# Chapter 35 \& 36 Physical Optics 

## Physical Optics

- Phase Difference \& Coherence
- Thin Film Interference
- 2-Slit Interference
- Single Slit Interference
- Diffraction Patterns
- Diffraction Grating
- Diffraction \& Resolution
- X-Ray Diffraction


## Periodic Waves

A periodic wave repeats the same pattern over and over.

- For periodic waves: $\mathrm{v}=\lambda \mathrm{f}$
- v is the wave's speed
- f is the wave's frequency
- $\lambda$ is the wave's wavelength

The period T is measured by the amount of time it takes for a point on the wave to go through one complete cycle of oscillations. The frequency is then $\mathrm{f}=1 / \mathrm{T}$.

## Periodic Waves

One way to determine the wavelength is by measuring the distance between two consecutive crests.

The maximum displacement from equilibrium is amplitude (A) of a wave.


## Phase Difference



When two waves travel different distances to reach the same point, the phase difference is determined by:

$$
\frac{d_{1}-d_{2}}{\lambda}=\frac{\text { phase difference }}{2 \pi}
$$

Note: This is a ratio comparison. $\lambda$ is not equal to $2 \pi$

## The Principle of Superposition

For small amplitudes, waves will pass through each other and emerge unchanged.

> Superposition Principle: When two or more waves overlap, the net disturbance at any point is the sum of the individual disturbances due to each wave.




# Two traveling wave pulses: left pulse travels right; right pulse travels left. 

## Interference

Two waves are considered coherent if they have the same frequency and maintain a fixed phase relationship.


Two coherent waves. The black wave is $\pi / 4$ radians behind the orange wave.

Two waves are considered incoherent if the phase relationship between them varies randomly.

When waves are in phase, their superposition gives constructive interference.

When waves are one-half a cycle out of phase, their superposition gives destructive interference.

This is referred to as:<br>"exactly out of phase" or " $180^{\circ}$ out of phase."


(a) Overlap begins

## Constructive Interference

(Slinky Example)

(b) Total overlap; the Slinky has twice the height of either pulse

(c) The receding pulses

(a) Overlap begins

# Destructive Interference (Slinky Example) 



(c) The receding pulses

Constructive Interference. Means that the waves ADD together and their amplitudes are in the same direction

Destructive Interference. Means that the waves ADD together and their amplitudes are in the opposite directions.

Interference $=$ Bad choice of words

- The two waves do not interfere with each other.
- They do not interact with each other.
- No energy or momentum is exchanged.


## Special Conditions for a Steady Optical Interference Pattern

- Same wavelength and frequency
- Phase difference independent of time
- Amplitudes approximately equal
- Same polarization

These conditions are most easily achieved by deriving the interfering light from the same source.

## Interference Patterns

Light from different sources can never yield a stationary interference pattern

Different sources are inherently incoherent
Successful observation of a stationary interference pattern.
$\Delta$ Optical Path length is small $\sim$ a few wavelengths

## Reflection and Refraction

At an abrupt boundary between two media, a
reflection will occur. A portion of the incident
wave will be reflected backward from the boundary.
A portion of the incident wave will be transmitted through the media. This is the refracted ray.

## Reflection and Refraction



When a wave is incident on the boundary between two different media, a portion of the wave is reflected, and a portion will be transmitted into the second medium. Reflected ray is $180^{\circ}$ out of phase with respect to the incident wave.

## The Frequency is Constant

The frequency of the transmitted wave remains the same. However, both the wave's speed and wavelength are changed such that:

$$
f=\frac{v_{1}}{\lambda_{1}}=\frac{v_{2}}{\lambda_{2}}
$$

The transmitted wave will also suffer a change in propagation direction (refraction). Described by Snell's Law.

## The Reflected Wave \& Phase Change

When you have a wave that travels from a "low density" medium to a "high density" medium, the reflected wave pulse will be inverted. ( $180^{\circ}$ phase shift.)

The frequency of the reflected wave remains the same.


## Interference Due to Phase Differences

## Phase Differences Result from

- Optical Path Differences
- Phase Shifting

The optical path length in a material of index of refraction n is: $d^{\prime}=d / n$

$$
\Delta(\text { Phase })=\Delta(\text { Optical Path Length })+\Delta(\text { Phase Shift })
$$

Constructive Interference: $\quad \Delta($ Phase $)=\mathrm{n} \lambda ; \mathrm{n}=0,1,2, \ldots$
Destructive Interference: $\quad \Delta($ Phase $)=\mathrm{m} \lambda / 2 ; \mathrm{m}=1,3,5, \ldots$

## Phase Shifting Upon Reflection

Ray 1 is phase shifted by $\lambda / 2$ at the air-film interface

Ray 2 is not phase shifted at the film-air interface

Extra Conditions:

- Monochromatic light
- Nearly normal incidence

$$
\begin{aligned}
& n_{\text {air }}=1.00 \\
& n_{\text {soap }}=1.33
\end{aligned}
$$

Incident

## Constructive Interference

Ray 1 is phase shifted by $\lambda / 2$ at the air-film interface

The difference in the optical path length is the distance the 2 nd ray travels in the soap film.

