## Chapter 34

Thin Lenses

## Thin Lenses

- Mirrors
- Lenses
- Optical Instruments


## Inversion

A right-handed coordinate system becomes a left-handed coordinate system


## Mirrors

Real Object
Mirror


Object and image distances measured from the plane of the mirror

## Plane Mirrors

Object - Source of light
Image - Destination of light
Vitual Image - Can't be imaged on a screen

Depth Inversion - R.H.S. => L.H.S. (Perverted image)

Lateral Distance $=y^{\prime} / \mathrm{y}$

Magnification: $m=y^{\prime} / \mathrm{y}=-\mathrm{s}^{\prime} / \mathrm{s}$

$$
m=-(-1) / 1=+1
$$



Object distance $=s$ Image distance $=s$ '

## Multiple Images



## Multiple Images in Plane Mirrors



## Curved Mirrors

Convention: The initial object is located to the left; the incident light rays travel from the left to the right.

Concave mirror - center of curvature on the same side of the mirror as object.

Convex mirror - center of curvature on the opposite side of the mirror as object

## Reflection from a Concave Mirror

Object at Infinity- Light Ray Picture


Construction method: incident angle $=$ reflection angle

Rays from an object at infinity will focus at a point in the Focal plane.

## Reflection from a Concave Mirror

## Object at Infinity- Wave Front Picture

Plane waves represent no wave front curvature. This is indicative of an object at infinity.

The curved wave front converges to a point.


Construction method: wave front construction using Huygens principle. The shape of the wave front changes due to the different arrival times of the flat wave front at the curved mirror surface.

## Reflection from Concave Mirrors



The light ray method is easier to apply and can yield quantitative results.


The wave front method is more difficult to apply and doesn't lend itself to quantitative results.

The light ray method above used the incident angle $=$ the reflected angle approach to construct the diagram. We will make some approximations to this approach to avoid the optical aberrations of curved surfaces.

## Reflection from a Concave Mirror

## Object outside center of curvature - Light Ray Picture



Two rays are needed to determine the location of the image.

Multiple rays are used for effect. Angle of incidence equal to angle of reflection violated.

The radius of curvature $R$ is equal to $2 \times$ Focal Length $=2 f$

## Conventions for Spherical Mirrors

1. The parallel ray, drawn parallel to the axis. This ray is reflected through the focal point.
2. The focal ray, drawn through the focal point. This ray is reflected parallel to the axis.
3. The radial ray, drawn through the center of curvature. This ray strikes the mirror perpendicular to its surface and is thus reflected back on itself.

Principal rays for a mirror

1. $s$ is positive if the object is on the incident-light side of the mirror
2. $s^{\prime}$ is positive if the image is on the reflected-light side of the mirror
3. $r(\operatorname{and} f)$ is positive if the mirror is concave so the center of curvature is on the reflected-light side of the mirror

Sign conventions for reflection

$$
m=\frac{y^{\prime}}{y}=-\frac{s^{\prime}}{s}
$$

Lateral magnification

## Conventions for Spherical Mirrors



Concave

$$
\begin{aligned}
& \mathrm{R}>0 \\
& \mathrm{f}>0
\end{aligned}
$$



Convex
$\mathrm{R}<0$
f $<0$

# Making Sense of the Sign Conventions 

(For mirrors and lenses)

The distances are positive if you are measuring where the light IS supposed to go.

The distances are negative if the you are measuring where the light IS NOT supposed to go.

## Making Sense of the Sign Conventions



Light can't go back here.

Distances negative behind the mirror.

Distances positive in front of the mirror.

## Concave Mirror



Incident rays converge to an real, inverted image in front of the concave mirror.

## Concave Mirror



1. Parallel ray - This ray passes through the focal point after reflecting from the mirror surface.
2. Focal ray - This ray passes through the focal point and is parallel after reflecting from the mirror surface.
3. Center ray - This ray is passes through the center of curvature and is reflected back on itself.

## Convex Mirror



Incident rays diverge as though coming from a point behind the surface of the convex mirror. Image is virtual and erect.

## Convex Mirror



1. Parallel ray - This ray is reflected at the surface at a trajectory that can be traced back through the focal point behind the mirror surface.
2. Focal ray - This ray is reflected at the surface at a parallel trajectory that can be traced straight back behind the mirror surface.
3. Center ray - This ray is reflected at the surface at a trajectory that can be traced back through the center of curvature behind the mirror surface.

