1633	.19	3
SSC5	15.2	5.0	4.4	7300	7.3	7.7	30	0.81	1.5	1186	.11	1
LHC	1.0	5.0	1.8	15090	10.0	10.3	26	1.29	1.6	1050	.15	40
LHC	1.0	5.0	4.2	11930	8.1	8.4	26	1.29	1.6	830	.09	5
D19	1.0	5.0	1.8	9800	10.1	10.6	30	0.81	1.5	1593	.2	8
D19	1.0	5.0	4.2	6910	7.6	8.0	30	0.81	1.5	1123	.1	1
SSC quad	5.2	4.0	4.4	8400	-	6.5	30	0.65	1.8	1829	.25	8

* Since the Isabelle braid was solder filled, α was calculated using the cable thickness (.8 mm) in place of the strand diameter d_s for the surface-to-volume ratio.

The Current Loop and Its Interactions with a Uniform Magnetic Field

The Current Loop

The current loop is a closed mathematical curve, C. It can be traversed in two directions: clockwise or counter clockwise. The curve encloses a two-sided surface S. The path shown below traverses the curve in the counter clockwise direction, when viewed from above. For this traversal direction the normal vector associated with the surface points up.

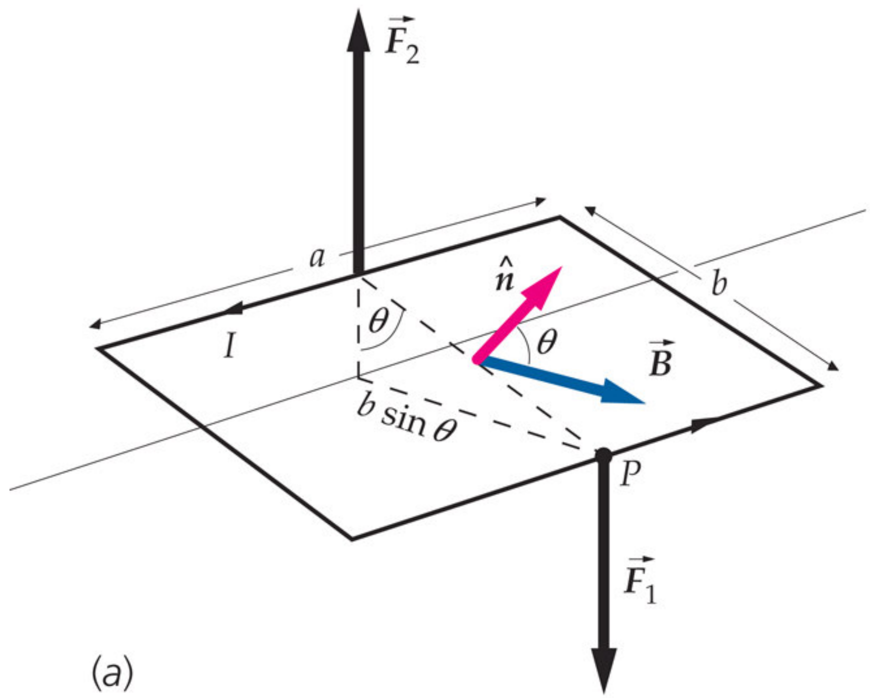


A right handed rule can be employed to determine the direction of the surface normal.

Magnetic Forces on the Current Loop

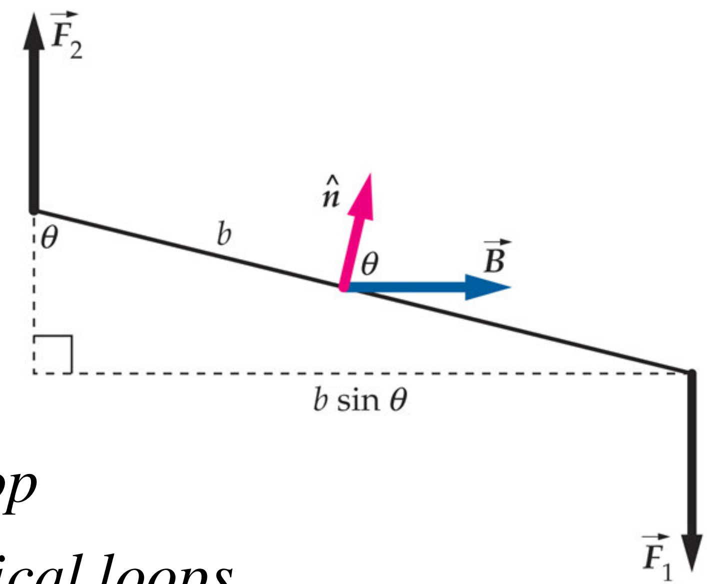
The current in the sides of the loop of length “b” are parallel to the direction of the magnetic field.

