Chapter 29

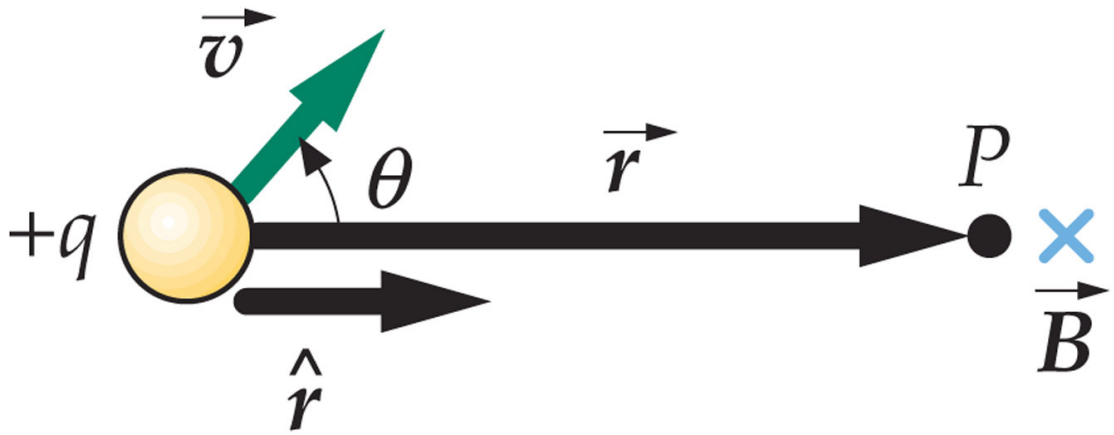
Sources of the Magnetic Field

Sources of the Magnetic Field

- Magnetic Field of Moving Point Charges
- Magnetic Field of Currents - Biot-Savart Law
- Gauss's Law for Magnetism
- Ampere's Law
- Magnetism in Matter

Magnetic Field Caused by a Moving Charge

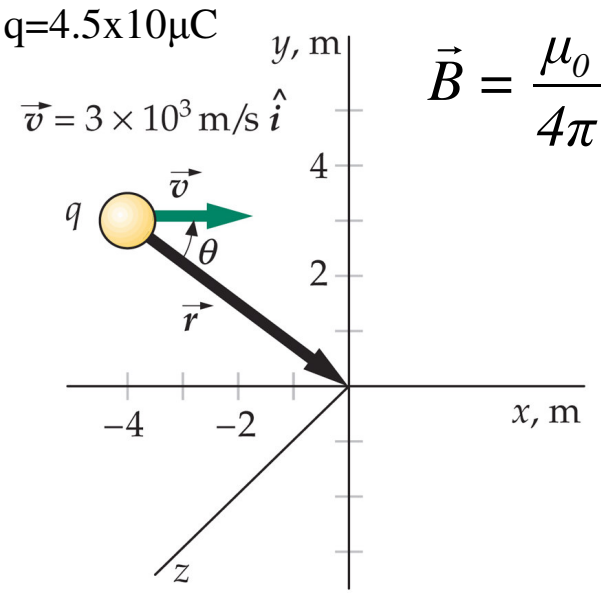
$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \hat{r}}{r^2}$$



This formula gives the magnetic field at the location P <u>*at the point in time*</u> when the charges particle is a distance r away.

Moving Charge Magnetic Field Example

q=4.5x10μC



$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \hat{r}}{r^2}$$

$$\vec{r} = 4\hat{i} - 3\hat{j}$$

$$r = \sqrt{4^2 + 3^2} = 5$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}$$

$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{(3 \times 10^3 \hat{i}) \times (0.8\hat{i} - 0.6\hat{j})}{5^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{(3 \times 10^3)(-0.6) \times (\hat{i} \times \hat{j})}{5^2}$$

$$\vec{B} = -\frac{\mu_0}{4\pi} q \frac{(1.8 \times 10^3)\hat{k}}{25}$$

$$\vec{B} = -10^{-7}(4.5 \times 10^{-6}) \frac{(1.8 \times 10^3)\hat{k}}{25}$$

$$\vec{B} = -\frac{(1.8)(4.5) \times 10^{-10}}{25} \hat{k}$$

$$\vec{B} = -3.24 \times 10^{-11}\ T\ \hat{k}$$

The example in the book is incorrect. The value of q was changed from nC to μC in going to the 6th Edition but they kept the answer from the 5th Edition.

Magnetic Units

The permeability is a property of matter related to the ease with which the substance can be magnetized.

μ_0 is the permeability of free space.

$$\mu_0 = 4\pi 10^{-7}\ \text{T-m/A} = 4\pi 10^{-7}\ \text{N/A}^2$$

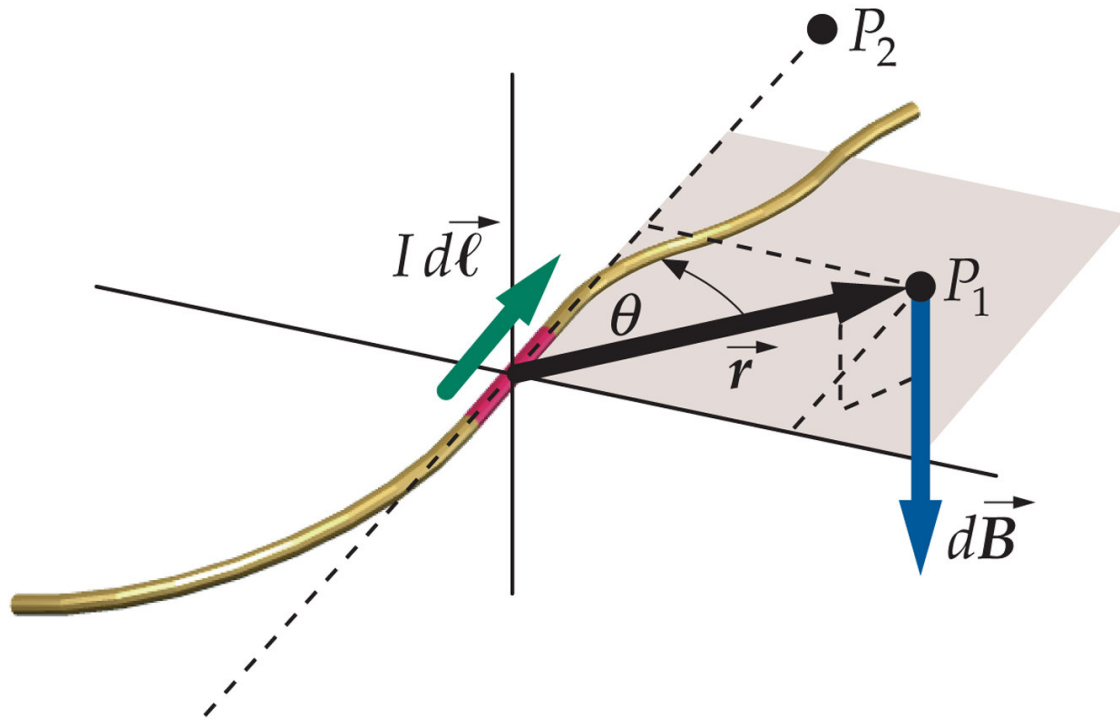
$$\frac{\mu_0}{4\pi} = 10^{-7}\ \frac{N}{A^2}$$

This is so easy to remember that we will try to keep this combination of factors together whenever possible.

Magnetic Field of Currents - Biot-Savart Law

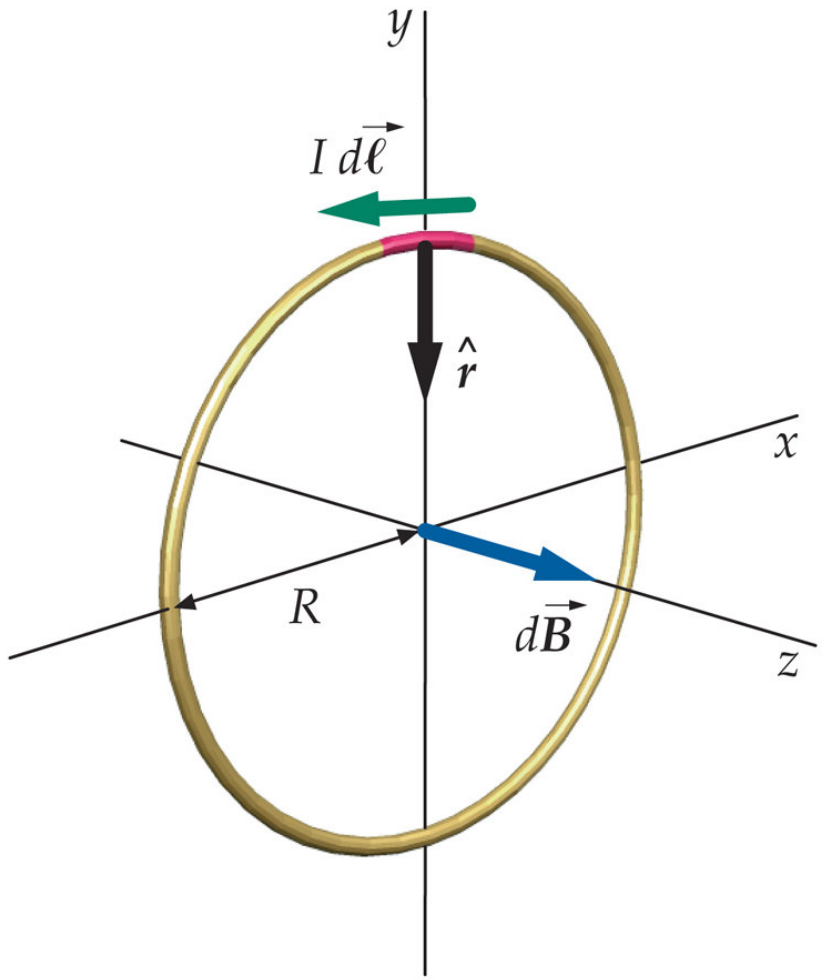
$$q\vec{v} \;\Rightarrow\; Id\vec{L}$$

$$\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{L} \times \hat{r}}{r^2}$$



This is a steady state field because even though there is charge movement the current is constant in time.

Ring Current Magnetic Field at Center



Ring Current Magnetic Field at Center

$$\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{L} \times \hat{r}}{r^2}$$

$$d\vec{L} \times \hat{r} = dL sin\theta \hat{k}$$

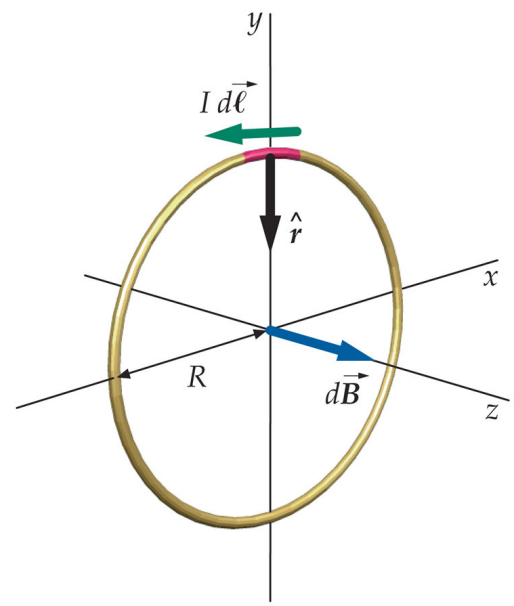
$$\theta = 90^0 \Rightarrow sin\theta = 1$$

$$d\vec{L} \times \hat{r} = dL\hat{k}$$

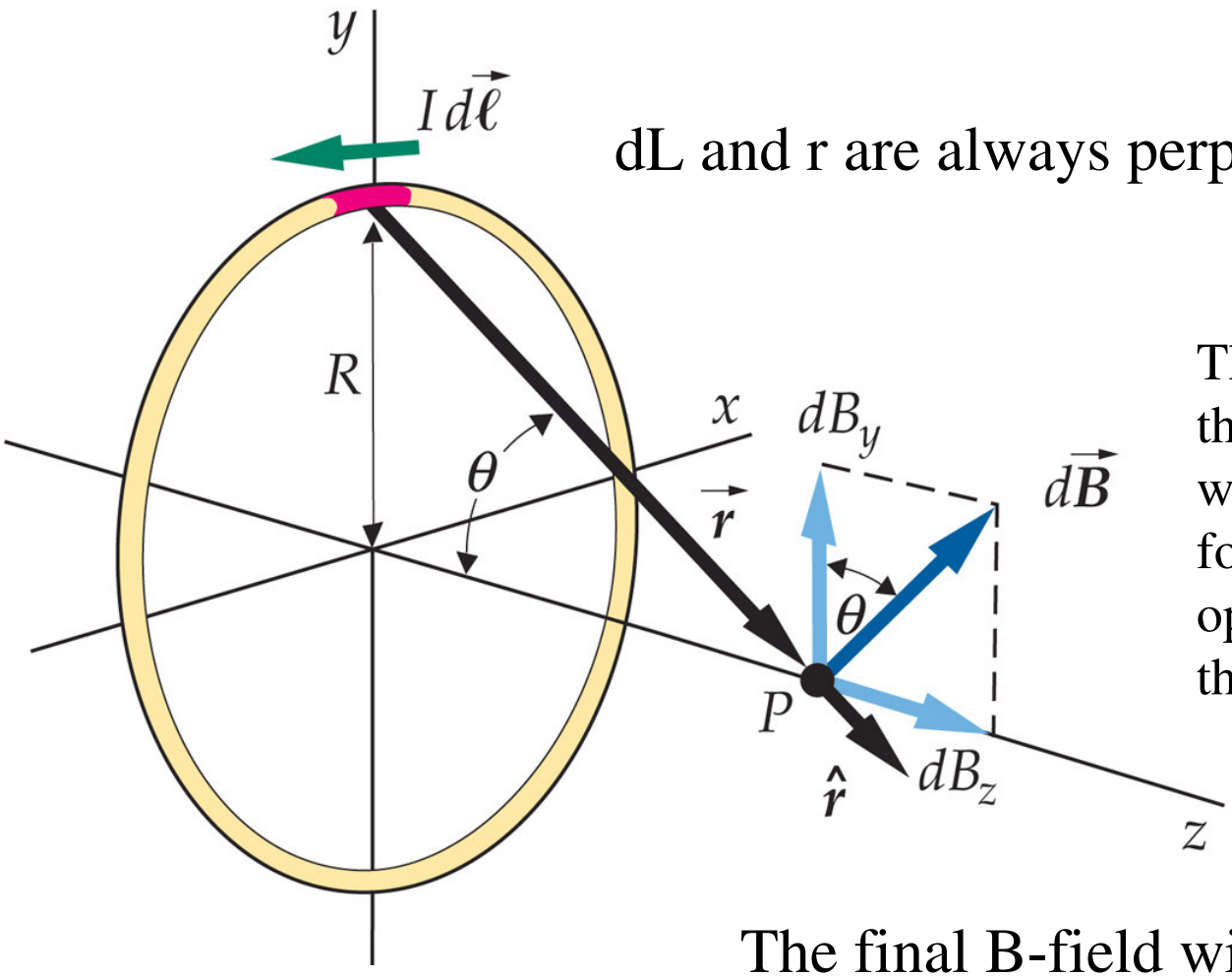
$$dB_z \hat{k} = \frac{\mu_0}{4\pi} \frac{IdLsin\theta}{R^2} \hat{k}$$

$$B_z = \frac{\mu_0}{4\pi} \frac{I}{R^2} \oint dL$$

$$B_z = \frac{\mu_0}{4\pi} \frac{I}{R^2} 2\pi R = \frac{\mu_0 I}{2R}$$



Ring Current Magnetic Field on Axis



dL and r are always perpendicular

The y-components of the magnetic field will cancel in pairs for diametrically opposed segments of the ring