## Rays From the Focal Point


(a)

( 1 )

Incident rays (a) diverge from the focal point of the concave mirror and reflect from the mirror as a parallel beam. Rays converging (b) on the focal point of the convex mirror reflect off the surface and form a parallel beam

## Rays From Infinity


(a)

(b)

Incident rays parallel to the axis (a) converge to the focal point $F$ of a concave mirror, (b) diverge as though coming from the focal point F of a convex mirror.

## Concave Mirror

\author{

1. Parallel ray <br> 2. Focal ray <br> 3. Center ray
}


## Concave Mirror

| 1. Parallel ray |
| :--- |
| 2. Focal ray |
| 3. Center ray |

(c)

(d)


## Some Equations

Approximation: Paraxial rays
We only consider rays that are approximately parallel to the axis of the lens system.

$$
\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{R} \quad \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \quad f=\frac{R}{2}
$$



Concave: $\mathrm{r}>0 ; \mathrm{f}>0$


Convex: $\mathrm{r}<0 ; \mathrm{f}<0$

## Some Equations

It is useful to specify the object distance as a multiple of the focal length

This picture has $\mathrm{s}=3 \mathrm{f}$

$$
\begin{aligned}
& \frac{1}{s}+\frac{l}{s^{\prime}}=\frac{1}{f} \\
& \frac{1}{s^{\prime}}=\frac{1}{f}-\frac{1}{s} \\
& s^{\prime}=\frac{f s}{s-f} \\
& s^{\prime}=\frac{f(3 f)}{3 f-f}=\frac{3 f^{2}}{2 f}=1.5 f
\end{aligned}
$$



$$
\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{R} \quad \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \quad f=\frac{R}{2}
$$

$$
\text { Concave: } \mathrm{r}>0 \text {; } \mathrm{f}>0
$$

$$
m=-\frac{s^{\prime}}{s}=-\frac{1.5 f}{3 f}=-\frac{1}{2}
$$

## Some Equations

It is useful to specify the object distance as a multiple of the focal length
This picture has $\mathrm{s}=2 \mathrm{f}$

$$
\begin{array}{ll}
\frac{1}{s}+\frac{l}{s^{\prime}}=-\frac{l}{f} & \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{R} \quad \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} \quad f= \\
\frac{1}{s^{\prime}}=-\frac{1}{f}-\frac{1}{s} & \text { Convex: } \mathrm{r}<0 ; \mathrm{f}<0 \\
s^{\prime}=\frac{-f s}{s+f} & m=-\frac{s^{\prime}}{s}=-\frac{(-2 f / 3)}{2 f}=+\frac{1}{3} \\
s^{\prime}=-\frac{f(2 f)}{2 f+f}=-\frac{2 f^{2}}{3 f}=-0.67 f &
\end{array}
$$

## Transparent Materials

## An Introduction to Lenses

## Basic Problem - Relate Image to Object



We want to derive a relationship between the image and the object using ray optics to derive the mathematical relationships.

We will then use a small angle approximation to simplify these equations.

## Magnification

$$
\tan \theta_{1}=\frac{y}{s} ; \tan \theta_{2}=\frac{-y^{\prime}}{s^{\prime}}
$$

Small angle approx $\tan \theta \cong \theta$
$\theta_{1}=\frac{y}{s} ; \quad \theta_{2}=\frac{-y^{\prime}}{s^{\prime}}$
Snell's Law $\quad n_{1} \theta_{1}=n_{2} \theta_{2}$

$n_{1}\left(\frac{y}{s}\right)=n_{2}\left(\frac{-y^{\prime}}{s^{\prime}}\right)$
$m=\frac{y^{\prime}}{y}=-\frac{n_{l} s^{\prime}}{n_{2} s}$

## Magnification

$$
\begin{aligned}
& n_{1}\left(\frac{y}{s}\right)=n_{2}\left(\frac{-y^{\prime}}{s^{\prime}}\right) \\
& m=\frac{y^{\prime}}{y}=-\frac{n_{l} s^{\prime}}{n_{2} s}
\end{aligned}
$$


$\mathrm{s}>0$ for an object on the incident side
s' $>0$ for an image on the refracted side
$\mathrm{R}>0$ if the center of curvature is on the refracred side of the glass.