Hence they experience no magnetic force.



Looking Along the Current Loop, Perpendicular to the Magnetic Field

Both wire segments of length "a" experience a magnetic forces of equal and opposite magnitude.



$$F_1 = F_2 = IaB$$

$$\tau = F_2 b\sin\theta = IaBb\sin\theta = IAB\sin\theta$$

$$\textit{where } A = ab \textit{ is the area of the loop}$$

$$\tau = NIAB\sin\theta \quad \textit{in the case of N identical loops}$$

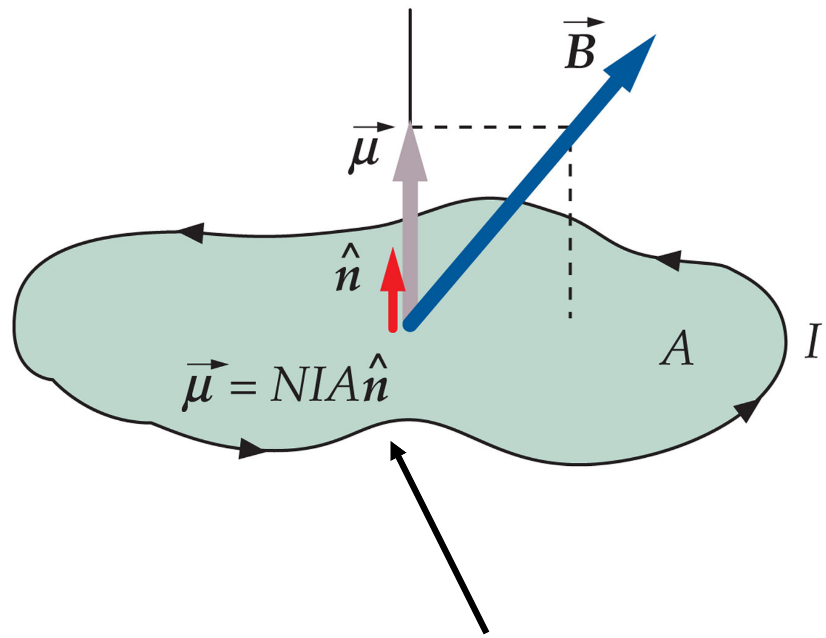
There is no translation of the current loop - only a torque, or twisting motion – because the magnetic field is uniform.