The final B-field will be B_z

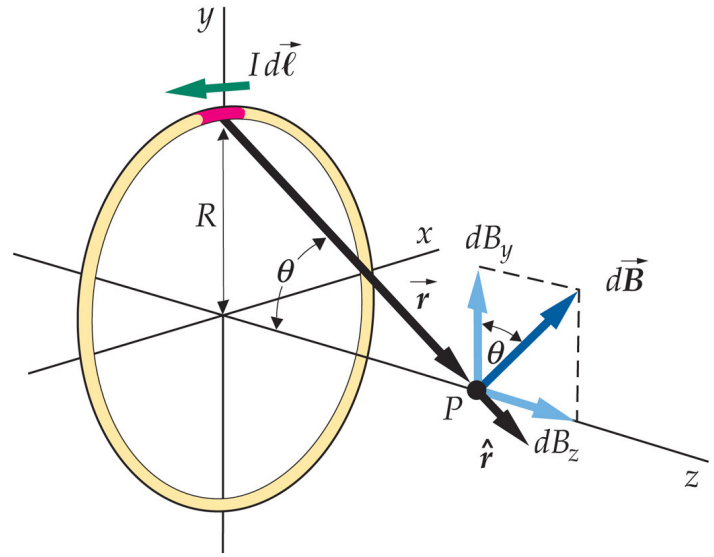
Ring Current Magnetic Field on Axis

$$\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{L} \times \hat{r}}{r^2}$$

$$\left|d\vec{B}\right| = dB = \frac{\mu_0}{4\pi} \frac{I\left|d\vec{L} \times \hat{r}\right|}{r^2} = \frac{\mu_0}{4\pi} \frac{IdL}{\left(z^2 + R^2\right)}$$

$$dB_z = dB\sin\theta = \frac{\mu_0}{4\pi} \frac{IdL}{\left(z^2 + R^2\right)} \left(\frac{R}{\sqrt{z^2 + R^2}} \right)$$

$$dB_z = \frac{\mu_0}{4\pi} \frac{IRdL}{\left(z^2 + R^2\right)^{3/2}}$$



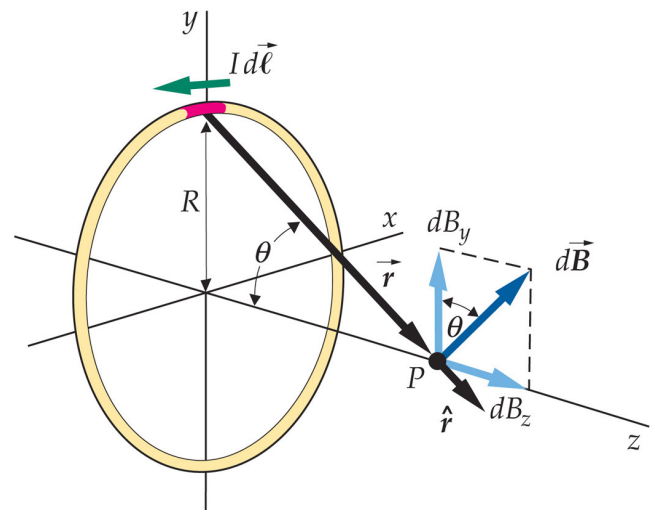
$$B_z = \oint dB_z = \oint \frac{\mu_0}{4\pi} \frac{IRdL}{\left(z^2 + R^2\right)^{3/2}} = \frac{\mu_0}{4\pi} \frac{IR}{\left(z^2 + R^2\right)^{3/2}} \oint dL$$

$$B_z = \frac{\mu_0}{4\pi} \frac{2\pi R^2 I}{\left(z^2 + R^2\right)^{3/2}}$$

Notice that the field is proportional to I and that the factor $\mu_0/4\pi$ is kept together despite the 2π above.

Ring Current Magnetic Field on Axis

$$B_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{\left(z^2 + R^2\right)^{3/2}}$$



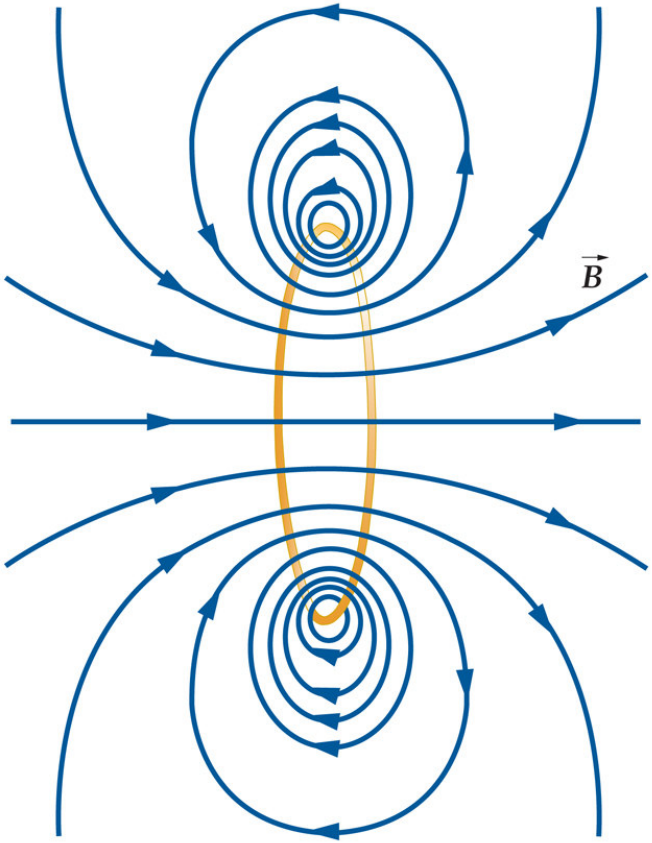
Far away from the loop the field takes on the form of a dipole field.

Z >> R

$$B_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{z^3} = \frac{\mu_0}{4\pi}\frac{2\mu}{z^3}$$

where $\mu = \pi R^2 I$ *is the magnetic moment*

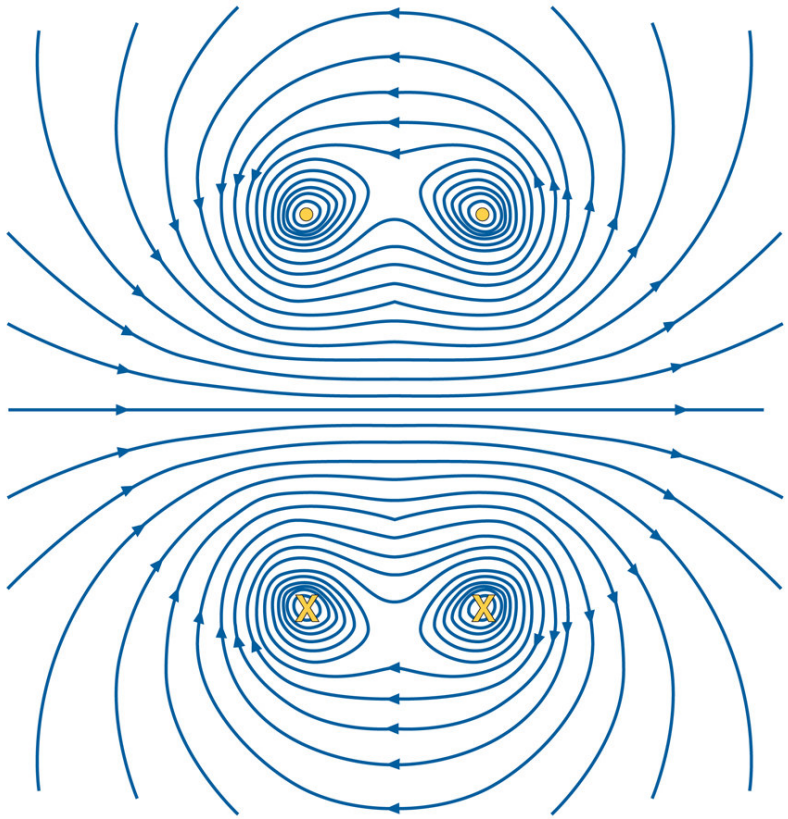
Ring Current Magnetic Field



(a)

(b)

Magnetic Field of Closely Spaced Rings

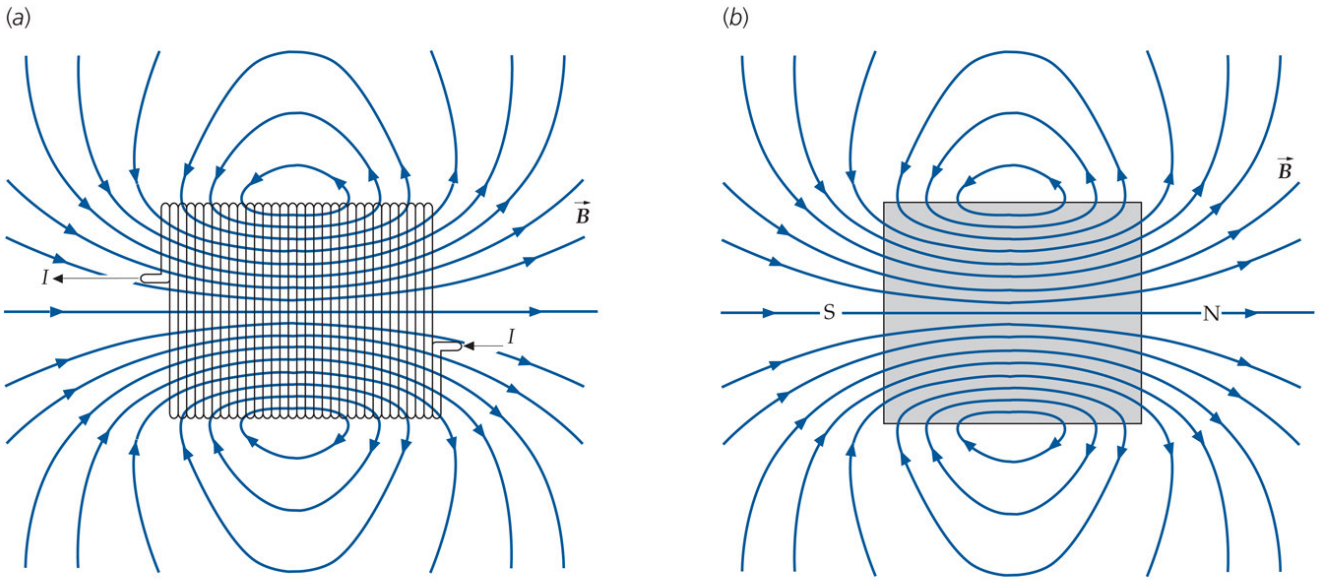


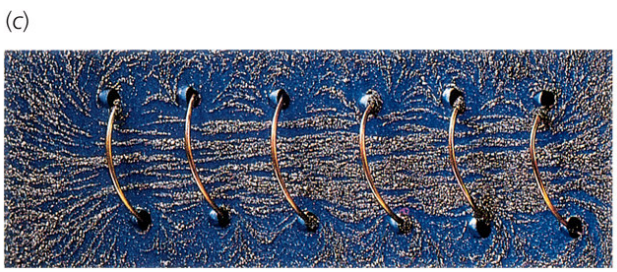
With the addition of a second current loop the magnetic field is becoming more uniform in the region of the center of the loops.