The film distance is 2 t .
The optical path length is $2 \mathrm{t} / \mathrm{n}_{\text {soap }}$
Looking for constructive interference.
This requires that the phase difference of 1 and 2 are an integral number of wavelengths.

Ray 2 is not phase shifted at the film-air interface


## Constructive Interference

## $\Delta($ Phase $)=\Delta($ Optical Path Length $)+\Delta($ Phase Shift $)$

Constructive interference requires that the phase difference of 1 and 2 be equal to an integral number of wavelengths.

Constructive interference requires a minimum phase difference equivalent to one wavelength $=\lambda$

Phase shifting contributes $\lambda / 2$ to the total phase difference. The optical path length difference needs to contribute another $\lambda / 2$.

The film distance is 2 t . The optical path length difference $\lambda^{\prime} / 2=2 \mathrm{t}$

$$
\frac{\lambda}{2 n_{\text {soap }}}=2 t \Rightarrow t=\frac{\lambda}{4 n_{\text {soap }}}
$$



## Destructive Interference

## $\Delta($ Phase $)=\Delta($ Optical Path Length $)+\Delta($ Phase Shift $)$

Destructive interference requires that the phase difference of 1 and 2 be equal to an odd multiple of half wavelengths.

Denstructive interference requires a minimum phase difference equivalent to one-half wavelength $=\lambda / 2$

Phase shifting contributes $\lambda / 2$ to the total phase difference. The optical path length difference needs to contribute $\lambda$.

The film distance is 2 t . The optical path length difference $\lambda^{\prime}=2 \mathrm{t}$

$$
\frac{\lambda}{n_{\text {soap }}}=2 t \Rightarrow t=\frac{\lambda}{2 n_{\text {soap }}}
$$



## Destructive Interference

## $\Delta($ Phase $)=\Delta($ Optical Path Length $)+\Delta($ Phase Shift $)$

Ray 1 is phase shifted by $\lambda / 2$ at the air-film interface

Ray 2 is also phase shifted by $\lambda / 2$ at the filmglass interface


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## Destructive Interference

## $\Delta($ Phase $)=\Delta($ Optical Path Length $)+\Delta($ Phase Shift $)$

Destructive interference requires that the phase difference of 1 and 2 be equal to a half-integral number of wavelengths.

Destructive interference requires a minimum phase difference equivalent to one wavelength $=\lambda / 2$

Phase shifting contributes 0 to the total phase difference. The optical path length difference needs to contribute the $\lambda / 2$.

The film distance is 2 t . The optical path length difference $\lambda^{\prime} / 2=2 \mathrm{t}$

$$
\frac{\lambda}{2 n_{f i l m}}=2 t \Rightarrow t=\frac{\lambda}{4 n_{\text {film }}}
$$



## Interference Summary

Destructive Interference - let more light in
Example - Camera Lens (anti-reflective).

Constructive Interference - "Reflects" more light
Example - eye glasses.

## Newton's Rings

Illuminated from above by monochromatic light

(a)


Dark rings - destructive interference
Bright rings - constructive interference

## Newton’s Rings



Center spot is dark because difference in optical path length is zero


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## Newton's Rings



Ray 1 is NOT phase shifted at the glassair interface

Ray 2 is phase shifted by $\lambda / 2$ at the airglass interface

Dark rings - destructive interference
Bright rings - constructive interference

For the first rings

$$
\begin{aligned}
& \text { Dark }=>2 t=\lambda \\
& \text { Bright }=>2 t=\lambda / 2
\end{aligned}
$$

## Newton's Rings



As one moves outward from the center from one dark ring to the next the path difference increases by the same amount $\lambda$, a corresponding increase occurs in the thickness of the air layer $\lambda / 2$. The bottom surface of the lens is curved and the slope of the lens surface increases. Therefore the separation of the rings gets smaller for the outer rings. The radius of the Nth bright ring is given by

$$
r_{N}=\left[\left(N-\frac{1}{2}\right) \lambda R\right]^{1 / 2}
$$

## Measurement Using Interference

Normal incidence of monochromatic light light.