## Magnification

$$
\frac{n_{1}}{s}+\frac{n_{2}}{s^{\prime}}=\frac{n_{2}-n_{1}}{R}
$$



For a single curved surface there isn't a simple relationship between the radius of curvature $R$ and the focal length $f$.

$$
\begin{gathered}
\mathrm{n}_{1}=1.00 \\
\mathrm{n}_{2}=1.33 \\
\mathrm{~s}=10 \mathrm{~cm} \\
\mathrm{R}=15 \mathrm{~cm} \\
\mathrm{~s}^{\prime}=? \\
\frac{n_{1}}{s}+\frac{n_{2}}{s^{\prime}}=\frac{n_{2}-n_{1}}{R} \\
\frac{1}{10}+\frac{1.33}{s^{\prime}}=\frac{1.33-1.00}{15}=\frac{0.33}{15} \\
\frac{1}{10}-\frac{0.33}{15}=-\frac{1.33}{s^{\prime}} \\
\frac{s^{\prime}}{1.33}=-\frac{150}{15-3.3}=-\frac{150}{11.7} \\
s^{\prime}=-\frac{150(1.33)}{11.7}=-17.1 \mathrm{~cm}
\end{gathered}
$$

## Example



The image is real, erect and magnified by $29 \%$

$$
\begin{gathered}
m=-\frac{n_{1} s^{\prime}}{n_{2} s}=\frac{(1)(-17.1)}{(1.33)(10)} \\
m=1.29
\end{gathered}
$$

Object

## Thin Lenses

## The Lens Maker's Equation



FIGURE 32-28 Refraction occurs at both surfaces of a lens. Here, the refraction at the first surface leads to a virtual image at $P_{1}^{\prime}$. The rays strike the second surface as if they came from $P_{1}^{\prime}$. Image distances are negative when the image is on the incident-light side of the surface, whereas object distances are positive for objects located on that side. Thus, $s_{2}\left(\approx-s_{1}^{\prime}\right)$ is the object distance for the second surface of the lens.

## For surface 1

$$
\frac{n_{a i r}}{s}+\frac{n}{s_{1}{ }^{\prime}}=\frac{n-n_{a i r}}{R_{1}}
$$

## For surface 2

$$
\frac{n}{-s_{1}^{\prime}}+\frac{n_{\text {air }}}{s^{\prime}}=\frac{n_{\text {air }}-n}{R_{2}}
$$

## Add these together

 and let $\mathrm{s}=>\infty$.$$
\frac{1}{f}=\left(\frac{n}{n_{\text {air }}}-1\right)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)
$$

## The Lens Maker's Equation

$$
\frac{1}{f}=\left(\frac{n}{n_{\text {air }}}-1\right)\left(\frac{1}{R_{1}}-\frac{1}{R_{2}}\right)
$$

The Thin Lens Equation

$$
\frac{l}{s}+\frac{l}{s^{\prime}}=\frac{l}{f}
$$

## Conventions for Thin Lenses

1. The parallel ray, drawn parallel to the axis. The emerging ray is directed toward (or away from) the second focal point of the lens.
2. The central ray, drawn through the center (the vertex) of the lens. This ray is undeflected. (The faces of the lens are parallel at this point, so the ray emerges in the same direction but displaced slightly. Since the lens is thin, the displacement is negligible.)
3. The focal ray, drawn through the first focal point. ${ }^{+}$This ray emerges parallel to the axis.

## Converging Thin Lens



This is also called a positive lens or a convex lens

## Object-Image Relationships - Convex Lens


parallel rays meet at a principal focus
object distance and image distance each equal to twice the focal length
object at principal focus, image at infinity
object inside principal focus, virtual image

## Thin Converging (Positive) Lens

Object distance outside the focal distance

(a)

There are two focal points. $F$ is the first focal point and $F^{\prime}$ is the second focal point. It is important to distinguish between these two points. It will prove essential for treating the negative lenses.

## Thin Converging (Positive) Lens

Object distance outside the focal distance

(a)

Ray $\mathbf{1}$ = Parallel ray: This ray approaches the lens parallel to the axis. It is deflected at the lens center line and passes through the second focus.

Ray $2=$ Center ray: This ray passes undeflected through the center of the lens.

Ray 3 = Focal ray: This ray passes through the first focal point, is deflected at the center line of the lens and travels parallel to the axis.