Magnetic Solenoid & Bar Magnet

Current Loops

Bar Magnet

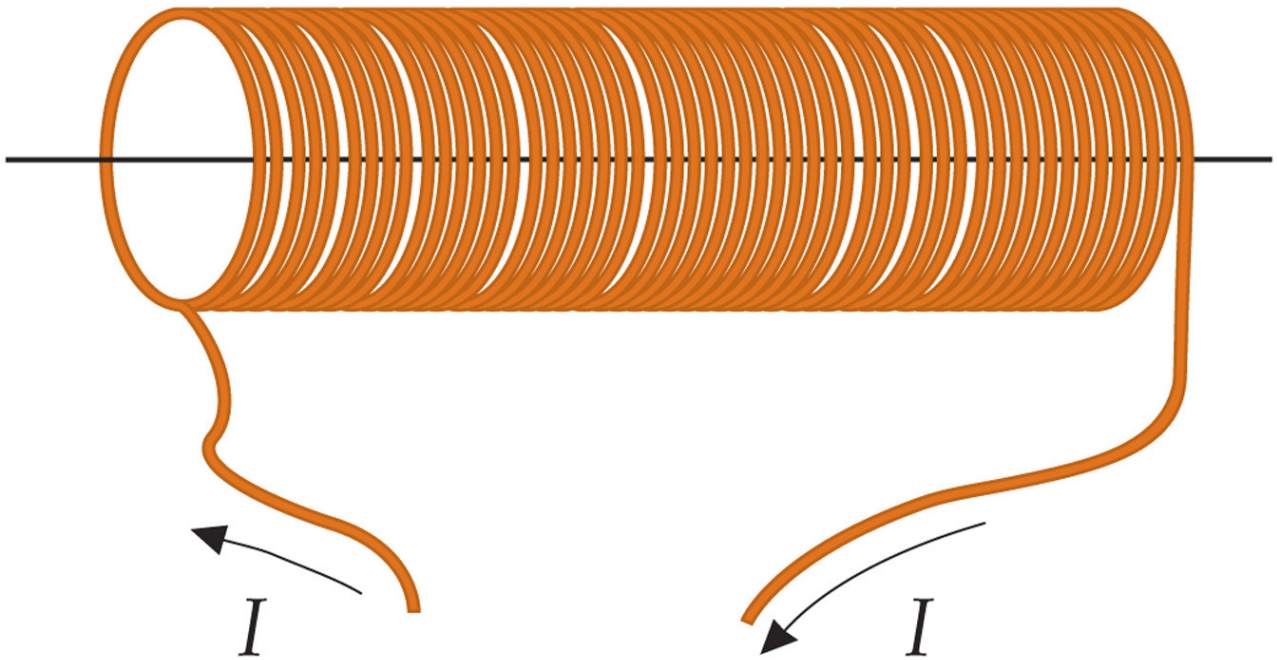




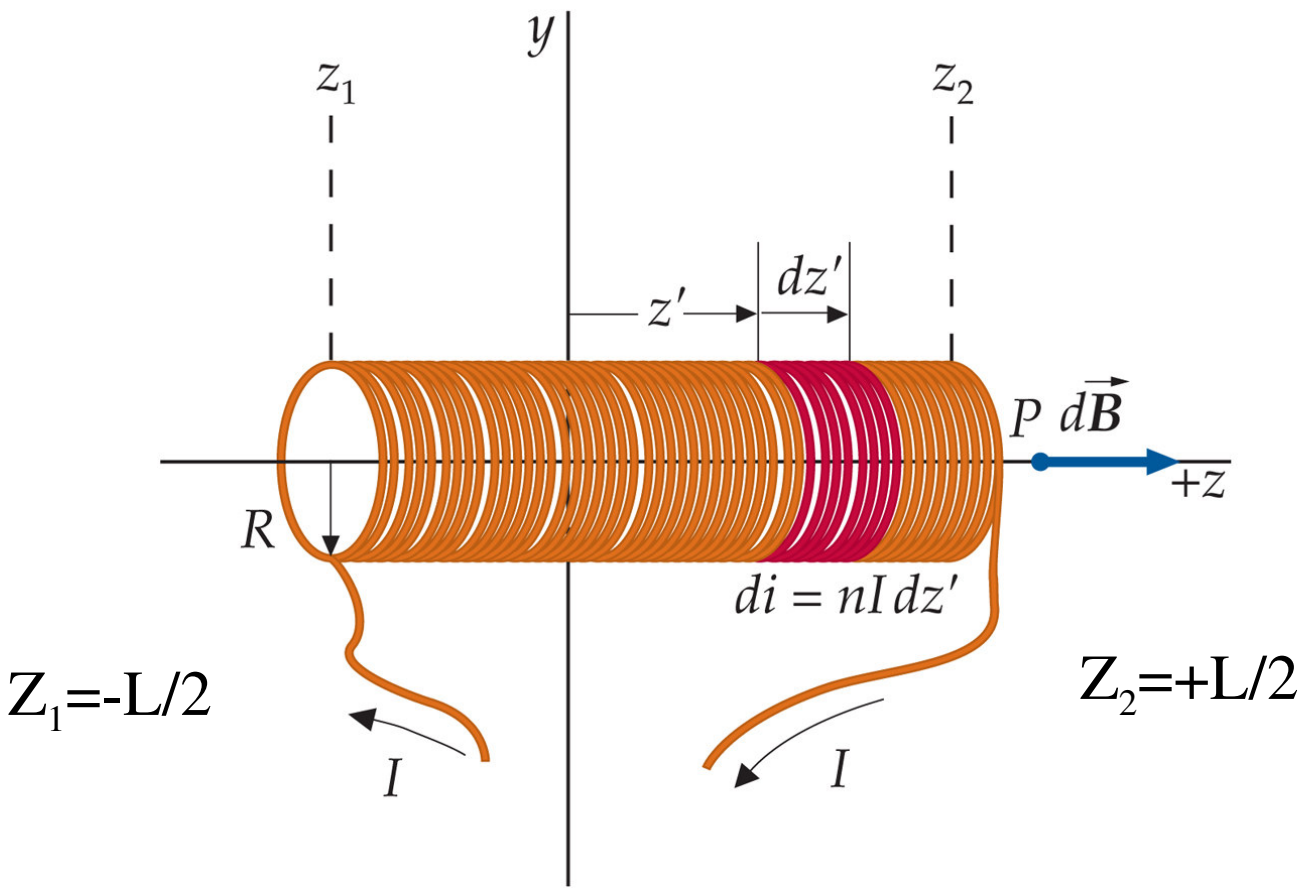
The fields are similar but you can't get access to the internal region of the bar magnet.

Magnetic Solenoid as Multiple Coils

Loop density = n = N/L= numbers of loops / length of coil



Magnetic Solenoid Geometry

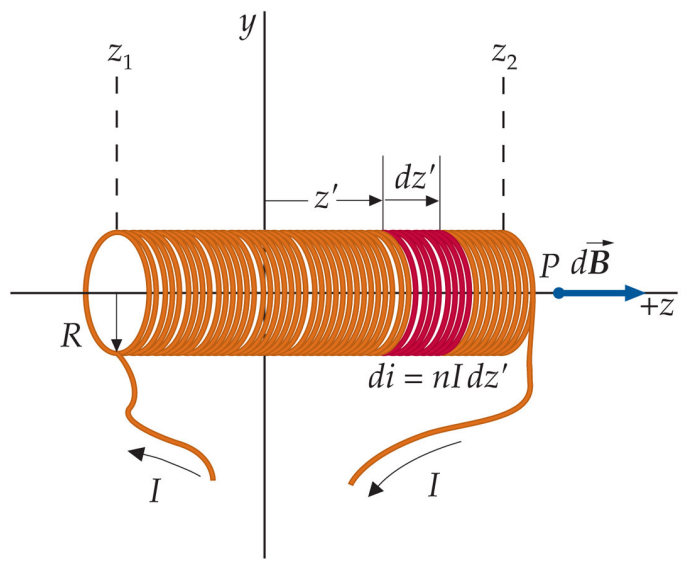


Magnetic Solenoid

$$z_2 - z_1 = L \quad n = \frac{N}{L}$$

In dz there are ndz turns of current I

$$di = nIdz$$



$$dB_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 di}{\left(z^2+R^2\right)^{3/2}} = \frac{\mu_0}{4\pi}\frac{2\pi R^2 nIdz}{\left(z^2+R^2\right)^{3/2}}$$

$$B_z = \frac{\mu_0}{4\pi}2\pi R^2 nI\int_{z_1}^{z_2}\frac{dz}{\left(z^2+R^2\right)^{3/2}}$$

← Use a trig substitution: z=Rtanθ

$$B_z = \mu_0\frac{1}{2}nI\left[\frac{z_2}{\sqrt{z_2^2+R^2}} - \frac{z_1}{\sqrt{z_1^2+R^2}}\right]$$

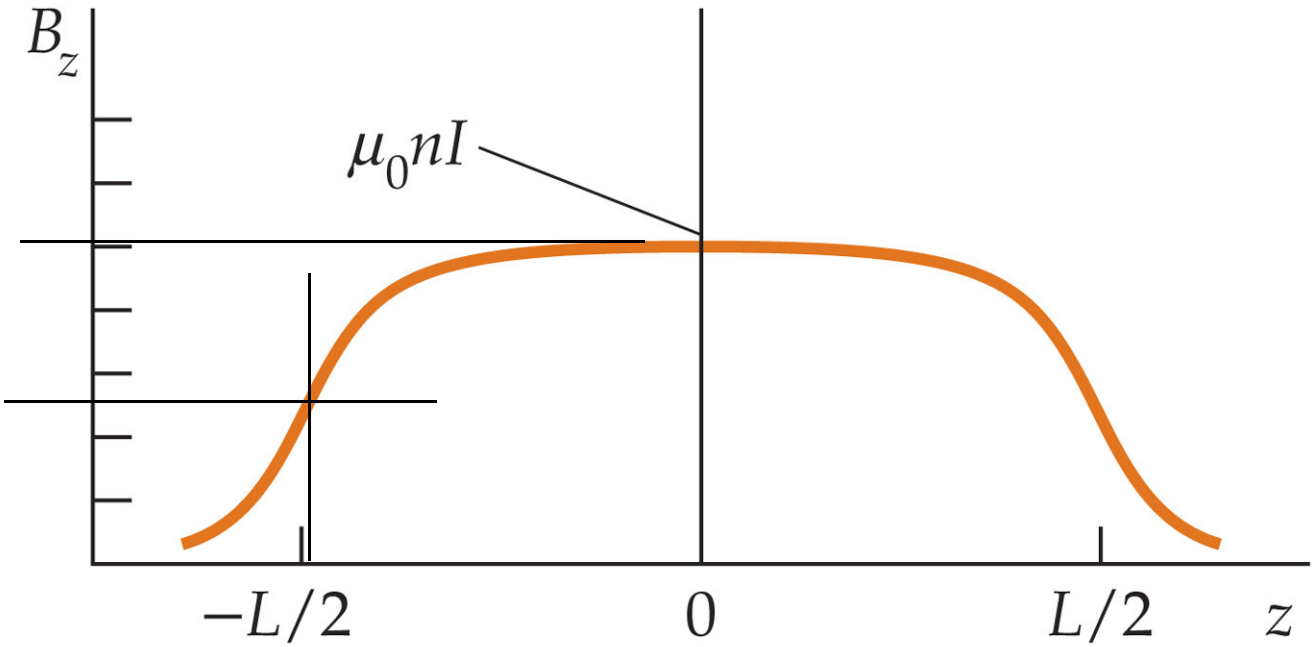
If $R << L$

$$B_z = \mu_0 nI = \mu_0 M$$

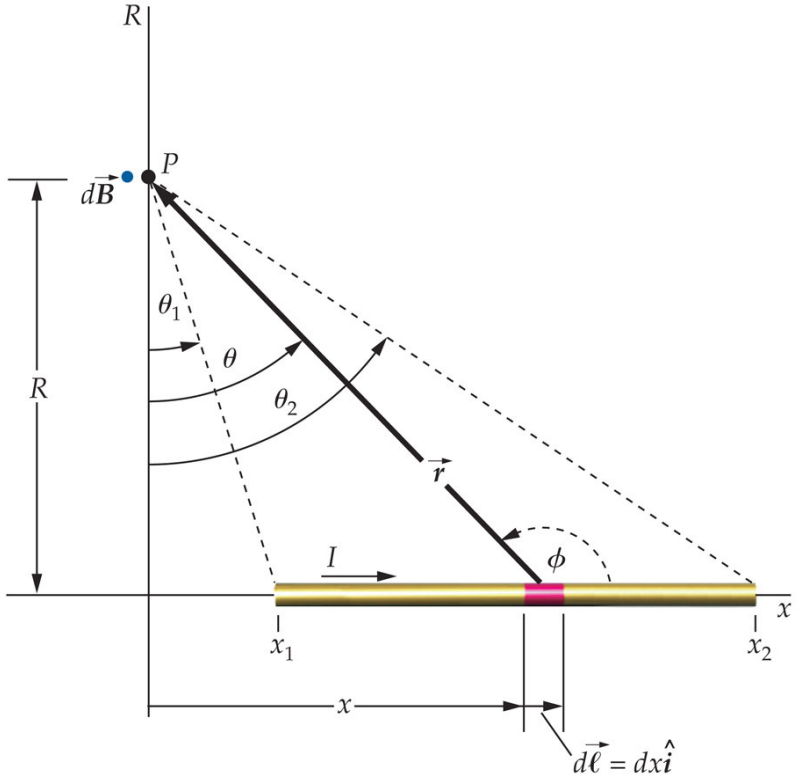
B-field at the center of the coil at z = 0.

Solenoid Magnetic Field

Why is the field strength at the end of the coil equal to one-half the interior value?



Magnetic Field of Current Segment



This is a calculation for a general position relative to the current segment but we will be using it for highly symmetric situatiuons.

Magnetic Field of Current Segment

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{L}\times\hat{r}}{r^2}$$

$d\vec{L}\times\hat{r}$ *is out of the paper*

$d\vec{L} = dx\hat{i};\ d\vec{L}\times\hat{r} = dL\sin\varphi\hat{k}$

$d\vec{L}\times\hat{r} = \sin\varphi dx\hat{k}$

$$dB = \frac{\mu_0}{4\pi}\frac{I}{r^2}\sin\varphi dx\hat{k} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\hat{k}\sin\theta dx$$

Need to relate $r, x,$ *and* θ

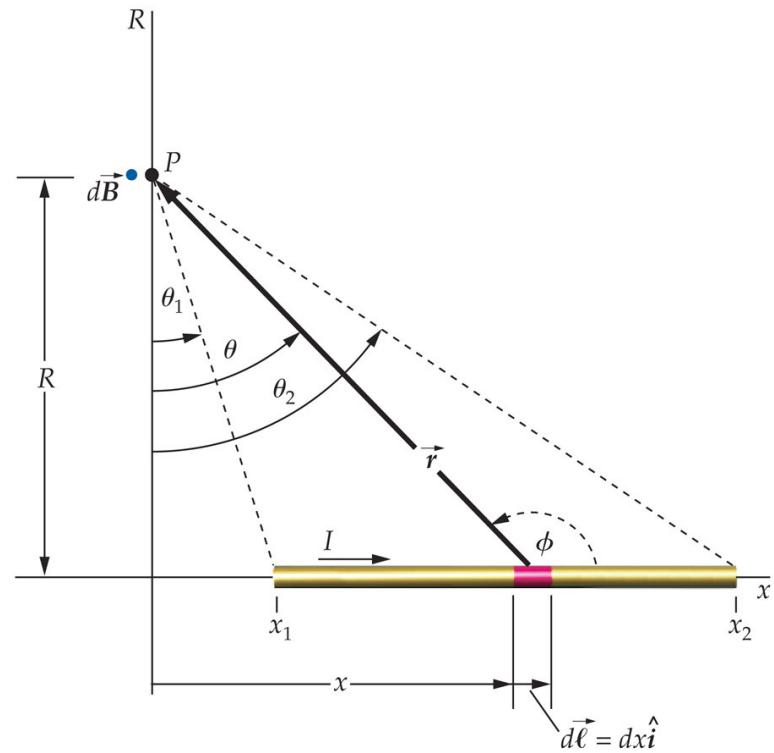
$x = R\tan\theta$

$dx = Rd(\tan\theta) = R\sec^2\theta d\theta$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{R}$$

$$dx = R\frac{r^2}{R^2}d\theta = \frac{r^2}{R}d\theta$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\cos\theta\frac{r^2}{R}d\theta\hat{k}$$



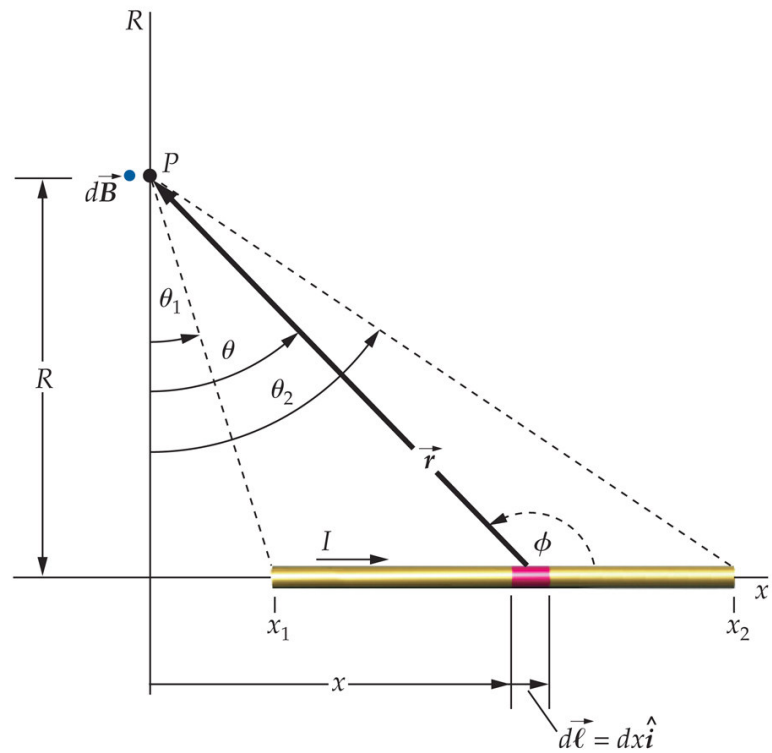
Magnetic Field of Current Segment

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\cos\theta\frac{r^2}{R}d\theta\hat{k}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{L}\times\hat{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\hat{k}}{R}\int_{\theta_1}^{\theta_2}\cos\theta d\theta = \frac{\mu_0}{4\pi}\frac{I}{R}\sin\theta\Big|_{\theta_1}^{\theta_2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_2 - \sin\theta_1\right)\hat{k}$$