The Magnetic Moment

$\vec{\mu} = NIA\hat{n} \longleftarrow$ Valid for any shape loop

$\tau = \mu B sin\theta$

$\vec{\tau} = \vec{\mu} \times \vec{B}$



A General Current Loop

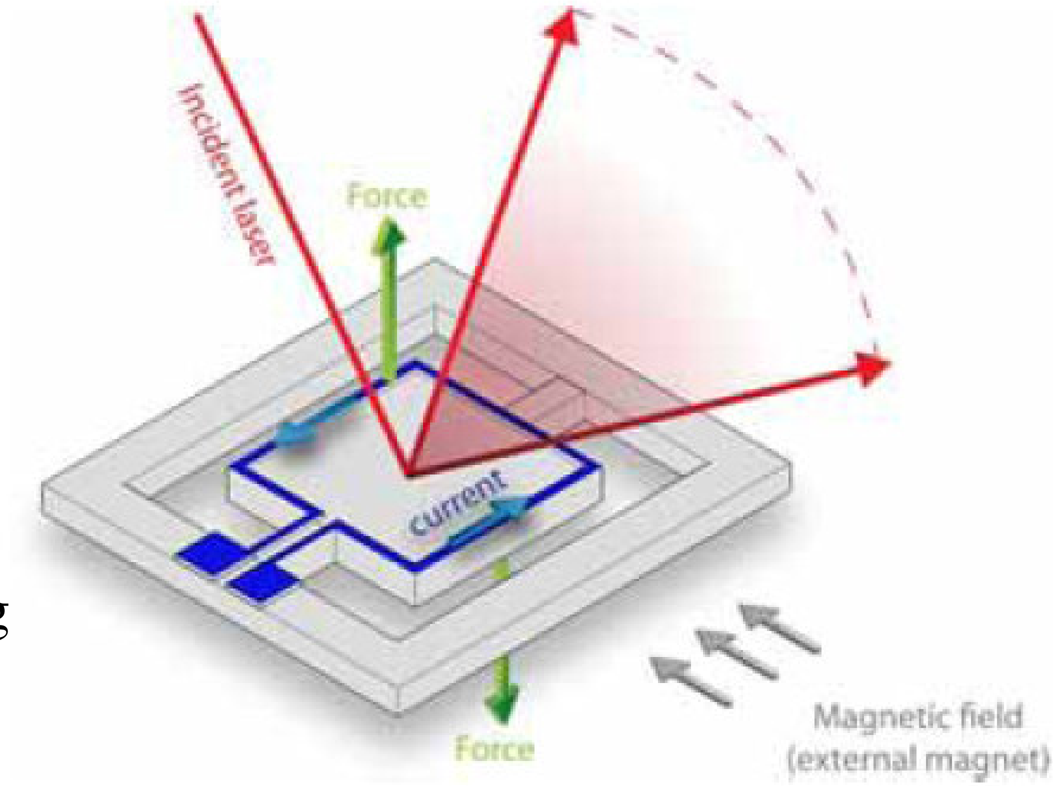
The net effect on a current loop in a uniform magnetic field is that it experiences a torque.

The magnetic moment wants to align itself with the direction of the magnetic field.

Commercial Application – Current Loop

APPLICATIONS

- Barcode scanners
- Laser printer
- Endoscopy/confocal microscopy
- Medical Imaging
- Optical Sensing
- Laser pointing – Steering of laser beams