Object at 3 f

Object at 2 f

Object at 1.5 f

Object at 1.0f
(1)

(2)


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Object at $2 \mathrm{f} / 3$

As the object distance approaches zero so does the image distance. The magnification approaches +1 . The physical meaning of this arrangement is questionable.

Object at -f/2
(5)
(6)

(7)


## Diverging Thin Lens


(a)

(b)

(c)

This is also called a negative lens or a concave lens

## Thin Diverging (Negative) Lens

Object distance outside the focal distance


The first ( F ) and second ( F ') focal points are reversed relative to the converging lens.

## Thin Diverging (Negative) Lens

Object distance outside the focal distance


Ray 1 = Parallel ray: This ray approaches the lens parallel to the axis. It is deflected at the lens center line and follows a trajectory that can be traced back through the second focus.

Ray 2 = Center ray: This ray passes undeflected through the center of the lens.
Ray 3 = Focal ray: This ray approaches the lens on a line that would pass through the first focal point. It is deflected at the center line of the lens and travels parallel to the axis.

## Thin Diverging (Negative) Lens



## Two Convex Lens System



33-9 Summary. The results of this chapter are conveniently summarized in Table 33-1. Note that by letting $R=\infty$, the equation for a plane surface follows immediately from the appropriate equation for a curved surface.

Table 33-1

|  | Plane mirror | Curved mirror | Plane refracting surface | Curved refracting surface |
| :---: | :---: | :---: | :---: | :---: |
| Object and image distances | $\frac{1}{s}+\frac{1}{s^{\prime}}=0$ | $\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{2}{R}=\frac{1}{f}$ | $\frac{n}{s}+\frac{n^{\prime}}{s^{\prime}}=0$ | $\frac{n}{s}+\frac{n^{\prime}}{s^{\prime}}=\frac{n^{\prime}-n}{R}$ |
| Magnification | $m=-\frac{s^{\prime}}{s}=1$ | $m=-\frac{s^{\prime}}{s}$ | $m=-\frac{n s^{\prime}}{n^{\prime} s}=1$ | $m=-\frac{n s^{\prime}}{n^{\prime} s}$ |

## Optical Instruments

## Similar Optical Systems



## The Human Eye



## The Human Eye



Figure 2.1 The human eye

## The Human Eye



The cornea, aquueous humor and the lens all affect the diffraction of the light rays.

The combination of all three is considered to be the "effective" lens.

The average length of the eyeball, defined as the lens to retina distance, is 2.50 cm .

## Object at Infinity - Imaged at the Focal Point

Fixed Image Distance


$$
\begin{aligned}
& \frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} ; \quad \frac{1}{\infty}+\frac{1}{0.025}=\frac{1}{f}=40 \text { Diopters } \\
& f=2.5 \mathrm{~cm}
\end{aligned}
$$

This situation is described as the "relaxed" eye

## Diopter Unit of Measure

The lens equation deals with inverse length units of measure. The diopter unit $\left(\mathrm{m}^{-1}\right)$ takes advantage of this and makes it easier to deal with the lens equation.

The quantities measured in diopters are referred to as curvatures and they can be easily added and subtracted.

$$
\frac{l}{s}+\frac{l}{s^{\prime}}=\frac{l}{f}
$$

$$
\frac{l}{d_{o}}+\frac{l}{d_{i}}=\frac{l}{f}
$$

## Convex Lens - Object Approaching

As the object approaches from infinity the image grows in size and moves further back from the focal point
(1)

(2)

(3)
(4)


## The Near Point $-\mathrm{X}_{\mathrm{np}}$

The near point $\mathrm{X}_{\mathrm{np}}$ is defined as the smallest object distance for which the human eye can focus an image on the retina.


$$
\frac{1}{s}+\frac{1}{s^{\prime}}=\frac{1}{f} ; \quad \frac{1}{0.25}+\frac{1}{0.025}=\frac{1}{f}=44 \text { Diopters }
$$

The near point is taken to be 25 cm .

## The Range of the Human Eye



The range of the human eye is from 40D to 44D.

Object at Infinity
Power $=40 \mathrm{D}$

Object at $X_{\text {np }}$
Power $=44 \mathrm{D}$

## Lens Combinations

For two thin lens that are in contact or spaced very close the inverse focal lengths can be added.