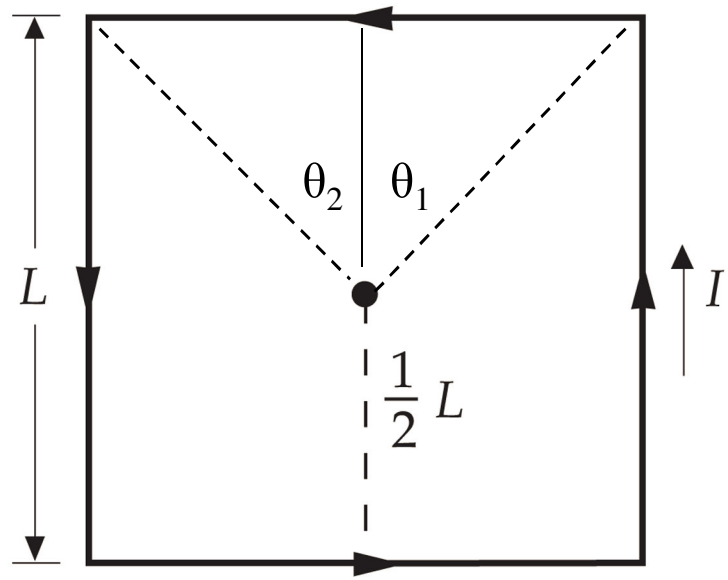
For an infinitely long wire

$$\theta_1 \to -\frac{\pi}{2};\ \theta_2 \to +\frac{\pi}{2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{R}\left(+1-(-1)\right)\hat{k} = \frac{\mu_0}{4\pi}\frac{2I}{R}\hat{k}$$

Current Carrying Loop as Line Segments

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\theta_2 - \sin\theta_1\right)\hat{k}$$



$$\theta_1 = {}^{-\pi}\!/_{4};\ \theta_2 = {}^{+\pi}\!/_{4}$$

$$\sin\left({}^{-\pi}\!/_{4}\right) = {}^{-\sqrt{2}}\!/_{2}\ ;\ \sin\left({}^{+\pi}\!/_{4}\right) = {}^{\sqrt{2}}\!/_{2}$$

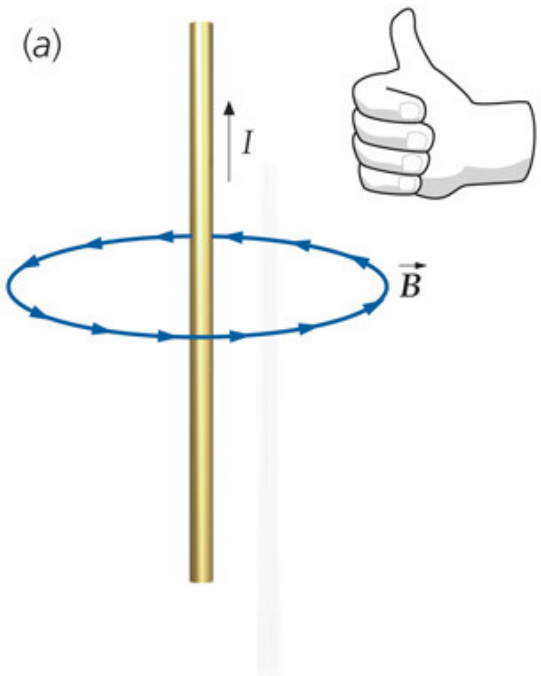
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)\hat{k} = \frac{\mu_0}{4\pi}\frac{I\sqrt{2}}{R}\hat{k}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{4I\sqrt{2}}{R}\hat{k}$$

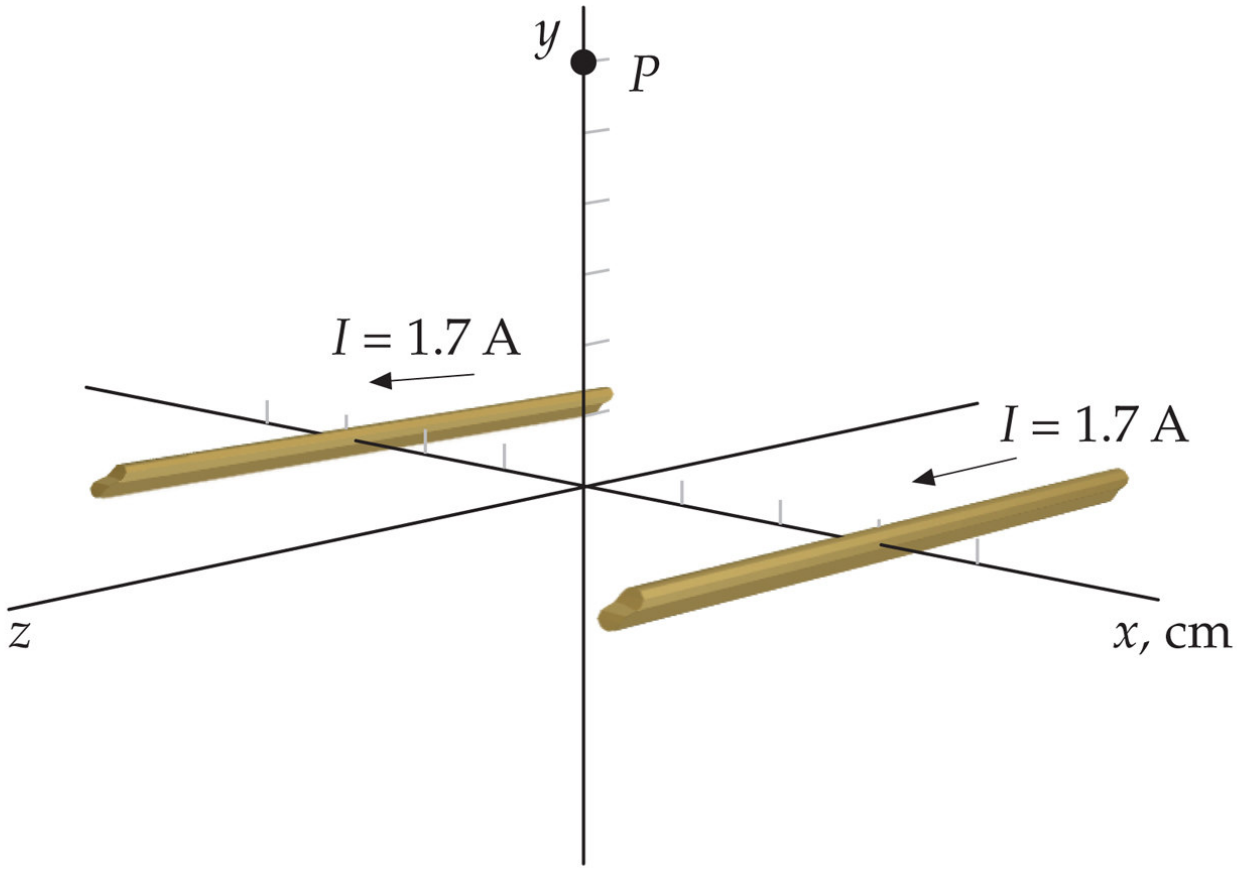
Each segments make an identical contribution

B-Field: Infinite Current Carrying Wire

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{R}\hat{k}$$

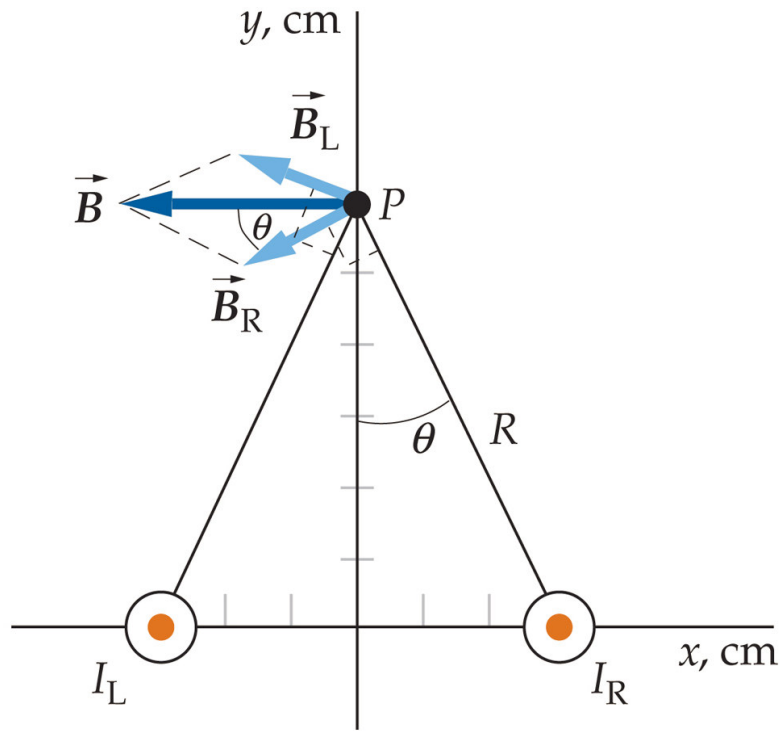


Magnetic Field from Two Infinite Current Carrying Wires



Magnetic Field from Two Infinite Current Carrying Wires

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{R}\hat{k}$$



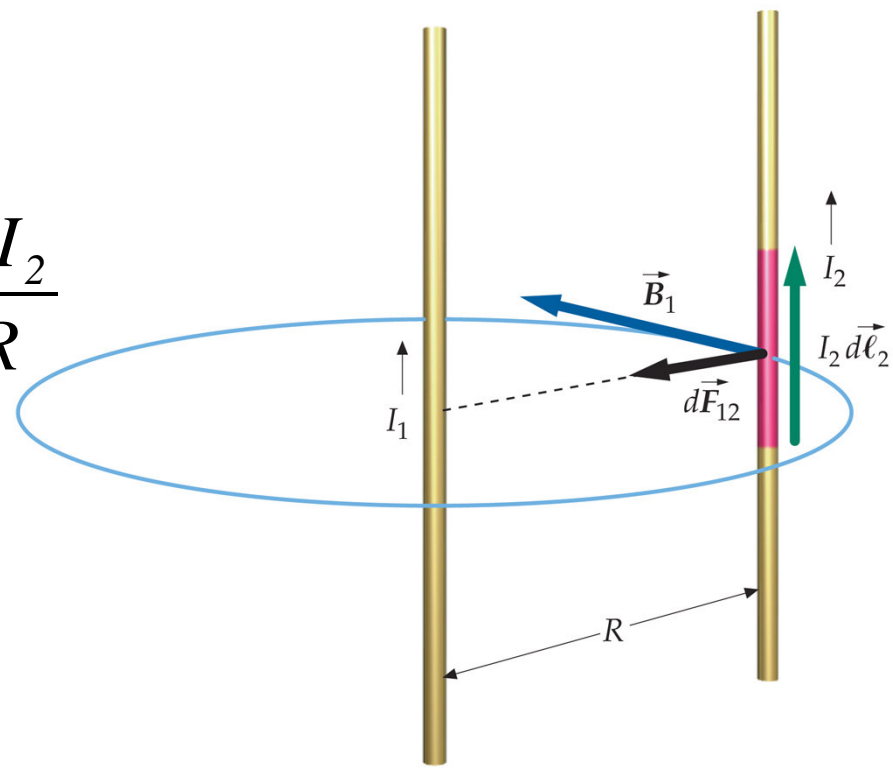
Force between Current Carrying Wires

$$d\vec{F}_2 = I_2 d\vec{L}_2 \times \vec{B}_1$$

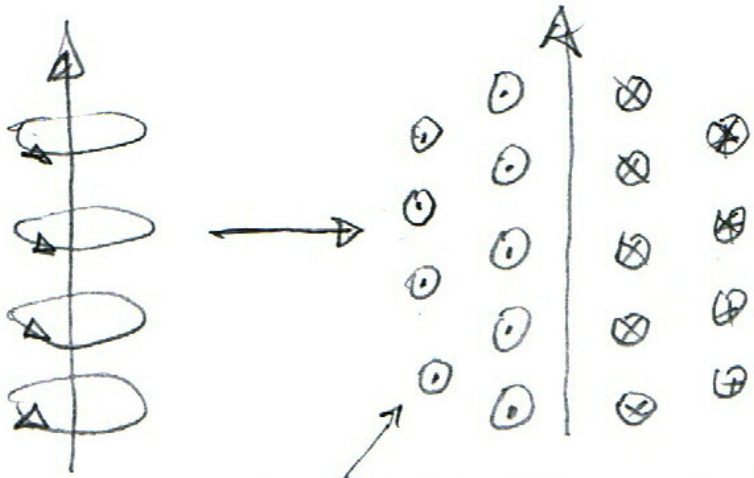
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{R}\hat{k}$$

$$dF_2 = I_2\left(\frac{\mu_0 I_1}{2\pi R}\right)dL_2$$

$$\frac{dF_2}{dL_2} = \frac{\mu_0 I_1 I_2}{2\pi R} = 2\frac{\mu_0}{4\pi}\frac{I_1 I_2}{R}$$



Current Carrying Wire Examples



MAG FIELD LINES COMING OUT OF THE PAGE

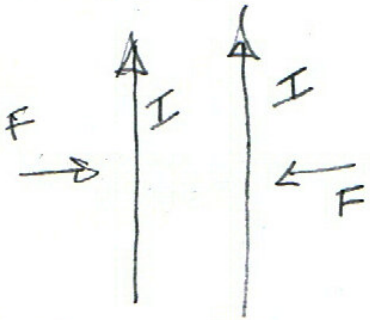
MAGNETIC FIELD LINES GOING INTO THE PAGE

USE ⊗ instead of ⊕ to avoid confusion with positive charges.