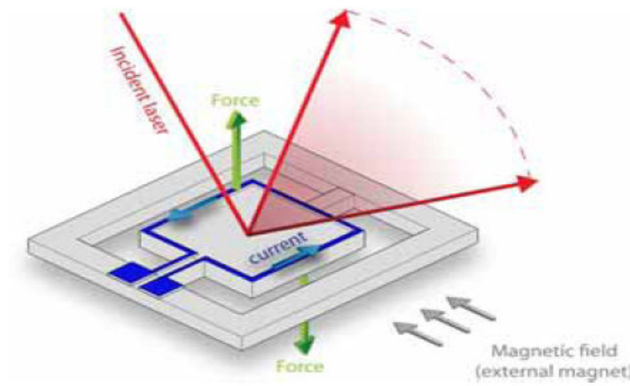


Lemoptix MEMS Scanning Micromirror Technology

Commercial Application – Current Loop

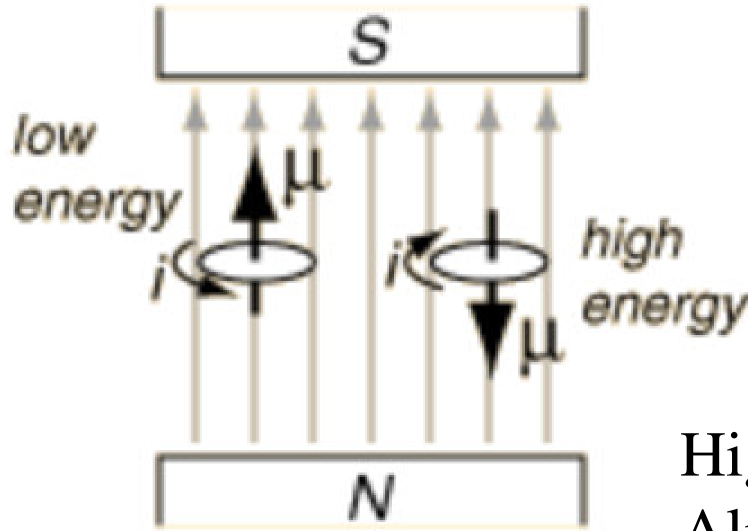
Achievable performance range

Actuation	1D (1 axis) or 2D (2 axis)
Micromirror size	Up to 6mm
Scanning angle	Up to 60° (optical)
Light reflection	> 90% in visible and IR
Shock resistance	> 2000g
Actuation voltage	< 5V
Resonant Frequency	From 500 Hz to 70 kHz
Static actuation	From fix steps to 400Hz
Consumption	From 0.1mW to 100mW
Chip size	Down to 3mm x 3.5mm



Current Loop in a Magnetic Field

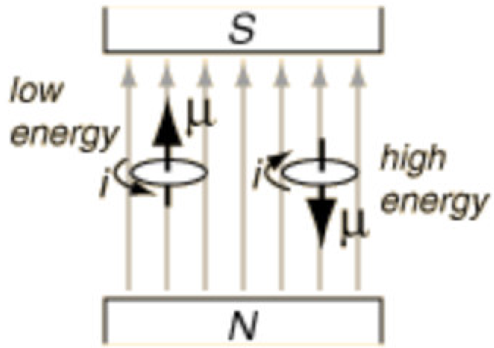
Low Energy:
Aligned with field

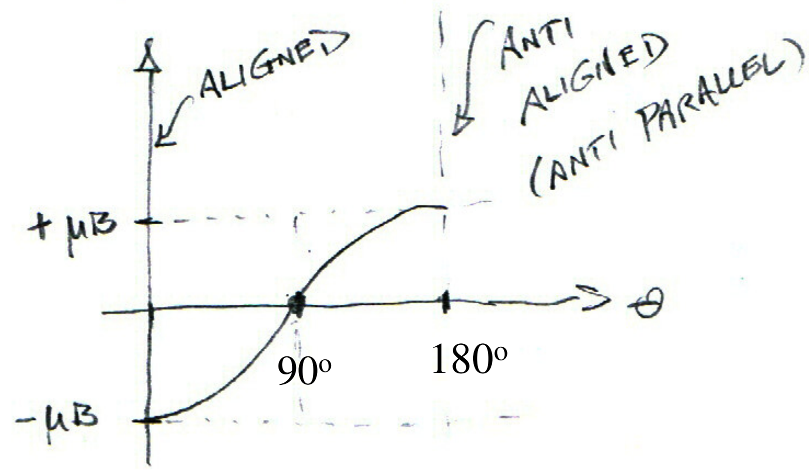


High Energy:
Aligned against
the field.

Work has to be done to reorient the magnetic moment from its aligned orientation to an anti-aligned orientation.

Current Loop in a Magnetic Field





$$dW = +\vec{\tau} \bullet d\vec{\theta} = +\mu B sin\theta d\theta$$

$$\Delta U = \int dW = \int_0^{\theta} \mu B sin\theta d\theta$$

$$\Delta U = \mu B \int_1^{cos\theta} (-d(cos\theta)) = -\mu B(cos\theta - 1)$$

Current Loop in a Magnetic Field

$$\Delta U = -\mu B(\cos\theta - 1)$$

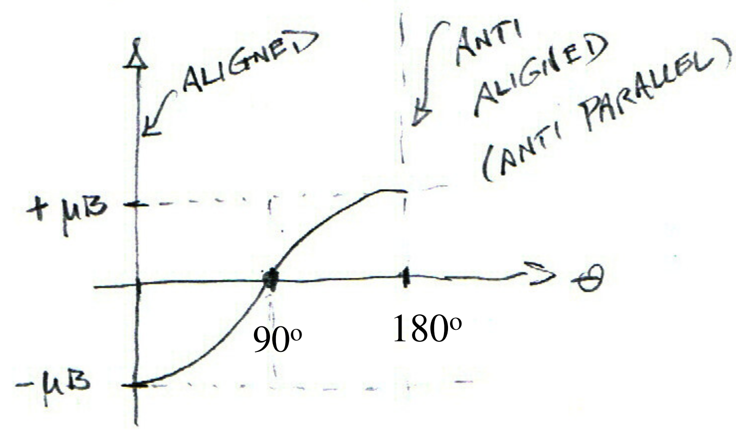
$$From \quad \theta = 0 \; to \; \theta = \pi$$