Since the inverse focal lengths are called powers then behavior means that the power are additive.

$$
\begin{aligned}
& \frac{l}{f_{1}}+\frac{l}{f_{2}}=\frac{l}{f_{e q}} \\
& f_{e q}=\frac{f_{1} f_{2}}{f_{1}+f_{2}}
\end{aligned}
$$

## The Hyperopic Eye - Far Sighted



As the object moves farther away the focus point moves in.
"It is easier to see far away objects"
The lens is too weak - the power needs to be increased.
The correction needs to be a positive lens

## The Hyperopic Eye

## Corrected \& Uncorrected

(a)

(b)


A person with hyperopia is far sighted. In other words, it is easier for them to focus on objects that are far away.

## The Myopic Eye - Nearsighted



As the object moves in closer the focus point moves back
"It is easier to see near objects"
The lens is too strong - the power needs to be reduced.
The correction needs to be a negative lens.

## The Myopic Eye

## Corrected \& Uncorrected



A person with myopia is near sighted. In other words, it is easier for them to focus on objects that are close by.

## The Myopic Eye - Nearsighted



Fig. 1 Comparison of a normal, myopic and corrected myopic eye. The far point and near point represent the limits of clear vision.

## Eye Correction with Reading Glasses



This person's uncorrected near point is 75 cm . The use of reading glasses allows the person to "move" an object at 25 cm out to 75 cm .

## The Non-Spherical Lens - Astigmatism


(a) Uncorrected astigmatism

(b) Test for astigmatism

Due to different radii of curvature in the horizontal and vertical planes, the astigmatic eye focuses horizontal and vertical images in different planes

(c) Corrected by lens

## Apparent Size

Apparent size is greater when the object is closer to the eye.


Image size proportional to $\theta$

(b)

$$
\varphi \approx \frac{y^{\prime}}{2.5 \mathrm{~cm}} \quad \theta \approx \frac{y}{s}
$$

Snell's Law $\quad n_{\text {air }} \sin \theta=n \sin \varphi$

$$
\theta \approx n \varphi
$$

$$
\frac{y}{s}=n \varphi=\frac{n y^{\prime}}{2.5 \mathrm{~cm}} \quad y^{\prime}=\frac{2.5 \mathrm{~cm}}{n} \frac{y}{s} \quad m=\frac{-y^{\prime}}{y}=\frac{-2.5 \mathrm{~cm}}{n} \frac{1}{s}
$$

## Apparent Size


(a)

Coompare with the general formula

$$
m=-\frac{n_{1}}{n_{2}} \frac{s^{\prime}}{s}
$$

(b)

This formula has been specialized for this particlar situation by placing the number 2.5 in it. The units have also been fixed by the choice of cm for the image distance.

## Angular Magnification

Goal: Push the image to infinity to yield parallel rays and easy viewing with the eye.

(a) The lens allows the object to be moved in closer.

$$
\theta_{0}=\frac{y}{x_{n p}} ; \quad \theta=\frac{y}{f}
$$

Angular Magnification = Magnifying Power $=\mathrm{M}$

$$
M=\frac{\theta}{\theta_{0}}=\frac{x_{n p}}{f}
$$

Simple magnifiers are used as

> eyepieces (oculars) on microscopes and telescopes

Problems: (1) Ignores the convention that the object ray always starts from the left. (2) The angle $\theta_{\mathrm{i}}$ is not unique. (3) The distance D isn't neeeded.

## A Simple Magnifying Glass



## Compound Microscope

Goal: Make a small object appear larger while viewing it with a relaxed eye.


Note: (1) The object is just outside the Objective lens focal length. (2) The focal lengths of the two lenses do not overlap. (3) The Eyepiece is placed so that the image from the Objective lens fall just inside its (the eyepiece) focal length.

## Compound Microscope



The object is just outside the Objective lens focal length to maximize the size of the image.

The Eyepiece is placed so that the image from the Objective lens falls just inside its (the eyepiece) focal length. This will give a virtual image at infinity. The rays from this image will be parallel and provide easy (relaxed lens) viewing by the eye.