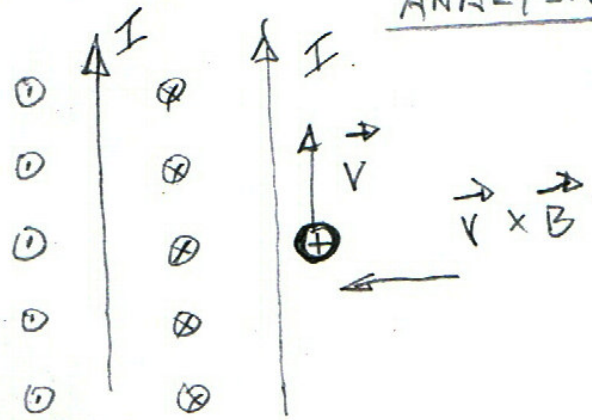
Current - Same Direction

CANCELLATION OF FIELD LINES IN REGION BETWEEN THE WIRE

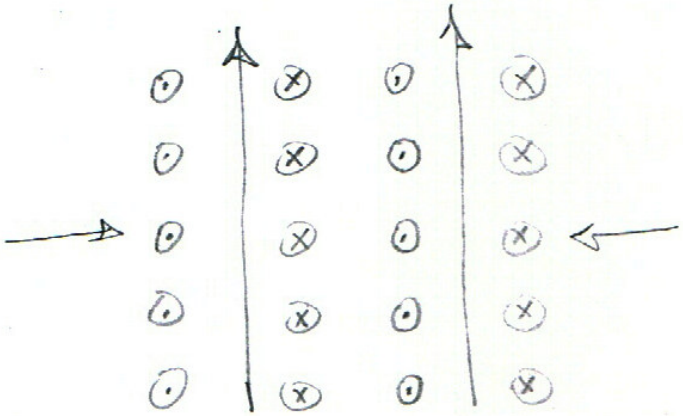
THIS REDUCES THE "PRESSURE" IN THE REGION AND THE FIELD LINES IN THE REGIONS OUTSIDE THE WIRES PUSH THE WIRES CLOSER TOGETHER



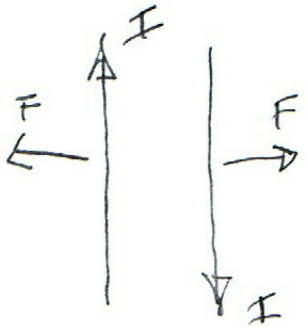
LORENTZ FORCE ANALYSIS



FIELD LINE ANALYSIS



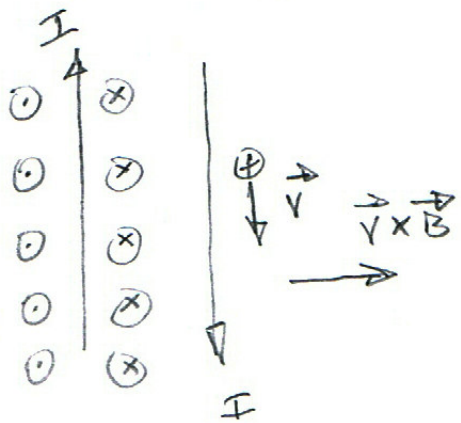
Current - Opposite Direction



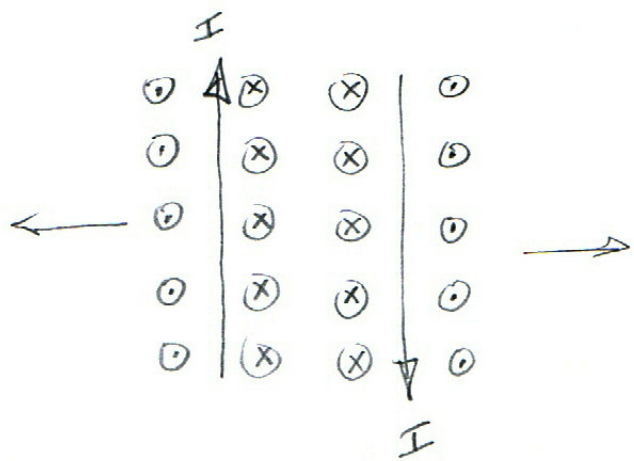
MAGNETIC FIELD LINE REINFORCE EACH OTHER IN REGION BETWEEN THESE WIRES.

INCREASE "MAGNETIC PRESSURE" CAUSES THE WIRES TO REPEL.

LORENZ FORCE ANALYSIS



FIELD LINE ANALYSIS

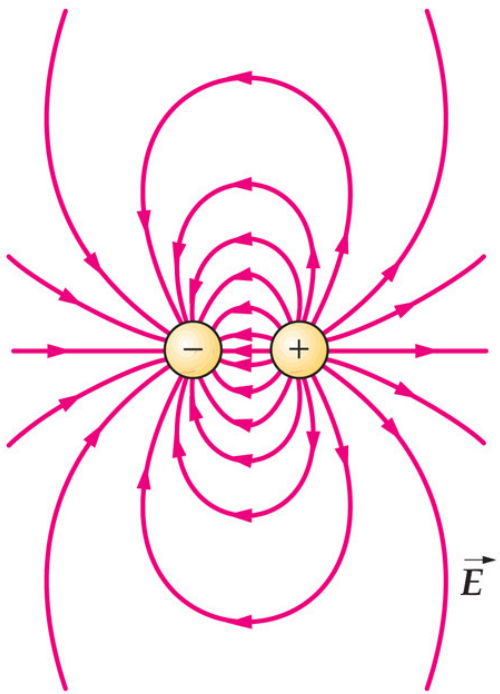


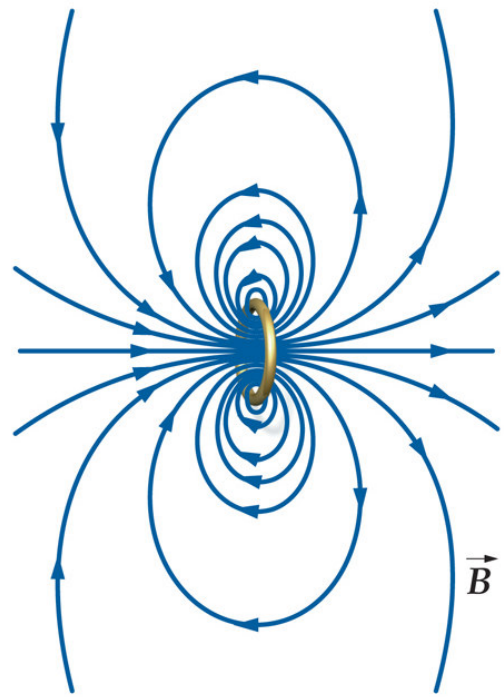
Gauss's Law for Magnetism

Neither situation has the high degree of symmetry needed for the practical use of Gauss's Law for field calculation.

$$\varphi_E = \oint_S \vec{E} \bullet d\vec{A} = \frac{Q_{inside}}{\varepsilon_0}$$

$$\varphi_B = \oint_S \vec{B} \bullet d\vec{A} = 0$$

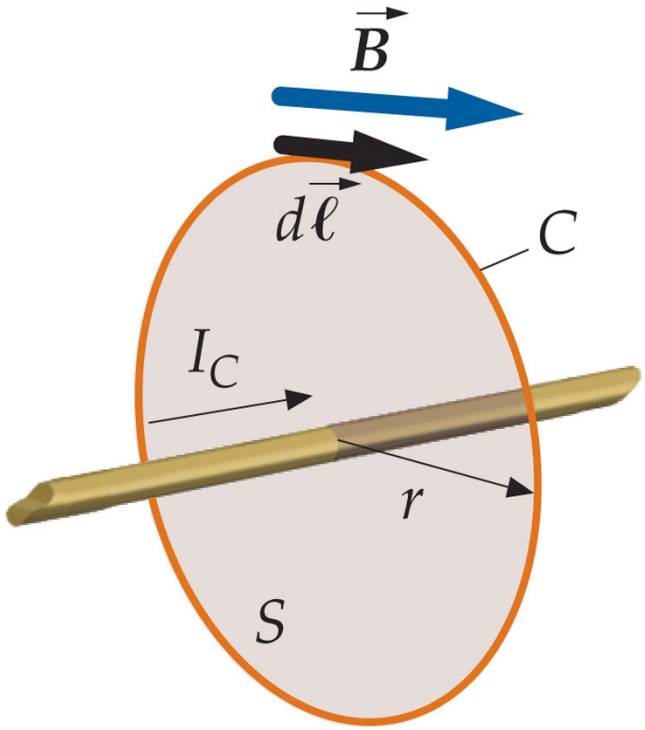




Ampere's Law

Ampere's Law

Ampere's Law does for magnetic field calculations what Gauss's Law does for electric field calculations.



$$\oint_C \vec{B} \bullet d\vec{L} = \oint_C B_t dL = \mu_0 I_C$$

For practical use in the calculation of magnetic fields we need to be able to take the magnetic field out from under the integral.

$$\oint_C B_t \bullet dL = B_t \oint_C dL$$

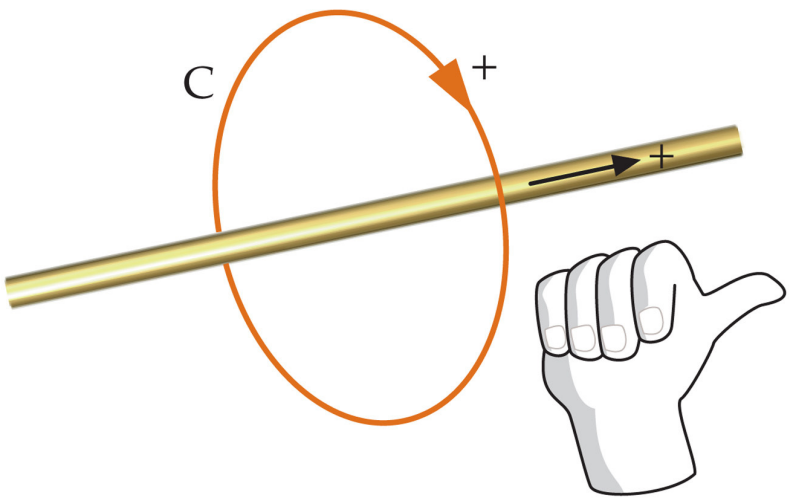
Positive Direction for Path Integral

The right-hand rule aligns the curve direction with the direction of the current.

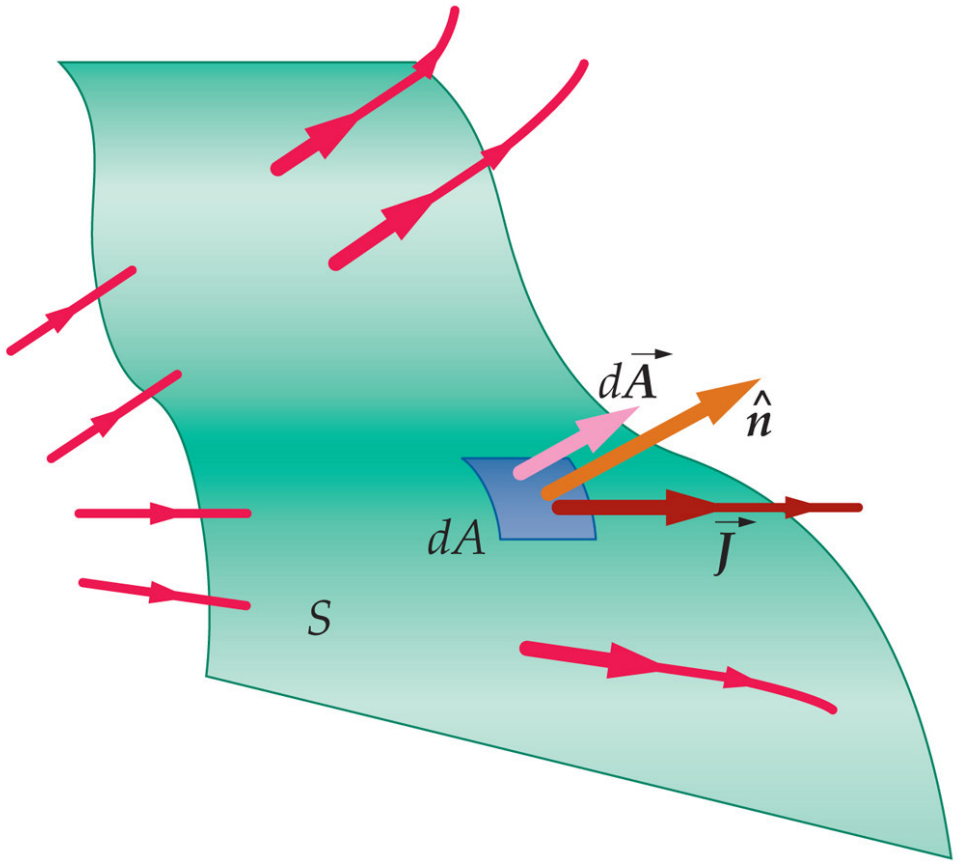
$$\oint_C \vec{B} \bullet d\vec{L} = \oint_C B_t dL = \mu_0 I_C$$

$$\oint_C B_t \bullet dL = B_t \oint_C dL$$

The ability to take the magnetic field out from under the integral requires that a curve C be found on which the magnetic field is constant.



Current Density

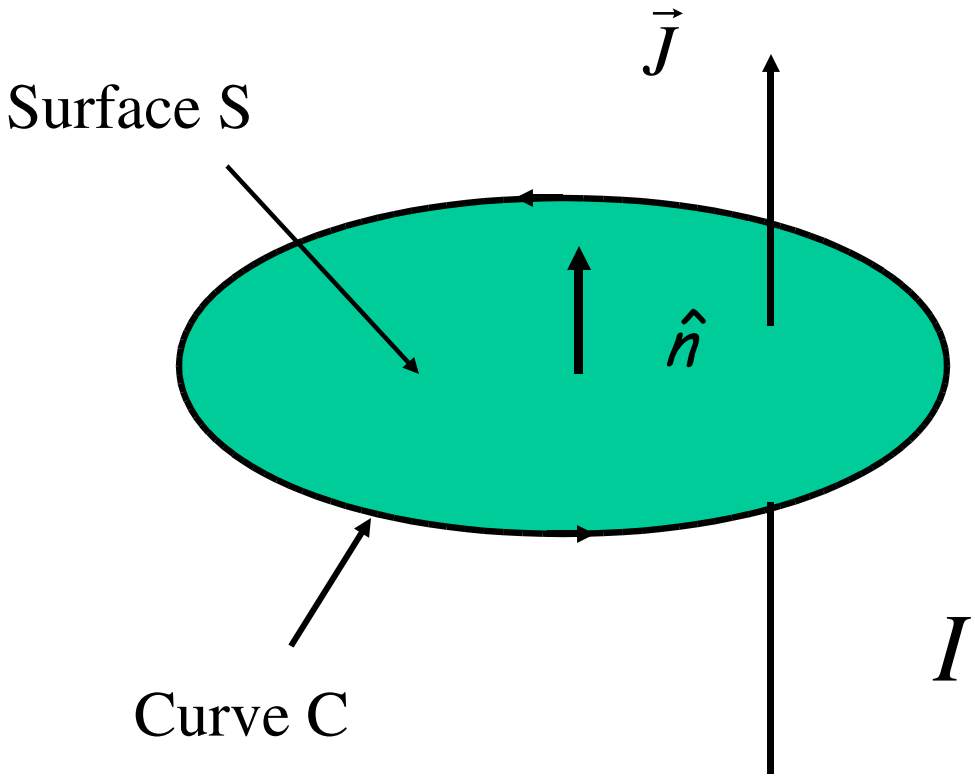


$$I = \int_S \vec{J} \bullet d\vec{A} = \int_S \vec{J} \bullet \hat{n} dA$$

The current density plays for magnetism the same role that charge densities played for electric fields.