$$\Delta U = 2\mu B$$

$$U = -\mu B(\cos\theta - 1) + U_0$$

Anticipating Quantum Mechanics

$$we \; define \; U(\theta = 90^0) = 0$$



$$U(90^0) = -\mu B(\cos(90^0) - 1) + U_0 = 0$$

$$\therefore U_0 = -\mu B$$

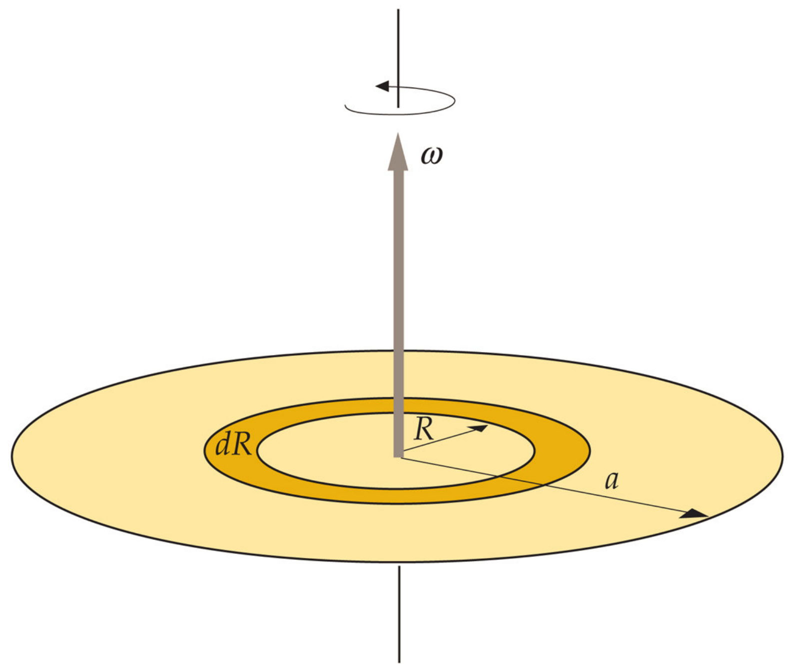
$$U(\theta) = -\mu B\cos\theta$$

$$U(\theta) = -\vec{\mu} \bullet \vec{B}$$

$$U = -\vec{\mu} \bullet \vec{B}$$

Magnetic Moment of a Rotating Charged Disk

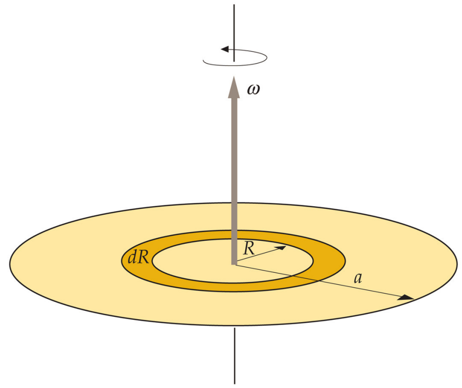
Magnetic Moment of a Rotating Charged Disk



A rotating charged disk can be treated as a collection of concentric current loops. Each loop has a magnetic moment. The sum of all these magnetic moments is the magnetic moment of the disk.

Rotating Charged Disk

The infinitesimal magnetic moment $d\mu$ is due to the rotation of the charge in infinitesimal area $2\pi RdR$. The area of this loop is πR^2



$$d\mu = \pi R^2 dI$$

$$dI = \frac{dq}{T} = \frac{\omega}{2\pi} dq = \frac{\omega}{2\pi} \sigma dA$$

$$dA = 2\pi R dR$$

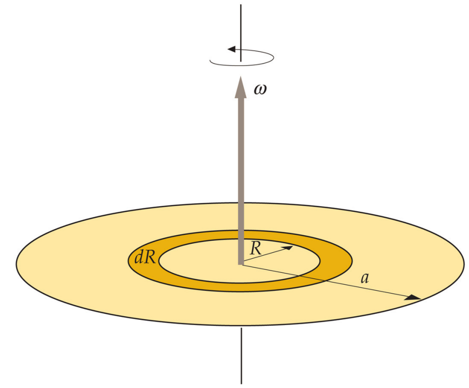
$$dI = \frac{\omega}{2\pi} \sigma 2\pi R dR = \omega\sigma R dR$$