(a)

## What happens on either side of the air wedge is what counts

Ray 1 is not phase shifted at the glass-air interface

Ray 2 is phase shifted at the air-glass interface

(b)

## First Bright and Dark Lines

## 1st Bright Line

Phase shift already contributes $\lambda / 2$. Path length delta needs to contribute another $\lambda / 2$.

$$
2 t=\frac{\lambda}{2} ; \quad t=\frac{\lambda}{4}
$$


(b)

## 1st Dark Line

Phase shift already contributes $\lambda / 2$. Path length delta needs to contribute $\lambda$.

$$
2 t=\lambda ; \quad t=\frac{\lambda}{2}
$$

What happens on either side of the air wedge is what counts

Ray 1 is not phase shifted at the top glass-air interface Ray 2 is phase shifted at the bottom air-glass interface

$$
\begin{array}{lll}
2 t=n \lambda=\lambda & \text { Small triangle } & \sin \theta=\frac{t}{s}=\frac{\lambda}{2 s} \\
t=\lambda / 2 & & \\
& \text { Large triangle } & \sin \theta=\frac{T}{L}
\end{array}
$$




| Small triangle | $\sin \theta=\frac{t}{s}=\frac{\lambda}{2 s}$ | $\frac{\lambda}{2 s}=\frac{T}{L}$ |
| :--- | :--- | :--- |
| Large triangle | $\sin \theta=\frac{T}{L}$ | $\frac{2}{\lambda} T=\frac{L}{s}=N_{D}=$ \#Dark lines |
|  | $T=\frac{\lambda}{2} N_{D}$ |  |

## Young's Experiment

## Double Slit Interference

## Double Slit Interference Pattern



## Double Slit Interference Pattern



The intensity equation implies that the bright peaks should all be of equal amplitudes. The decrease in intensity is due to the single slit diffraction pattern of each slit.

$$
I=4 I_{0} \cos ^{2} \frac{1}{2} \delta
$$

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INTENSITY IN TERMS OF PHASE DIFFERENCE

## Double Slit Calculation



## Double Slit Calculation



## Both slits need to be open to observe the interference pattern



## Diffraction

Diffraction is the spreading of a wave around an obstacle in its path and it is common to all types of waves.

The size of the obstacle must be similar to the wavelength of the wave for the diffraction to be observed.

Larger by 10 x is too big and smaller by $(1 / 10) \mathrm{x}$ is too small.

## Poisson Spot - Arago Spot




## Arago Spot

An Arago spot is a bright point that appears at the center of the shadow of a circular object in light from a point source. The spot occurs within the geometrical shadow and no particle theory of light could account for it. Its discovery provided strong evidence for the wave nature of light,


In 1818, Siméon Poisson deduced from Augustin Fresnel's theory, the necessity of a bright spot at the centre of the shadow of a circular opaque obstacle. With his counterintuitive result Poisson hoped to disprove the wave theory.

Much to Poisson's embarrassment Dominique Arago experimentally verifed the prediction. Dominique Arago supported Jean-Augustin Fresnel's optical theories, and his observation of what is now known as the spot of Arago helped to confirm Fresnel's wave theory of light.

## Single Slit Diffraction Pattern

## Single Slit Interference Pattern



## Single Slit - Minimum (Dark)

Imagine a number of equally separated points on the wavefront in the single slit opening. Each point is a source of a spherical wavelet. The angle is chosen such that the difference in pathlength from each wavelet source is as shown in the diagram. The waves from points a half slit width apart are $180^{\circ}$ out of phase and cancel.

Moving to the next pair, a half slit width apart, they also cancel and so on...

The result is a minimum at this special angle


## Single Slit - Bright Line



Imagine 5 equally separated points on the wavefront in the single slit opening. Each point is a source of a spherical wavelet. The angle is chosen such that the difference in path length from each wavelet source is as shown in the diagram.

Following the same argument as in the case of the minimum, these waves reinforce in pairs and result in a bright spot on the screen.

## Single Slit Interference Pattern



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$$
\begin{aligned}
& \tan \theta_{m}=\frac{y_{m}}{L} \\
& \theta_{m}=\tan ^{-1}\left[\frac{y_{m}}{L}\right]
\end{aligned}
$$



Location of the minima

$$
\begin{aligned}
& \frac{w}{2} \sin \theta_{m}=m \frac{\lambda}{2} \\
& w \sin \theta_{m}=m \lambda ; \quad m=1,2, \ldots
\end{aligned}
$$

## Single Slit Modulates the Double Slit Pattern



## Interference \& Diffraction Demos

Double-slit Interference $\triangle$

http://www.pas.rochester.edu/~ksmcf/p100/java/Optics/Diffraction.html

## Interference \& Diffraction Demos


http://www.austincc.edu/mmcgraw/physics_simulations.htm

## Diffraction Gratings

## Grating Information

Lab gratings: 100, 300 and 600 lines $/ \mathrm{mm}$
( $0.100,0.300$ and .600 lines $/ \mu \mathrm{m}$ )
These correspond to slit spacings of :

$$
10.0,3.33 \text { and } 1.67 \mu \mathrm{~m}
$$

Visible light spectrum
$400 \mathrm{~nm}-700 \mathrm{~nm}$
$0.400 \mu \mathrm{~m}-0.700 \mu \mathrm{~m}$
$\square$
Human Hair
Diam ~ 75-100 $\mu \mathrm{m}$