In some situations all the current is not contained within the curve C and current might be a function of position.

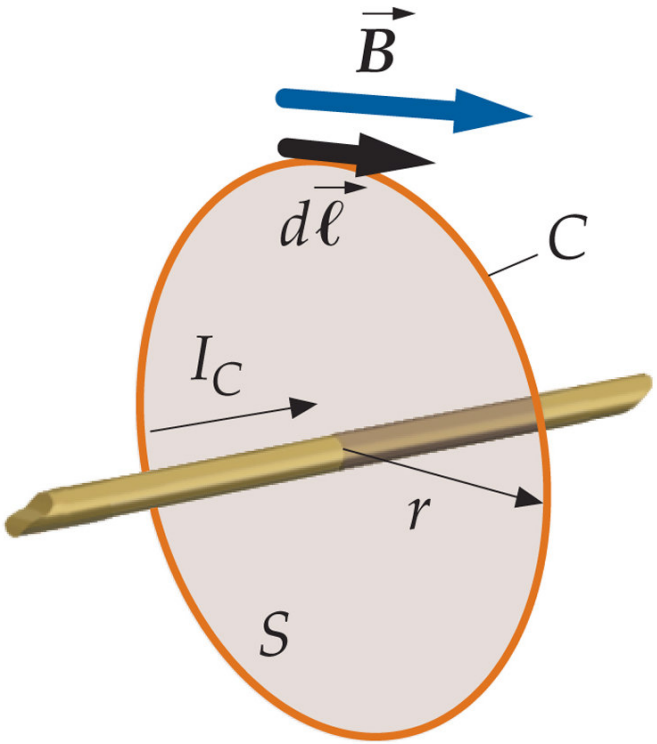
Positive Direction for Path Integral



$$I_C = \int_S \vec{J} \bullet \hat{n} dA$$

Curve C is a mathematical object. It does not have any current flowing in it.

Application of Ampere's Law



$$\oint_C \vec{B} \bullet d\vec{L} = \oint_C B_t dL = \mu_0 I_C$$

$$B_t \oint_C dL = B_t 2\pi r = \mu_0 I_C$$

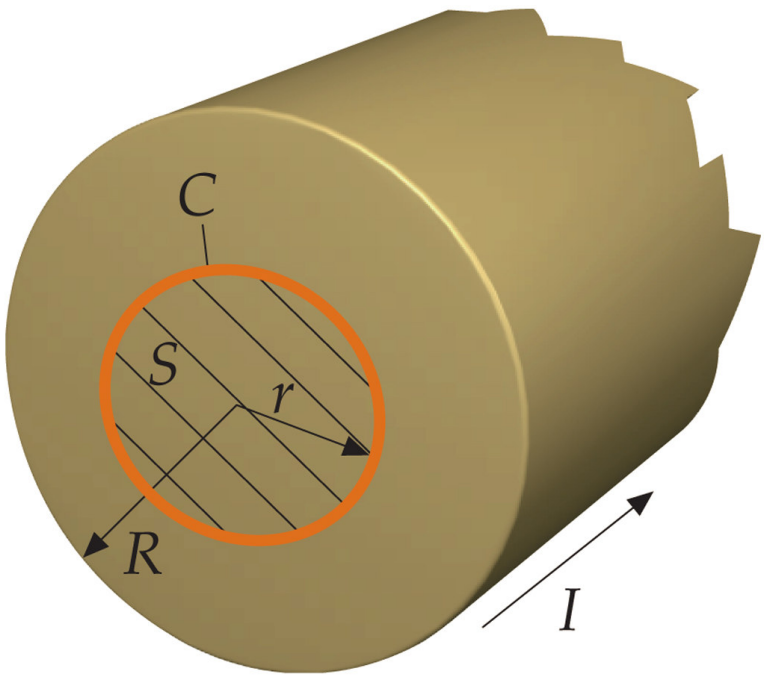
$$B_t = \frac{\mu_0 I_C}{2\pi r} = \frac{\mu_0}{4\pi}\frac{2I}{r}$$

The direction of the curve C is chosen so that the unit normal to the surface S is in the direction of the current I.

Magnetic Field Inside & Outside

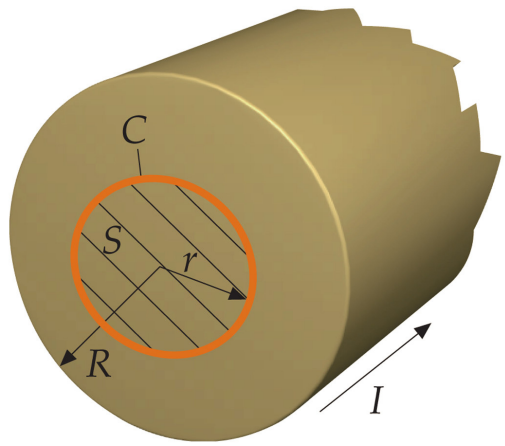
There is a need to coordinate the direction of the curve C and the direction of the current (current density).

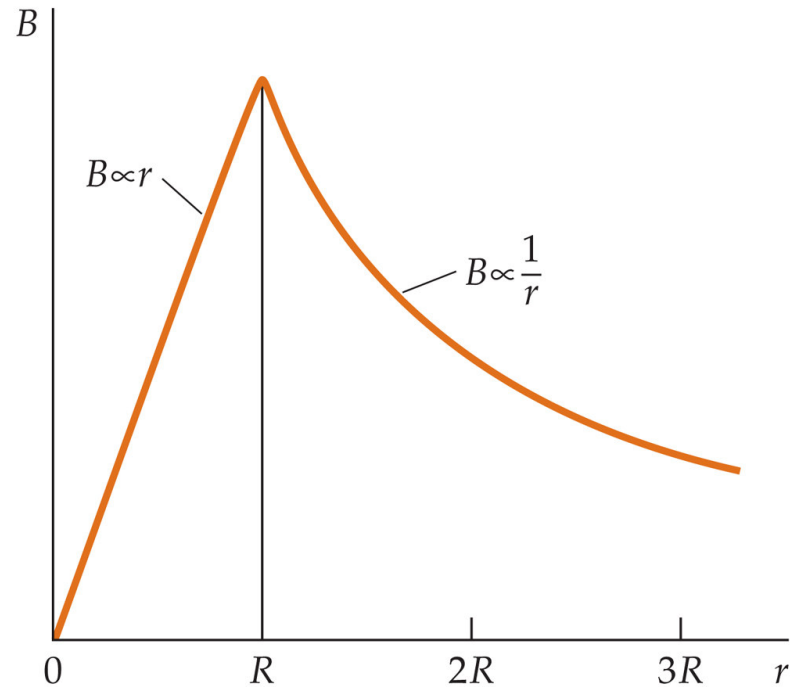
$$I = \int_S \vec{J} \bullet d\vec{A} = \int_S \vec{J} \bullet \hat{n} dA$$



The reason is the desire to have the unit normal for the surface S point in the same direction as the current density J.

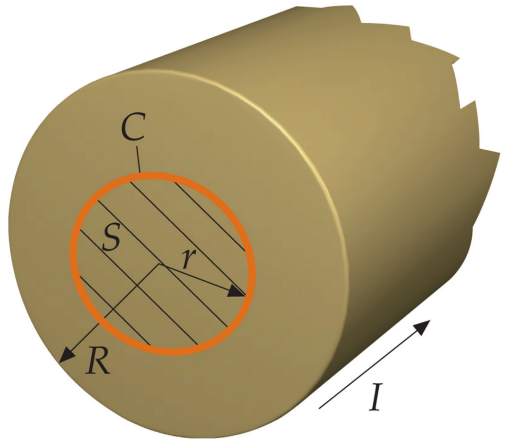
Magnetic Field Inside & Outside





This problem can be divided into an internal and external part. The external part was already shown on page 36.

Magnetic Field Inside & Outside



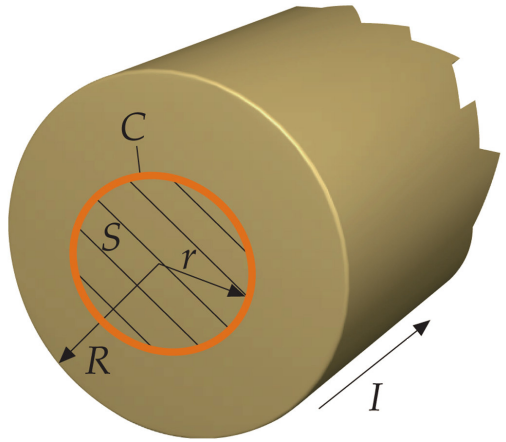
The curve C for the internal problem is shown above. The direction of the curve C is clockwise as viewed in the direction of the current.

$$B_t \oint_C dL = B_t 2\pi r = \mu_0 \oint_S J \bullet \hat{n} dA$$

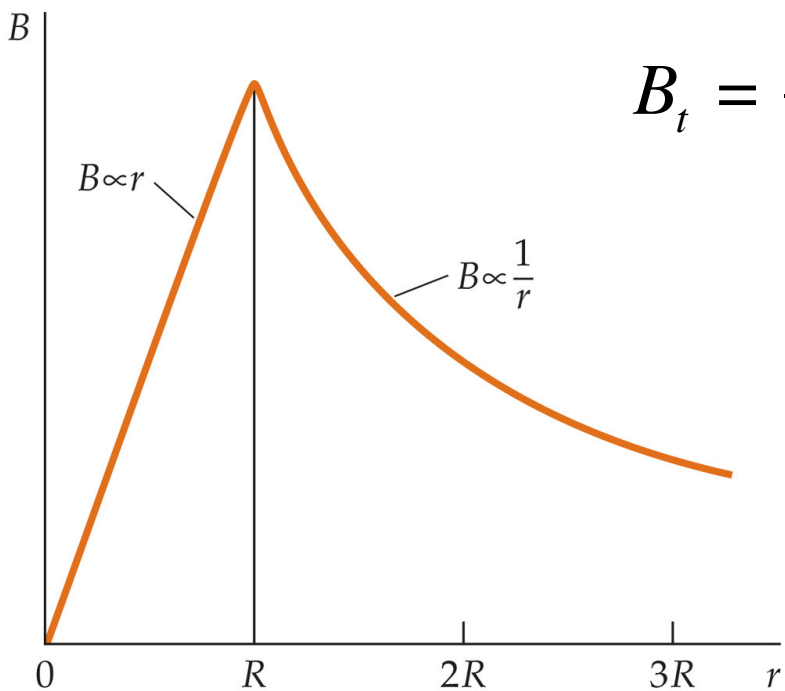
$$B_t 2\pi r = \mu_0 J \oint_S dA = \mu_0 J \pi r^2$$

$$B_t = \frac{\mu_0 J \pi r^2}{2\pi r} = \frac{\mu_0 \pi r^2}{2\pi r} \frac{I}{\pi R^2} = \frac{\mu_0}{4\pi} \frac{2I}{R^2} r$$

Magnetic Field Inside & Outside



$$B_t = \frac{\mu_0}{4\pi}\frac{2I}{R^2}r$$

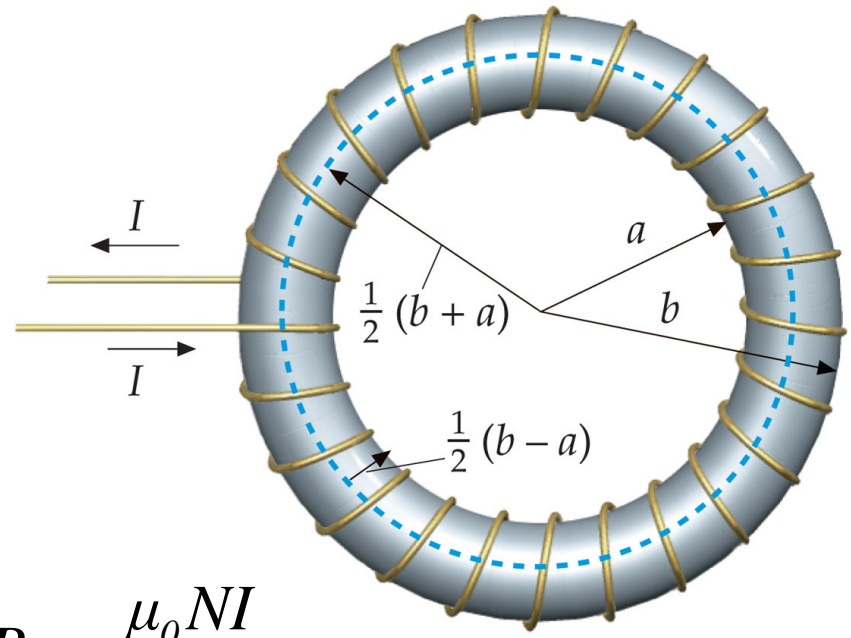


$$B_t = \frac{\mu_0}{4\pi}\frac{2I}{r}$$

Toroidal Magnetic Field

$$B_t \oint_C dL = B_t 2\pi r = \mu_0 I_C$$

$$B_t = \frac{\mu_0 I_C}{2\pi r}$$

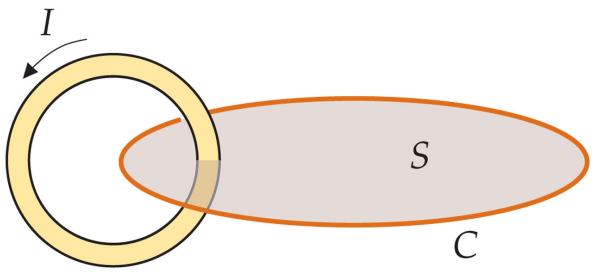


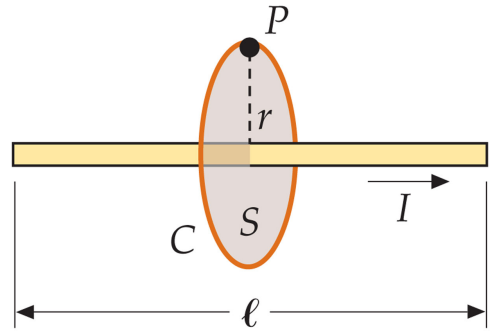
$$for\ r < a,\quad I_C = 0 \Rightarrow B_t = 0$$

$$for\ a < r < b,\quad I_C = NI \Rightarrow B_t = \frac{\mu_0 NI}{2\pi r}$$

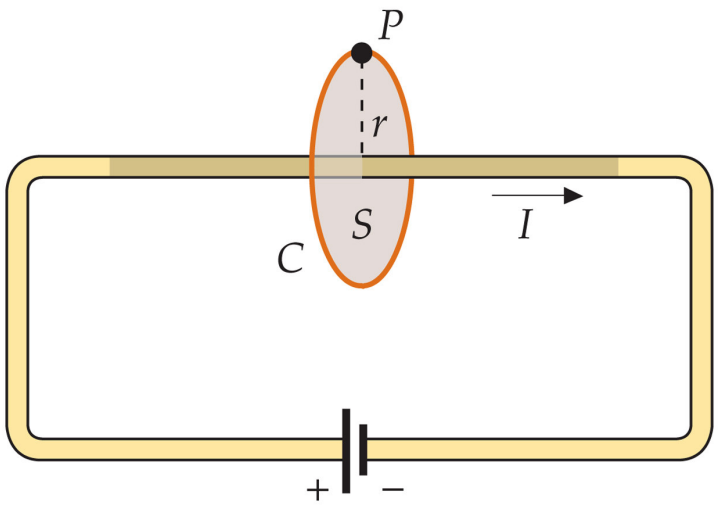
$$for\ b < r,\quad I_C = 0 \Rightarrow B_t = 0$$

Situations Where Ampere's Law Won't Work





Current might or might not be continuous.