Rotating Charged Disk

$$d\mu = \pi R^2 \omega\sigma R dR = \pi\omega\sigma R^3 dR$$

$$\mu = \int_0^a \pi\omega\sigma R^3 dR = \pi\omega\sigma \int_0^a R^3 dR$$

$$\mu = \frac{1}{4}\pi\omega\sigma a^4 = \frac{\omega a^2}{4}\sigma\pi a^2 = \frac{\omega a^2}{4} Q$$



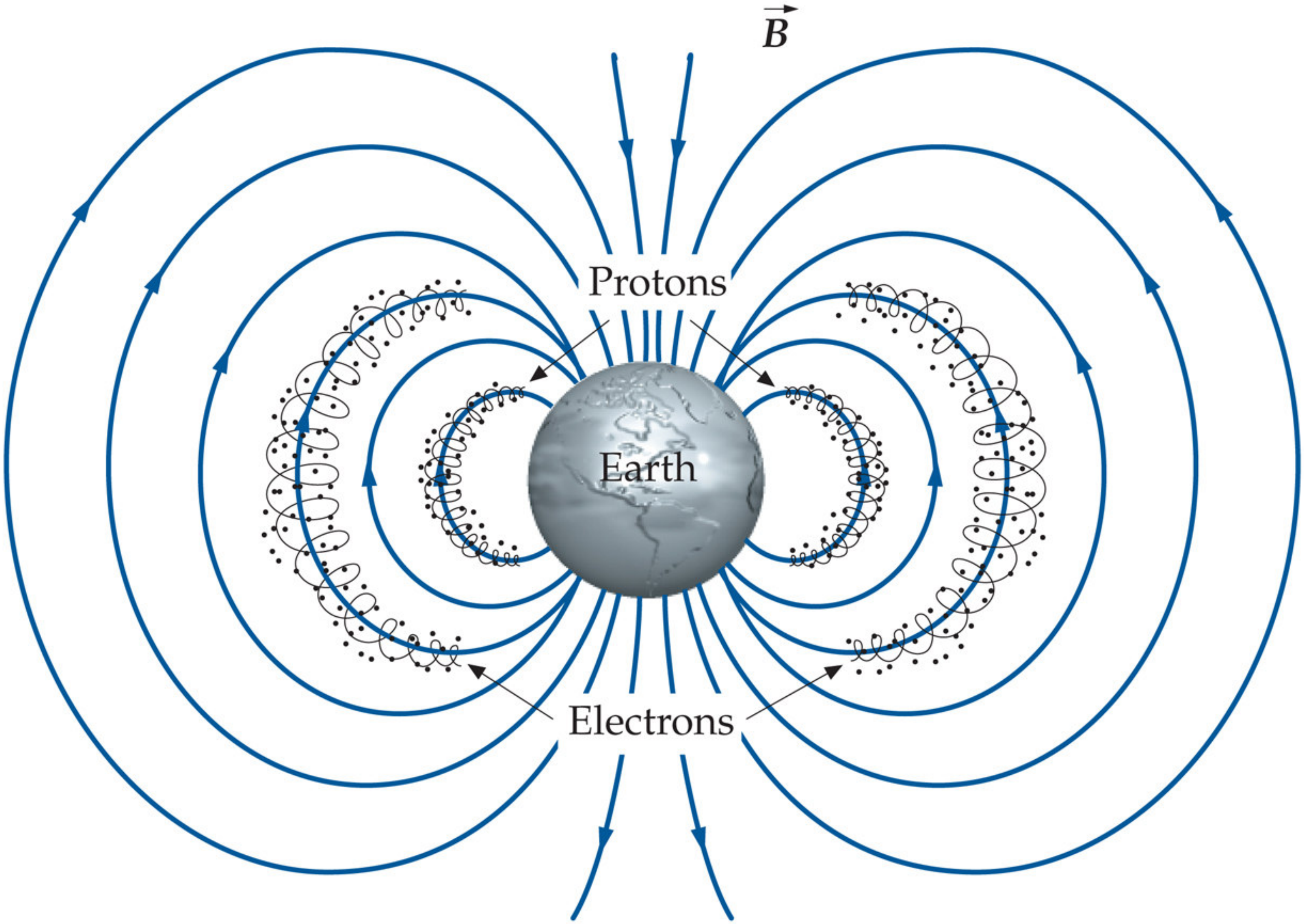
This can be cast into a more general form by remembering