## Compound Microscope

$$
\begin{array}{ll}
\tan \beta=\frac{y}{f_{o}}=\frac{-y^{\prime}}{L} & \begin{array}{l}
\text { Objective } \\
m_{o}=\frac{y^{\prime}}{y}=\frac{-L}{f_{0}} \\
m_{e}=\frac{x_{n p}}{f_{e}} \\
M=m_{0} m_{e}=\frac{-L x_{n p}}{f_{0} f_{e}}
\end{array} \begin{array}{l}
\text { Compound microscope requirement: } \\
\begin{array}{l}
\text { The image of the objective lens needs } \\
\text { to be just inside the focal point of the } \\
\text { eyepiece }
\end{array}
\end{array}
\end{array}
$$

## The Telescope

The goal of the objective lens is not magnification but to place an image inside the focal length of the eyepiece.

Note: (1) The object is at infinity (the rays enter from the left and are parallel.
(2) The distance between the lenses is just about the sum of the focal lengths.
(3) The purpose of the eyepiece is again to place the final image at a convenient viewing distance.


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## The Diagram for Calculating the Magnification of the Telescope



With the telescope it is the angular magnification that is important. If you are looking at the moon it is obvious that you haven't magnified the size of the moon. What has happened is that the angular that the image subtends on the back of your eye has increased - the moon appears bigger.

## Magnification of the Telescope



Object at infinity images at focal point. This point coincides with the focal point of the eyepiece. The image from the objective becomes the object of the eyepiece and is imaged out to infinty.

## Telescopes


(a) Galilean telescope

(b) Terrestrial telescope

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## The Evolution of the Telescope

Once it was apparent that the telescopes allowed one to see other far away worlds, people wanted MORE.

This meant more magnification and they wanted to see things still further away.

The first telescopes were hand held and most of those used for navigation remained so. (Check the Pirates of the Caribbean.)

But for astronomy there seems to be no end in the quest for MORE.

## Evolution of the Astronomical Telescope

Larger meant an increase the magnification

$$
M=-f_{o} / f_{e}
$$

This meant an increase in $\mathrm{f}_{\mathrm{o}}$ and longer telescopes
More distant objects meant larger diameters and bigger objective lenses.

## Refractor Telescope



## Longer and Bigger also Meant Heavier

Reflector scope from the 1780's used by Friedrich Wilhelm Herschel


## The Reflecting Telescope Eliminates the Need for Large Lenses

Folding the optical pathway to shorten the length.

No lens requirements.
Requirements moved to the objective mirror.


## Different Viewing Options


(a)

(b)

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## Cross Section of a Charge Coupled Device



This device contributes one pixel to the final picture.

An array of millions of these devices forms the actual CCD device.

## Charged Coupled Device Array



Figure 2

## Quantum Efficiency

Quantum efficiency (QE) is a quantity defined for a photosensitive device such as photographic film or a charge-coupled device (CCD) as the percentage of photons hitting the photoreactive surface that will produce an electron-hole pair.

Photographic film typically has a QE of much less than $10 \%$, while CCDs can have a QE of well over $90 \%$ at some wavelengths.

## Quantum Efficiency



## Telescope - CCD Camera System System Diagram



## Hubble Image of M42, the Orion Nebula



## No More Long Cold Nights in the Telescope



Edwin Hubble in the observer's seat in the 5.08 m Hale reflecting scope.

## Appendix

## Sample Problem

Example 1. One end of a cylindrical glass rod (Fig. 33-19) is ground to a hemispherical surface of radius $R=20 \mathrm{~mm}$. Find the image distance of a point object on the axis of the rod, 80 mm to the left of the vertex. The rod is in air.

$$
\begin{gathered}
n=1, \quad n^{\prime}=1.5, \quad R=+20 \mathrm{~mm}, \quad s=+80 \mathrm{~mm} \\
\frac{1}{80 \mathrm{~mm}}+\frac{1.5}{s^{\prime}}=\frac{1.5-1}{+20 \mathrm{~mm}} \\
s^{\prime}=+120 \mathrm{~mm} .
\end{gathered}
$$

The image is therefore formed at the right of the vertex ( $s^{\prime}$ is positive) and at a distance of 120 mm from it. Suppose that the object is an arrow 1 mm high, perpendicular to the axis. Then

$$
n=1, \quad n^{\prime}=1.5, \quad s=+80 \mathrm{~mm}, \quad s^{\prime}=+120 \mathrm{~mm}
$$

Hence

$$
m=-\frac{n s^{\prime}}{n^{\prime} s}=-\frac{1 \times 120 \mathrm{~mm}}{1.5 \times 80 \mathrm{~mm}}=-1
$$

That is, the image is the same height as the object, but is inverted.


Figure 33-19


Figure 33-20