## Transmission Diffraction Grating



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## Diffraction Grating

Location of diffraction maxima


$$
d \sin \theta_{m}=m \lambda ; \quad m=0,1,2, \ldots
$$

At the location of a diffraction maximum the differences in the path lengths from the adjacent slits are equal to a wavelength

$$
\begin{aligned}
& \tan \theta_{m}=\frac{y_{m}}{L} \\
& \theta_{m}=\tan ^{-1}\left[\frac{y_{m}}{L}\right]
\end{aligned}
$$

## Diffraction Grating - Line Width



## Optical Spectroscopy

Transmission diffraction grating illuminated with white light


In the lab we used a HeNe laser of wavelength 632.8 nm so we did not observed this rainbow effect.

The observational technique displayed above is not recommended for use with a laser source

## Diffraction Grating Resolving Power

The resolving power ( $R$ ) of a grating is a measure of its ability to separate the light of two very close wavelengths


$$
\begin{aligned}
& R \equiv \frac{\lambda}{|\Delta \lambda|}=m N \\
& N=\frac{\lambda}{m|\Delta \lambda|}
\end{aligned}
$$

R is defined as the average of the two wavelengths divided by the absolute value of the difference in the two wavelengths.

The second equation, a rearrangement of the first, gives the number of slits needed for a given $\mathrm{m}, \lambda$, and $\Delta \lambda$.

## Diffraction Limited Optics

## Optical Limits Due to Diffraction

Question: How close together can two objects be located and still be resolved by an observer as two distinct objects?

An image formed on the retina, on photographic film (or CCD) or viewed through a microscope or telescope passes through at least one circular aperture.

This aperature acts as a single "slit" and causes light from an object to spread out in a single "slit" diffraction pattern.

## Diffraction Limited Optics



As the two point sources move closer together, the angle $\alpha$ decreases and the overlap of the diffraction patterns increases.

## Limits of Resolution - Rayleigh's Criterion



Question: How much overlap is possible in the diffraction patterns if the observer is to still be able to resolve the patterns as due to two distinct objects?

Rayleigh's Criterion - The limit for resolving two objects as being distinct occurs when the maximum of one diffraction pattern is at the location of the first minimum of the other diffraction pattern.

## Limits of Resolution - Rayleigh's Criterion



This is similar to the result for the first minimum of the single slit diffraction pattern: $\sin \theta=\lambda / \mathrm{a}$. The 1.22 factor is due to the difference between a linear slit and a circular aperture.

## Limits of Resolution - Rayleigh's Criterion

The illustration below is for 2 objects separated by the Rayleigh criterion.



The illustration above is for objects separated by $0.8,1.0$ and 1.2 times the Rayleigh criterion.

## X-Ray Scattering

## X-Ray Crystal Scattering Geometry


(a)
"Diffraction occurs when each object in a periodic array scatters radiation coherently, producing concerted constructive interference at specific angles."

Basics of X-Ray Diffraction, Scott A Speakman, Ph.D., http://prism.mit.edu/xray/BasicsofXRD.ppt

## Bragg's Law



## $2 d \sin \Theta=n \lambda$

"Diffraction occurs when each object in a periodic array scatters radiation coherently, producing concerted constructive interference at specific angles."

## X-Ray Scattering Geometry


$\left(F_{o} e^{i \vec{k} \cdot \vec{\rho}}\right)\left(\frac{e^{i k r}}{r}\right)$
The incident beam amplitude and phase is represented by the first factor. The second factor represents the the spatial variation of the radiation scattered from the atom at $\rho$.

## X-Ray Scattering Wave Vectors


ddsfgddg

$$
\begin{aligned}
& \Delta \vec{k}=\vec{k}^{\prime}-\vec{k} \\
& |\Delta \vec{k}|=2 k \sin \Theta=(4 \pi / \lambda) \sin \Theta
\end{aligned}
$$

## Ewald Sphere



The discrete points represent the lattice points of the reciprocal lattice of the crystal. The Bragg condition is shown here as a requirement that the difference of the incident and scattered wavevectors be a reciprocal lattice vector.

## X-Ray Diffraction Pattern - Positive


"Every point in the reciprocal lattice corresponds to a possible reflection from the crystal lattice." X-ray diffraction experiments provide the reciprocal lattice points and these allow the reconstruction of the crystal lattice.

## X-Ray Powder Diffraction Camera