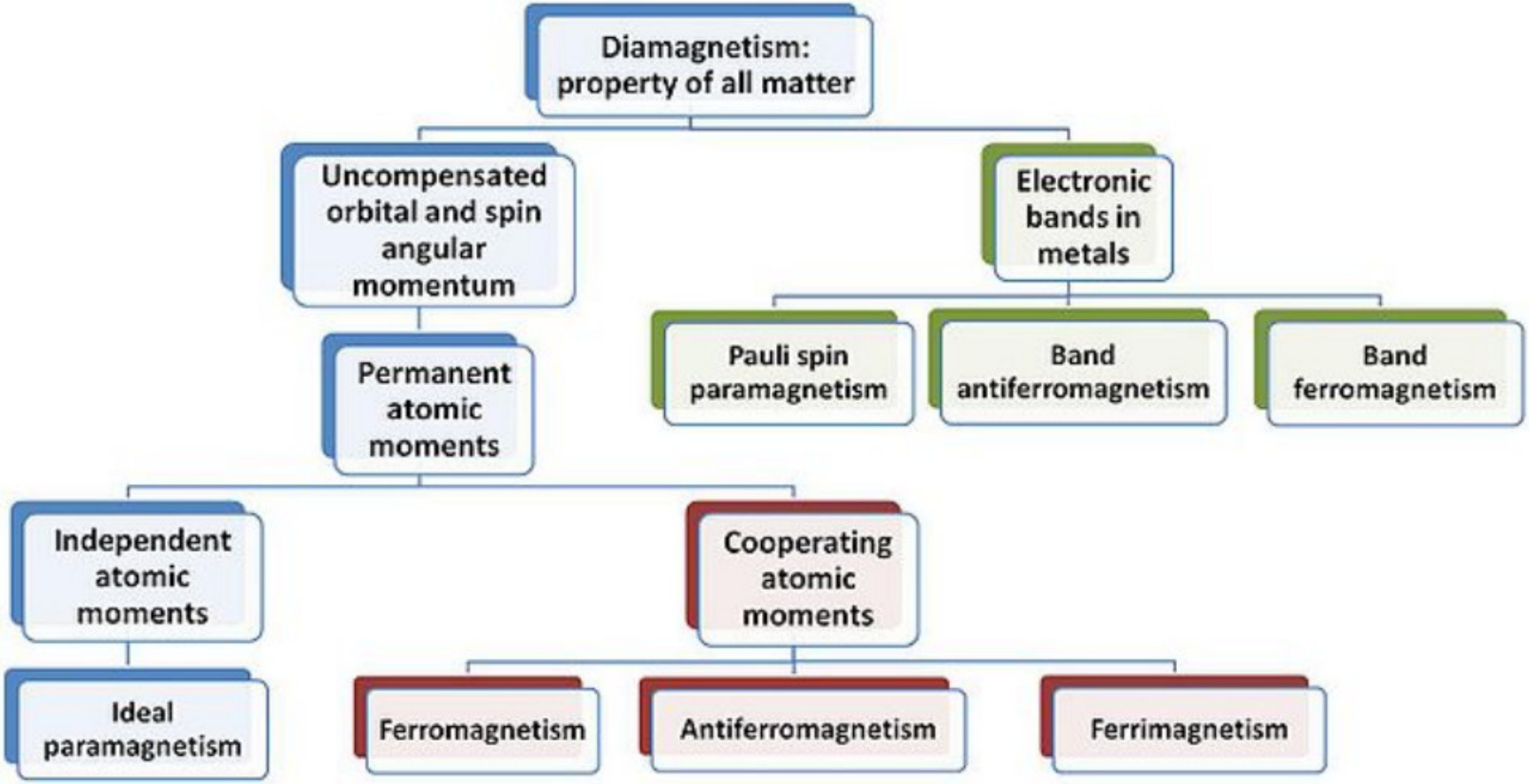
In all these cases there isn't enough there is not enough symmetry to allow the magnetic field to be taken outside of the integral.

Magnetism in Matter

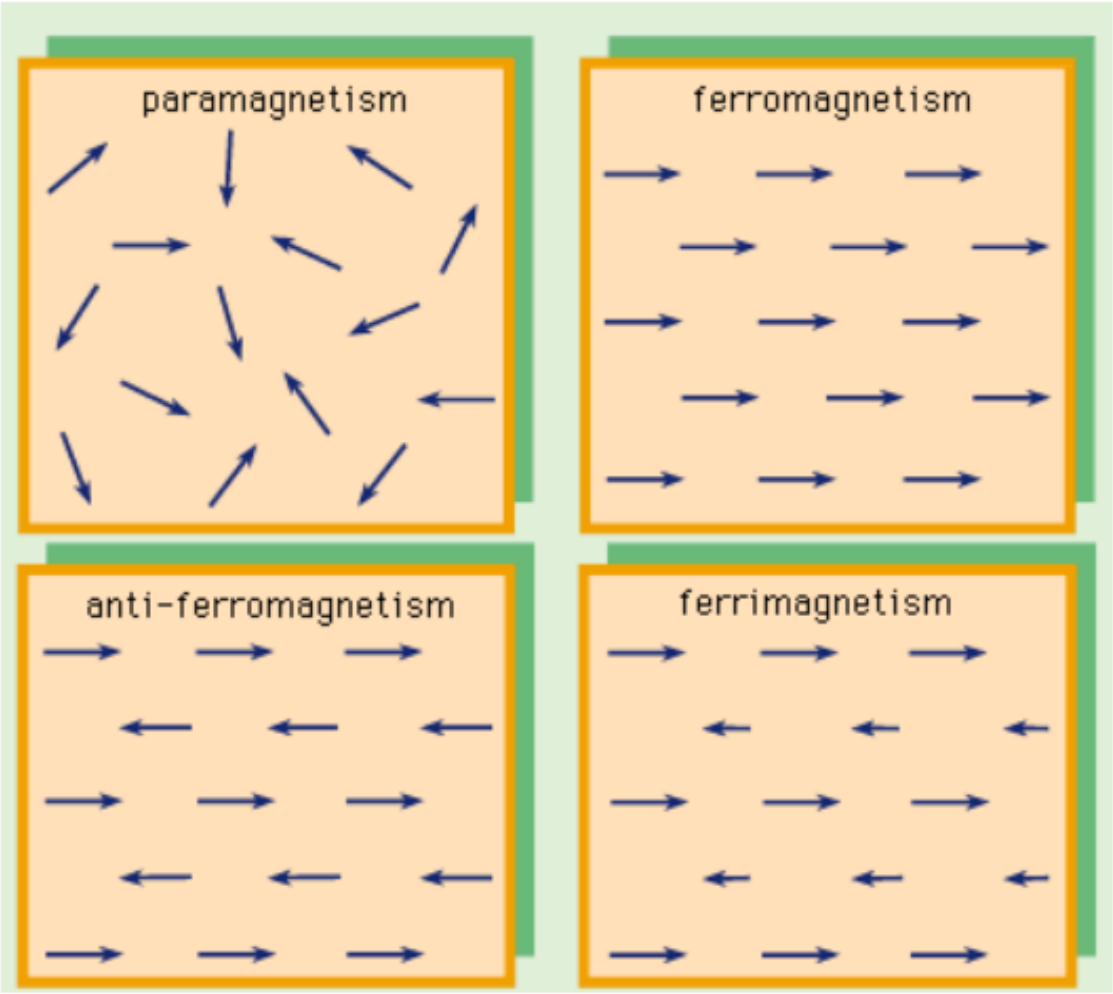
Magnetism in Matter

- Diamagnetism
  - Paramagnetism
  - Ferromagnetism
    - Ferromagnetism
    - Ferromagnetism

Magnetism in Matter

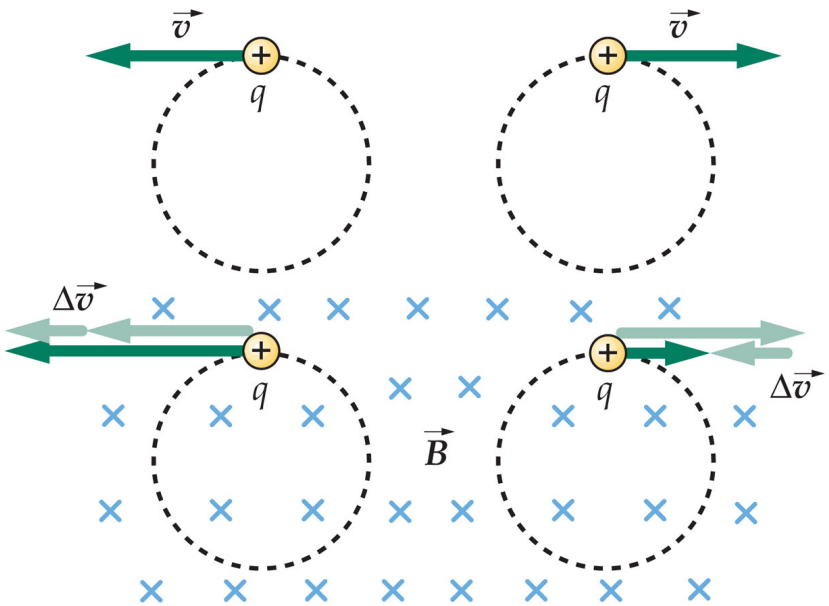


Magnetism in Matter



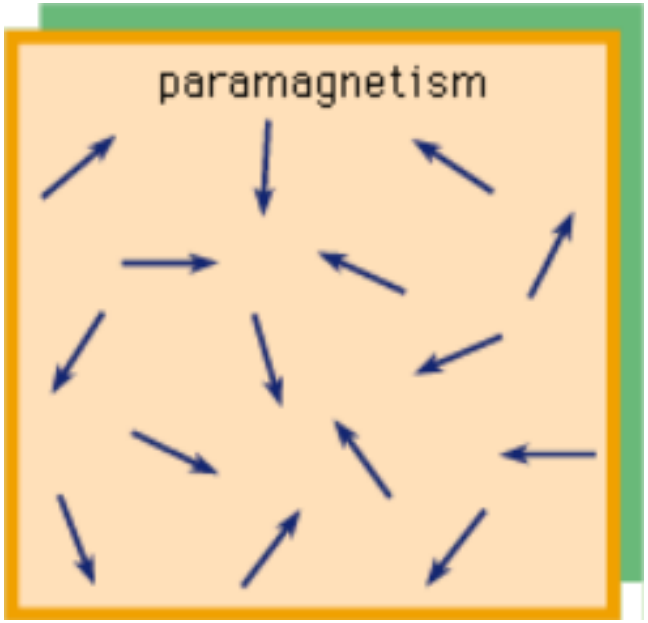
Diamagnetism

- Present in all materials but weak.
- Arises from orbital dipole moments induced by an applied magnetic field.
- Opposite direction of the applied field - decreases the external field.
- Only observed in materials with no permanent magnetic moments.



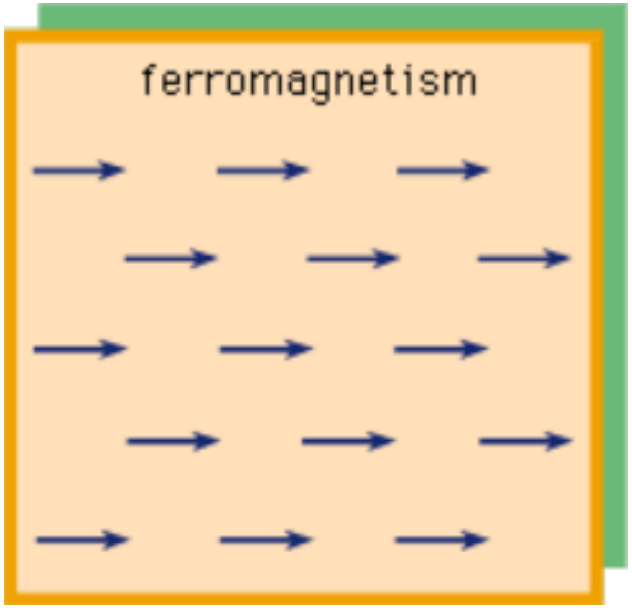
Paramagnetism

- Partial alignment of electron spins in metals or atomic or molecular moments
- No strong interaction among the magnetic moments
- Thermal motion randomizes the orientation of the magnetic moments.



Ferromagnetism

- Strong interaction between neighboring magnetic dipoles.
- High degree of alignment even in weak fields.
- May align at lower temperatures even in the absence of an external magnetic field.

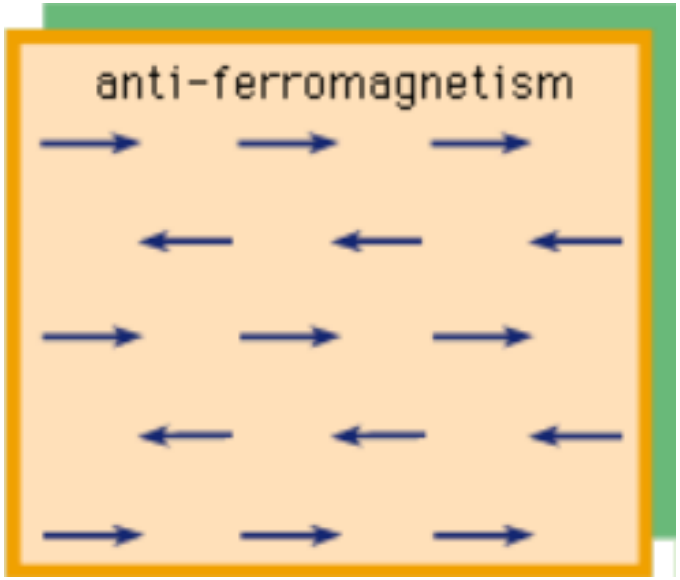


Anti-Ferromagnetism

In antiferromagnets, the magnetic moments, usually related to the spins of electrons, align with neighboring spins, on different sublattices, pointing in opposite directions.

Antiferromagnetic order exists at sufficiently low temperatures and vanishes above a certain temperature, the Néel temperature.

Above the Néel temperature, the material is typically paramagnetic.