$$\vec{L} = I\vec{\omega} = \frac{1}{2}ma^2\vec{\omega}$$

$$\vec{\mu} = \frac{Q\vec{\omega}a^2}{4} = \frac{Q}{2m}\frac{ma^2}{2}\vec{\omega} = \frac{Q}{2m}\vec{L}$$

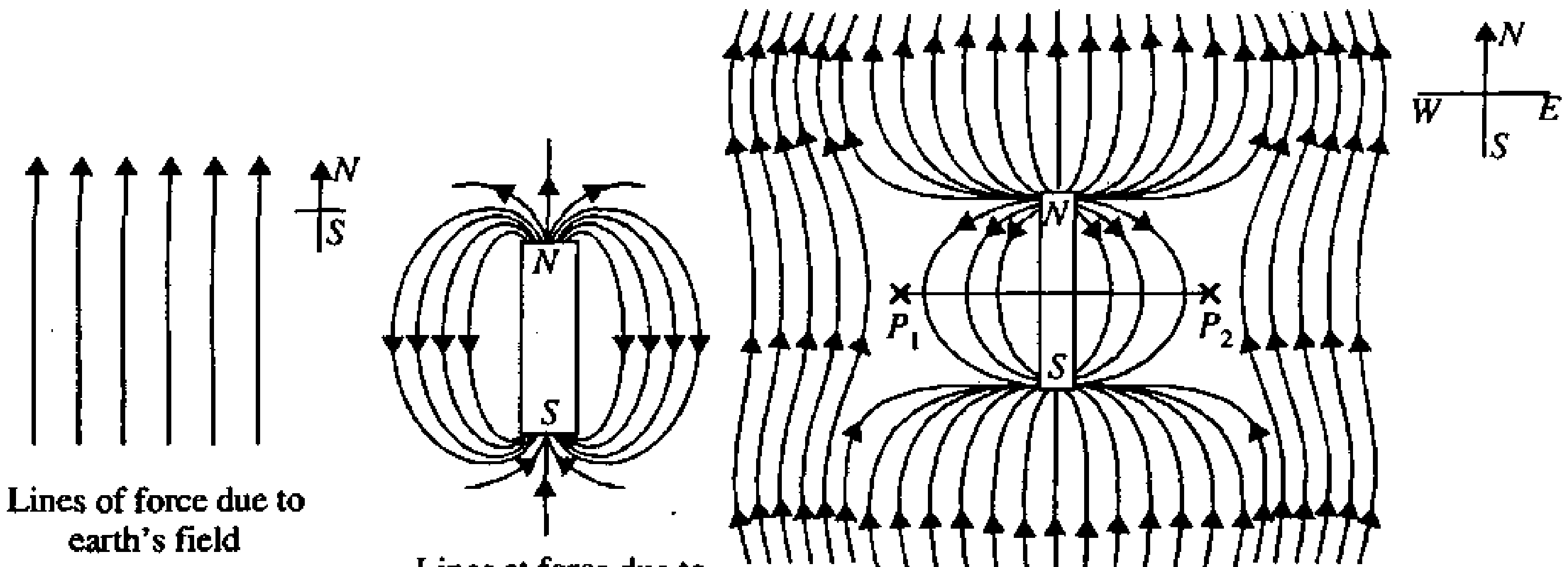
The “I” is the moment of inertia. The “Q/m” ratio is not a real charge to mass ratio.

Extra Slides



$\vec{B}$
Protons
Earth
Electrons

Superposition of Magnetic Fields



Lines of force due to earth's field

Lines at force due to a bar magnet alone

(a) (b) (c)

Fig. 13.10

Magnetic & Geographic Confusion