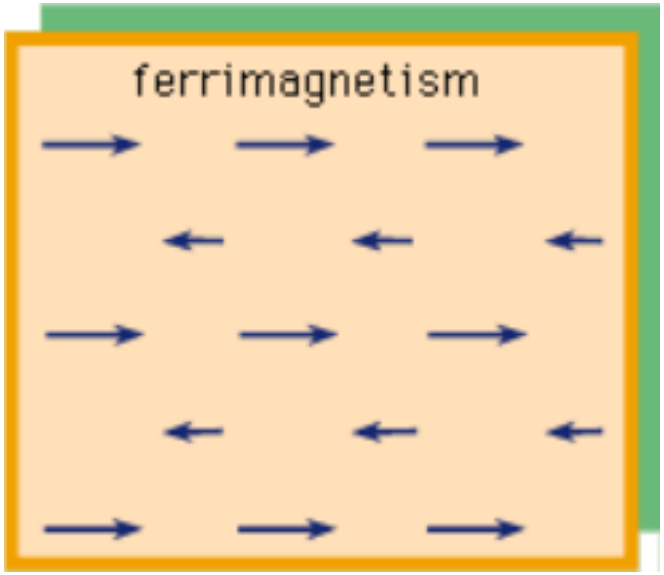


Ferrimagnetism

A ferrimagnetic material is one in which the *magnetic moments of the atoms on different sublattices are opposed*, as in antiferromagnetism.

In ferrimagnetic materials, the *opposing moments are unequal* and a spontaneous magnetization remains.

This happens when the sublattices consist of different materials or ions (such as Fe2+ and Fe3+).



Magnetic Susceptibility

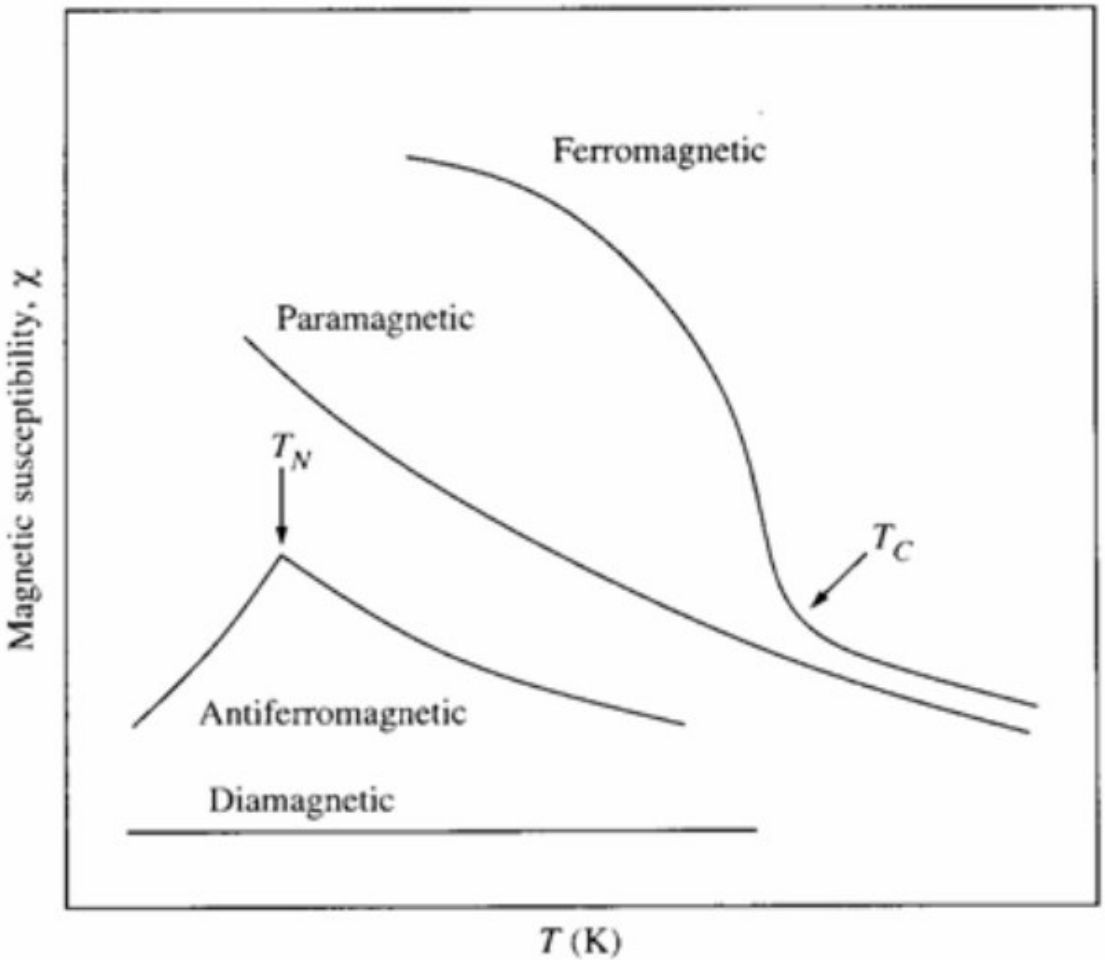
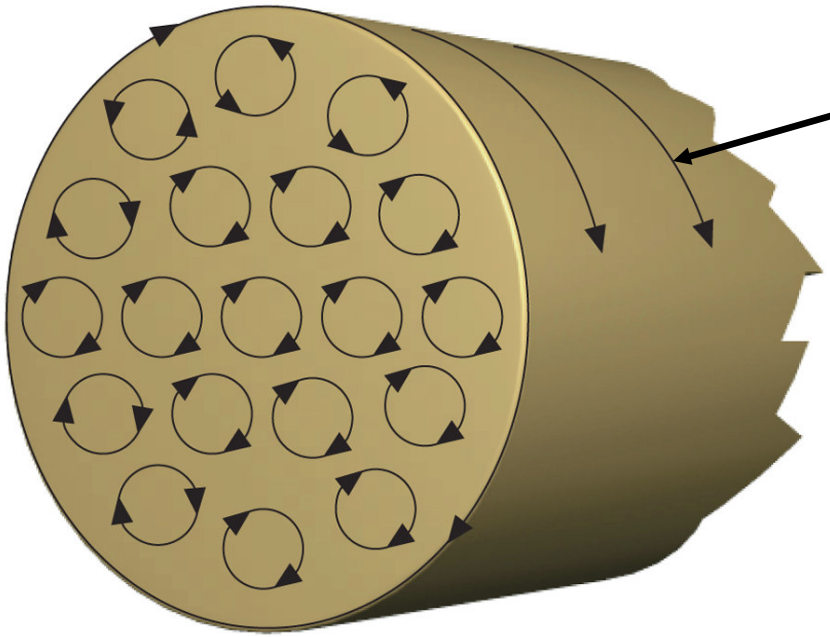


Fig. 11.59 Variation of magnetic susceptibility with temperature for diamagnetic, paramagnetic, ferromagnetic, and antiferromagnetic substances. Transitions to paramagnetic behavior for ferromagnetic and antiferromagnetic substances occur at the Curie (T_C) and Neel (T_N) temperatures, respectively.

Magnetism in Matter



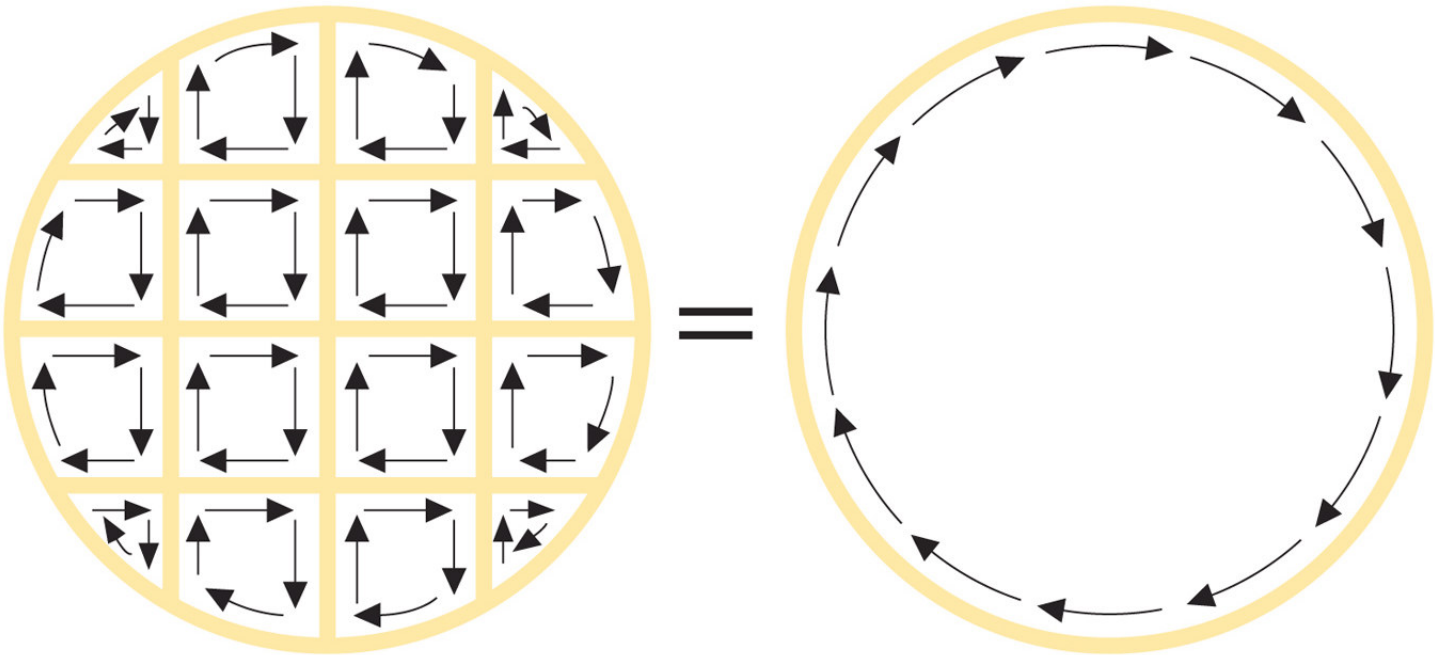
Amperian Current

Simplistic model of atomic current loops all aligned with the axis of the cylinder

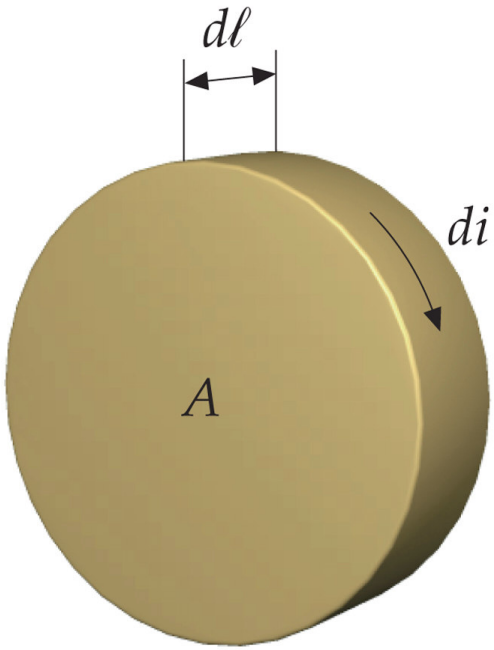
Internally the net current is zero due to the cancellation of current from neighboring atoms.

Amperian Current

Due to the internal cancellation of loop currents the net effect of all the current loops is only a surface current.



Incremental Slice of a Permeable Material



Define the magnetization M as the net magnetic dipole moment per unit volume

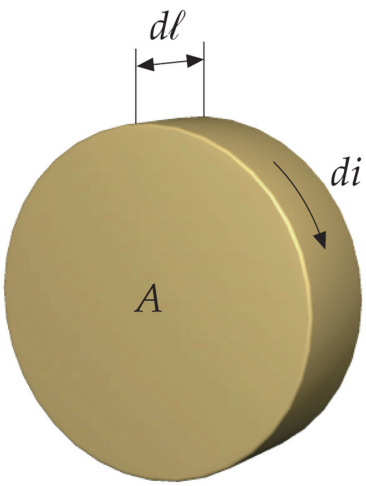
$$\vec{M} = \frac{d\vec{\mu}}{dV}$$

$$d\mu = Adi$$

$$M = \frac{d\mu}{dV} = \frac{Adi}{AdL} = \frac{di}{dL}$$

Magnetization is the amperian current per unit length

Magnetic Susceptibility



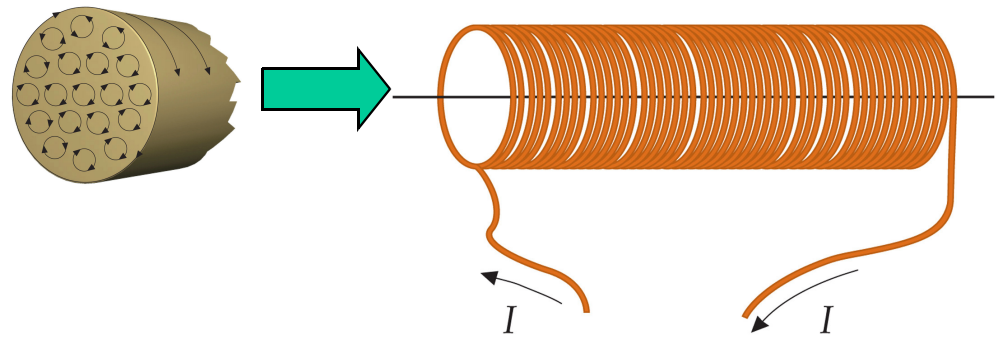
$$M = \frac{di}{dL}$$

Combining many of these small disks will form a cylinder which can be compared to a solenoid that has its current flowing around its periphery.

M plays the role of nI, where n = N/L, the loop density. For the B-field created in the solenoid B = nIL. By comparison

$$B_m = \mu_0 M$$

Magnetic Susceptibility



$$\vec{B} = \vec{B}_{app} + \mu_0 \vec{M}$$

$$\mu_0 \vec{M} = \chi_m \vec{B}_{app}$$

M is proportional to B_{app}

$$\vec{B} = \vec{B}_{app} + \chi_m \vec{B}_{app}$$

$$\vec{B} = \vec{B}_{app}\left(1 + \chi_m\right) = K_m \vec{B}_{app}$$

K_m is the relative permeability

$$K_m = \left(1 + \chi_m\right)$$

$K_m > 1$ Paramagnets

$K_m < 1$ Diamagnets

Magnetic Susceptibility of Various Materials at 20^0C

Material	χ_m
Aluminum	2.3×10^{-5}
Bismuth	-1.66×10^{-5}
Copper	-0.98×10^{-5}
Diamond	-2.2×10^{-5}
Gold	-3.6×10^{-5}
Magnesium	1.2×10^{-5}
Mercury	-3.2×10^{-5}
Silver	-2.6×10^{-5}
Sodium	-0.24×10^{-5}
Titanium	7.06×10^{-5}
Tungsten	6.8×10^{-5}
Hydrogen (1 atm)	-9.9×10^{-9}
Carbon dioxide (1 atm)	-2.3×10^{-9}
Nitrogen (1 atm)	-5.0×10^{-9}
Oxygen (1 atm)	2090×10^{-9}

Extra Slides